A PARAMETER SPACE APPROACH FOR STATE SPACE INDUCTION MACHINE MODELLING AND ROBUST CONTROL

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Resumo: Este artigo trata sobre o problema de modelagem da máquina de indução através da representação das não-estacionariedades e das não-linearidades como incertezas do sistema. O modelo resultante em espaço de estado é utilizado em um novo esquema de controle digital de velocidade por orientação do fluxo de rotor. A solução do problema de controle é obtida pela síntese de um ganho de realimentação de estado que é robusto às variações da planta.

ABSTRACT: This paper deals with the problem of the induction machine modelling by representation of non-stationarity and of non-linearity as system uncertainties. The resulting state space model is applied in a new scheme of robust digital speed control by orientation of the rotor flux. The solution of the control problem is performed by the synthesis of a state feedback gain which is robust to such plant variation.

1 INTRODUCTION

The development of microprocessors and their applications in systems engineering has caused quick technical advances that guarantee some performance improvement in control systems. In the case of motor drive systems, it has allowed the employment of sophisticated methods of analysis and control of the induction machine (Krause, 1986; Novotny and Lipo, 1995).

A very commonly accepted method is the vector control of the induction machine (Hasse, 1969; Blaschke,1972). The three-phase electrical variables for rotor flux and stator current are projected on two orthogonal axes, represented by *d* and *q*. The resulting decoupled components are related to the magnetization and torque production of the motor, respectively. Setting the *q*-component of the rotor flux to zero, the torque response can be controlled independently of the magnetization dynamics.

Many proposals have been presented in the literature for the implementation of vector control by the application of control systems theory. Methodologies employing the classic optimal control (Murata *et alii*, 1990) and frequency-domain optimal control (Kao and Liu, 1992) have been used in this sense. However, the complexity of the induction machine models, that in general are nonlinear and non-stationary, requires that these methods be applied to linearized models, which are obtained for an unique machine operating point.

In order to provide the machine to work in a wide speed range, there are several models whose parameters are updated every sampling period. These approaches require the controllers being computed in real time (Akagi and Nabae, 1986) and this computational efforts frequently demands the use of digital signal processors (DSP) (Vainio *et alii*, 1992). An alternative solution for this problem (Ben-Brahim and Kawamura, 1992) consists in the off-line computation of the system matrices and in the storage of these values in a look-up table.

In this work, we propose a new methodology for the representation of non-stationarity and non-linearity of the induction machine state space discrete-time models. The basic fourth order model (Bottura *et alii*, 1993) is composed by electrical equations for stator current and rotor flux variables, represented in the *dq* synchronously rotating reference frame. Then, a speed equation is added to this model. Once we consider wide changes in the machine operation, the electrical equations become non-stationary while the motion equation is naturally nonlinear. Besides, all the equations depend on changes in the machine parameters mainly because of temperature variation.

The non-stationarity and non-linearity characteristics of the model described above are dealt as systems uncertainties. The specification of limit values for the stator and slip frequencies, as well as for the *dq* components of the rotor flux, defines a set of the machine operating points in the parameter space. The convex combination of such points results in a polytope that

Artigo Submetido em 16/07/99
1a. Revisão em 28/10/99; 2a. Revisão em 20/03/2000
Aceito sob recomendação do Ed. Consultor Prof. Dr. Denizar Cruz Martins

contains all possible trajectories of the system parameters either in steady state or in transient conditions.

The uncertainty representation via convex polyhedral sets leads to an uncertain linear model which is very suitable for the application of robust control techniques. The primary objective of providing the systems quadratic stability all over the parametric domain is guaranteed by the verification of the asymptotic stability of the nominal system at the vertices of the polyhedral sets (Horisberger and Belanger, 1976). Besides that, the convex nature of the system representation guarantees the synthesis of globally optimal controllers (Geromel *et alii*, 1991).

Based on the above concepts, we also propose an optimal robust controller for the field oriented speed regulation of the induction machine. This controller is constituted by a state feedback gain, computed off-line, that minimizes an upper bound for the H_2 norm of the transfer function from the disturbance input to the output of the system (Geromel *et alii*, 1993). The controller synthesis is done in the basis of the so called *full information* problem (Zhou *et alii*, 1996; Colaneri *et alii*, 1997), so the state vector, and consequently the rotor flux, is considered known.

The validation of the proposal is verified through some simulation results.

2 THE INDUCTION MACHINE MODEL

Consider the non-stationary discrete time model of the induction machine in *dq* synchronously rotating reference frame, proposed in Bottura *et alii* (1993):

$$\begin{bmatrix} i_s(k+1) \\ \lambda_r(k+1) \end{bmatrix} = A \begin{bmatrix} i_s(k) \\ \lambda_r(k) \end{bmatrix} + \begin{bmatrix} aB_1^d \\ 0 \end{bmatrix} V_s(k) \tag{1}$$

where:

$$A = \begin{bmatrix} A_1^d - aR_s B_1^d + (1-\frac{1}{\sigma})\frac{B_2^d}{T_r} & c(A_1^d - A_2^d + \frac{B_2^d}{T_r}) \\ \frac{M}{T_r} B_2^d & A_2^d - \frac{B_2^d}{T_r} \end{bmatrix}$$

The vectors $i_s = \begin{bmatrix} i_{qs} & i_{ds} \end{bmatrix}^T$, $\lambda_r = \begin{bmatrix} \lambda_{qr} & \lambda_{dr} \end{bmatrix}^T$ and $V_s = \begin{bmatrix} V_{qs} & V_{ds} \end{bmatrix}^T$ are the stator current, rotor flux and stator voltage, respectively, and

$$A_1^d = \begin{bmatrix} \cos(\omega h) & -sin(\omega h) \\ sin(\omega h) & \cos(\omega h) \end{bmatrix}$$

$$A_2^d = \begin{bmatrix} \cos(\omega_s h) & -sin(\omega_s h) \\ sin(\omega_s h) & \cos(\omega_s h) \end{bmatrix}$$

$$B_1^d = \frac{1}{\omega} \begin{bmatrix} sin(\omega h) & -(1-\cos(\omega h)) \\ 1-\cos(\omega h) & sin(\omega h) \end{bmatrix}$$

$$B_2^d = \frac{1}{\omega_s} \begin{bmatrix} sin(\omega_s h) & -(1-\cos(\omega_s h)) \\ 1-\cos(\omega_s h) & sin(\omega_s h) \end{bmatrix}$$

where ω, ω_s are the stator and slip frequencies, R_s, L_s, R_r and L_r are the stator and rotor resistances and inductances, M is the mutual inductance and h is the sampling period.

The constants are:

$$a = \frac{1}{\sigma L_s}, \qquad b = \frac{1}{\sigma L_r}, \qquad c = \frac{M}{L_s L_r - M^2}$$

$$\sigma = 1 - \frac{M^2}{L_s L_r} \qquad T_r = \frac{L_r}{R_r}$$

This model is completed with the torque and motion equations:

$$T_e(k) = \frac{3P}{2}\frac{M}{L_r}\left(\lambda_{dr} i_{qs} - \lambda_{qr} i_{ds}\right) \tag{2}$$

$$\omega_r(k+1) = e^{hC_2}\omega_r(k) - \frac{1}{C_1}\left(e^{hC_2} - 1\right)\left(T_e(k) - T_L\right) \tag{3}$$

where ω_r is the rotor frequency, P is the number of pairs of poles, C_1 is the viscous friction, T_L is the load torque, J_m is the system inertia and $C_2 = -PC_1/J_m$.

In order to derive a fifth-order state space representation with

$$X = \begin{bmatrix} i_{qs} & i_{ds} & \lambda_{qr} & \lambda_{dr} & \omega_r \end{bmatrix}^T$$

as the state vector, let us substitute (2) in (3) and rewrite (1) to get:

$$\begin{bmatrix} i_{qs}(k+1) \\ i_{ds}(k+1) \\ \lambda_{qr}(k+1) \\ \lambda_{dr}(k+1) \\ \omega_r(k+1) \end{bmatrix} = \begin{bmatrix} \varphi_1 & -\varphi_2 & \varphi_3 & -\varphi_4 & 0 \\ \varphi_2 & \varphi_1 & \varphi_4 & \varphi_3 & 0 \\ \varphi_5 & -\varphi_6 & \varphi_7 & -\varphi_8 & 0 \\ \varphi_6 & \varphi_5 & \varphi_8 & \varphi_7 & 0 \\ -\varphi_9 & \varphi_{10} & 0 & 0 & \varphi_{11} \end{bmatrix} \begin{bmatrix} i_{qs}(k) \\ i_{ds}(k) \\ \lambda_{qr}(k) \\ \lambda_{dr}(k) \\ \omega_r(k) \end{bmatrix}$$

$$+ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & s_1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ T_L \end{bmatrix} + \begin{bmatrix} \gamma_1 & -\gamma_2 \\ \gamma_2 & \gamma_1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{qs} \\ V_{ds} \end{bmatrix} \tag{4}$$

where:

$$\varphi_1 = \cos(\omega h) - \frac{aR_s}{\omega} sin(\omega h) + \left(1 - \frac{1}{\sigma}\right)\frac{1}{\omega_s T_r} sin(\omega_s h)$$

$$\varphi_2 = sin(\omega h) - \frac{aR_s}{\omega}\left(1 - \cos(\omega h)\right) + \left(1 - \frac{1}{\sigma}\right)\frac{1}{\omega_s T_r}\left(1 - \cos(\omega_s h)\right)$$

$$\varphi_3 = c\left(\cos(\omega h) - \cos(\omega_s h) + \frac{1}{\omega_s T_r} sin(\omega_s h)\right)$$

$$\varphi_4 = c\left(sin(\omega h) - sin(\omega_s h) + \frac{1}{\omega_s T_r}(1 - \cos(\omega_s h))\right)$$

$$\varphi_5 = \frac{M}{\omega_s T_r} sin(\omega_s h)$$

$$\varphi_6 = \frac{M}{\omega_s T_r}\left(1-\cos(\omega_s h)\right)$$

$$\varphi_7 = \cos(\omega_s h) - \frac{1}{\omega_s T_r} sin(\omega_s h)$$

$$\varphi_8 = sin(\omega_s h) - \frac{1}{\omega_s T_r}(1-\cos(\omega_s h))$$

$$\varphi_9 = \frac{1}{C_1}\left(e^{hC_2}-1\right)\frac{3P}{2}\frac{M}{L_r}\lambda_{dr}$$

$$\varphi_{10} = \frac{1}{C_1}\left(e^{hC_2}-1\right)\frac{3P}{2}\frac{M}{L_r}\lambda_{qr}$$

$$\varphi_{11} = e^{hC_2}$$

$$\gamma_1 = \frac{a}{\omega} sin(\omega h)$$

$$\gamma_2 = \frac{a}{\omega}(1-\cos(\omega h))$$

$$s_1 = \frac{1}{C_1}\left(e^{hC_2}-1\right)$$

or, in a compact form:

$$X(k+1) = \Phi X(k) + Sw(k) + \Gamma V_s(k) \tag{5}$$

where Φ and Γ are the state and input matrices, respectively, and $w(k)$, which contain the load torque, is the perturbation vector pondered by the matrix S.

Note that the above model is non-linear and non-stationary once $\varphi_i, i=1,\ldots,8$ and $\gamma_i, i=1,2$, vary with ω and ω_s, and φ_9 and φ_{10} depend on the *d*-axis and on the *q*-axis components of the rotor flux vector. Conversely, this model is very suitable for the representation of those characteristics as system's uncertainties. Defining maximum and minimum values for ω and ω_s, as well as the maximum value for the rotor flux (based on the machine saturation), we obtain the variation range of each element of the matrices of the model. By combining these values, we find the matrices that determine the vertices of a convex polytope in the parameter space, which contain all the possible matrices of the system. This polytope is represented by the convex combination of the vertices, as follows:

$$\Phi \in D_\Phi = \left\{\Phi : \Phi = \sum_{i=1}^{N} \xi_i \Phi_i, .\xi_i \geq 0, \sum_{i=1}^{N} .\xi_i = 1\right\} \tag{6}$$

$$\Gamma \in D_\Gamma = \left\{\Gamma : \Gamma = \sum_{i=1}^{N} \xi_i \Gamma_i, .\xi_i \geq 0, \sum_{i=1}^{N} .\xi_i = 1\right\} \tag{7}$$

where N is the number of vertices.

3 SPEED CONTROL

The proposed digital speed control method for the induction machine allows its employment in applications which require a wide operation range. This technique constitutes a development of the oriented flux torque regulation scheme presented in Bottura *et alii* (1996). The objective is to calculate a voltage V_s that, applied to the machine, minimizes the rotor flux and rotor frequency errors, given the references $\lambda_{ro} = [0 \quad \lambda_{dro}]^T$ and ω_{ro}. This voltage is computed from the stator current equation. In order to find an error equation, we define a stator current reference $i_{so}(k)$, calculated every sampling period, as follows. From (4), in steady state:

$$\omega_{ro} = -\varphi_{9o} i_{qso}(k) + \varphi_{11}\omega_{ro} + s_1 T_L \tag{8}$$

$$0 = \varphi_5\, i_{qso}(k) - \varphi_6\, i_{dso}(k) - \varphi_8\, \lambda_{dro} \tag{9}$$

$$\lambda_{dro} = \varphi_6\, i_{qso}(k) + \varphi_5\, i_{dso}(k) + \varphi_7\, \lambda_{dro} \tag{10}$$

where $\varphi_{9o} = \frac{1}{C_1}\left(e^{hC_2}-1\right)\frac{3P}{2}\frac{M}{L_r}\lambda_{dro}$.

From (8), the component $i_{qso}(k)$ is obtained:

$$i_{qso}(k) = -\frac{\omega_{ro} - \varphi_{11}\omega_{ro} - s_1 T_L}{\varphi_{9o}}$$

Note that the computation of $i_{qso}(k)$ depends on the value of the load torque, which can be estimated from the measurement of the rotor speed and its reference. By decomposing the load torque in two parts, the first one constant (T_{Lo}) and the other one variable due to the exogenous perturbation (ΔT_L), the motion equation can be rewritten, respectively, in a generic instant and in steady state, as:

$$\omega_r(k+1) = \varphi_{11}\omega_r(k) - s_1(T_e(k) - (T_{Lo} + \Delta T_L(k))$$

$$\omega_{ro} = \varphi_{11}\omega_{ro} - s_1(T_{eo} - T_{Lo})$$

Subtracting one equation from the other one, gives:

$$\omega_r(k+1) - \omega_{ro} = \varphi_{11}(\omega_r(k) - \omega_{ro}) - s_1(\Delta T_e(k) - \Delta T_L(k))$$

where $\Delta T_e(k) = T_e(k) - T_{eo}$. Supposing that the electromagnetic torque at the instant *k* compensates the load torque variation at the instant (*k*-1), the following equality holds: $\Delta T_e(k) = \Delta T_L(k-1)$. Then, applying one period delay, the iteractive relation follows:

$$\Delta T_L(k) = \Delta T_L(k-1) + \frac{\omega_r(k) - \omega_{ro} - \varphi_{11}(\omega_r(k-1) - \omega_{ro})}{s_1} \tag{11}$$

and, considering $T_{Lo} = 0$, $i_{qso}(k)$ becomes

$$i_{qso}(k) = -\frac{\omega_{ro} - \varphi_{11}\omega_{ro} - s_1\Delta T_L(k)}{\varphi_{9o}} \tag{12}$$

From (9) and (10):

$$i_{dso}(k) = \frac{\lambda_{dro} - (\varphi_7 - \varphi_8)\lambda_{dro} - (\varphi_5 + \varphi_6) i_{qso}(k)}{(\varphi_5 - \varphi_6)} \tag{13}$$

Then, $i_{so}(k) = \left[i_{qso}(k) \quad i_{dso}(k)\right]^T$.

Remark 1: The stability of equation (11) depends on the stability of the system. When ω_r reaches its reference value, the speed error is zero and $\Delta T_L(k) = \Delta T_L(k-1)$. Next section presents a robust control method that guarantees the system stability all over the speed operating range.

Remark 2: The assumption that $\Delta T_e(k) = \Delta T_L(k-1)$ is reasonable once the stator current reference is calculated based on the variation of the load torque. As the electrical dynamics is faster than the mechanical one, it is commonly expected that the generated torque evolves to compensate such variation.

Remark 3: The slip frequency evaluation is done by $\omega_s = R_r i_{qs} / L_r i_{ds}$

The equation for X_o is obtained by partitioning the matrices of (4):

$$X_o(k+1) = \Phi X_o(k) + \Gamma V_{so}(k) \tag{14}$$

$$= \begin{bmatrix} \Phi_1 & \Phi_2 & 0 \\ \Phi_3 & \Phi_4 & 0 \\ \Phi_5 & 0 & \Phi_6 \end{bmatrix} X_o(k) + \begin{bmatrix} \Gamma_1 \\ 0 \end{bmatrix} V_{so}(k) \tag{15}$$

where $\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Gamma_1 \in \Re^{2\times 2}$, $\Phi_5 \in \Re^{1\times 2}$, $\Phi_6 \in \Re$ and $X_o(k) = \begin{bmatrix} i_{so}^T(k) & \lambda_{ro}^T & \omega_{ro} \end{bmatrix}^T$.

By applying an one period delay in (14), the value of $V_{so}(k)$ is calculated by:

$$V_{so}(k) = \Gamma_1^{-1}\Big(i_{so}(k) - \Phi_1\big(\Phi_3^{-1}(I - \Phi_4)\lambda_{ro}\big) - \Phi_2 \lambda_{ro}\Big) \tag{16}$$

where the non-singularity of Γ_1 and Φ_3 is guaranteed in Bottura et alii (1993). By subtracting (14) from (5) we derive the error equation:

$$e(k+1) = \Phi e(k) + S w(k) + \Gamma V(k) \tag{17}$$

where

$$e(k) = X(k) - X_o(k) \tag{18}$$

$$V(k) = V_s(k) - V_{so}(k) \tag{19}$$

for which asymptotic stability can be attained by the application of state feedback. So,

$$V(k) = -K e(k) \tag{20}$$

Equations (16), (19) and (20) determine the voltage $V_s(k)$:

$$V_s(k) = V(k) + V_{so}(k) \tag{21}$$

Remark 4: The state feedback gain *K*, considered in the control law (20), determines the dynamics of the servomechanism problem represented by equation (17). The estimation of the load torque is done in equation (11), through the difference between the actual and the reference motion equations, whose dynamics is affected by the control gain. Therefore, system rejection to disturbances in the load torque depends on the location of the closed-loop poles of the whole system.

Once the matrices entries of the error equation are uncertain, the synthesis of the state feedback controller must guarantee the asymptotic stability for every combination of $\Phi \in D_\Phi$ and $\Gamma \in D_\Gamma$. The search for a gain which is robust to the plant variation and to the nonlinearity is the aim of the next section.

Besides the stability and robustness characteristics, another objective of the following synthesis procedure is to compute a control gain that minimizes an upper bound to the H_2 norm of the transfer function from the disturbance input to the output of the system.

Note that the control law (18)-(20) requires the knowledge of the rotor flux, which is not available for measurement. Many results has been presented in the literature to try to overcome this problem through estimation theory. Although the discrete model used in this paper is apropriate for the application of state observers (Bottura *et alii*, 1993), the solution of the rotor flux estimation problem is out of the scope of this paper. Therefore, the results of the following section are obtained under the hypothesis of full information, that is, the state vector is completely known.

4 ROBUST DISCRETE CONTROLLER

The definition of an output equation leads to the following generic representation for the error equation (17). We consider the presence of impulsive disturbances in the state equation, so the following system is defined:

$$\begin{cases} e(k+1) = \Phi e(k) + S w(k) + \Gamma u(k) \\ y(k) = C e(k) + D u(k) \\ u(k) = -K e(k) \end{cases} \tag{22}$$

where $\Phi^{n\times n} \in D_\Phi$, $\Gamma^{n\times m} \in D_\Gamma$, $C^T D=0$ and $D^T D>0$.

The stability of the system (22) is discussed based on the closed loop equation:

$$e(k+1) = (\Phi - \Gamma K) e(k) = \Phi_f e(k) \tag{23}$$

This system is quadratically stabilizable via linear control for any $\Phi \in D_\Phi$ and $\Gamma \in D_\Gamma$ if and only if there exists a matrix $P = P^T > 0$, such that:

$$\Phi_f^T P \Phi_f - P < 0, \qquad \forall \Phi \in D_\Phi, \forall \Gamma \in D_\Gamma$$

If D_Φ and D_Γ are polyhedral sets, this quadratic stability condition has to be verified only in its *N* vertices (Horisberger and Belanger, 1976). Then,

$$\Phi_{f\,i}^T P \Phi_{f\,i} - P < 0, \quad i = 1, \ldots, N$$

It is equivalent to say that the augmented system associated to (22), proposed in Barmish (1983), given by

$$z(k+1) = F z(k) + G r(k)$$

is quadratically stable, where $F \in \Re^{p\times p}$, *p=n+m,* and the constant matrix $G \in \Re^{p\times m}$ are defined by:

$$F = \begin{bmatrix} \Phi & -\Gamma \\ 0 & 0 \end{bmatrix} \in D_F \qquad G = \begin{bmatrix} 0 \\ I \end{bmatrix} \in \Re^{p\times m}$$

That is, if the set D_F is polyedral, there exists a matrix $W=W^T>0$, such that $F_i W F_i^T - W < 0$, *i=1...N,* and the stabilizing gain of the closed loop system (23) can be evaluated by $K = W_2^T W_1^{-1}$, where:

$$W = \begin{bmatrix} W_1 & W_2 \\ W_2{}^T & W_3 \end{bmatrix} \tag{24}$$

with $W_1 > 0 \in \Re^{n\times n}$ and $W_2 \in \Re^{n\times m}$.

The above necessary and sufficient conditions guarantee that the poles of Φ_f are allocated in the interior of the unity circle. Based on these concepts, the robust gain K can be computed by the convex optimisation problem proposed in Geromel *et alii* (1993).

$$\begin{aligned} & \underset{W}{Min}(Trace(RW)) \\ s.t. \quad & v^T(FWF^T - W)v < 0 \end{aligned} \qquad (25)$$

where:

$$R = \begin{bmatrix} C^T C & 0 \\ 0 & D^T D \end{bmatrix}$$

$$v = \left\{ v \neq 0 \colon G^T v = 0, v^T = \begin{bmatrix} x^T & 0 \end{bmatrix}, x \in \Re^{n \times n} \right\}$$

Consider the transfer function from *w(k)* to *y(k)* of the closed loop system defined by:

$$H(s) = (C - DK)(sI - (\Phi - \Gamma K))^{-1} S$$

The H2 norm of *H(s)* is defined in function of the controlability Grammian, *Lc*, by:

$$\|H\|_2^2 = Tr\left\{(C - DK)L_C(C - DK)^T\right\}$$

where

$$L_C = \sum_{k=0}^{\infty} \Phi_f(k) S S^T \Phi_f^T(k)$$

The optimisation problem (25) constitutes na upper bound for the H2 norm of the transfer function *H(s)*. Besides that, it is the LQR problem for the certain systems (N=1). In this case, the cost function is equivalent to:

$$J = \sum_{k=0}^{\infty} (y(k)^T y(k))$$

where $y(k)=Ce(k)+Du(k)$ and its minimisation is obtained by the solution of the discrete Riccati equation.

5 SIMULATION RESULTS

This section presents the simulation results for an 1 Hp, 230/380V, 60Hz, 4 poles squirrel cage induction machine, whose parameters are: $R_S = 7.1\Omega$, $R_r = 5.8\Omega$, $L_s = L_r = 310.5mH$, $M = 284.56mH$, $J_m = 0.0038kgm^2$ and $C_1 = 0.0015Nms$. The first step is to characterize the plant variation in order to obtain the model based on the parameter space. For this machine, the parameters range are listed below, for the sampling period h=2ms.

$$\begin{array}{ll} 0.2707 \le \varphi_1 \le 0.5189 & -0.0784 \le \varphi_8 \le 0.0784 \\ 0.0080 \le \varphi_2 \le 0.5776 & -4.3379 \le \varphi_9 \le 0 \\ -4.3867 \le \varphi_3 \le 0.7447 & -1.4460 \le \varphi_{10} \le 1.4460 \\ -1.4418 \le \varphi_4 \le 14.1454 & 0.0365 \le \gamma_1 \le 0.0402 \\ 00105 \le \varphi_5 \le 0.0106 & 4.0233 \times 10^6 \le \gamma_2 \le 0.0146 \\ -0.0004 \le \varphi_6 \le 0.0004 & \varphi_{11} = 0.9984 \\ 0.9596 \le \varphi_7 \le 0.9628 & s_1 = -1.0518 \end{array}$$

These intervals were stated by defining the machine operating range restricted to $\omega \in [0,380]\,rad/s$, $\omega_s \in [-40,40]\,rad/s$, $\lambda_{qr} \in [-0.5,0.5]$ and $\lambda_{dr} \in [0,1.5]$. The non-stationarity and the non-linearity of the discrete time model are represented by the combination of the extremes values of the above parameters.

A important computational aspect of the definition of the optimisation problem is with respect to determining the set of constraints. If all these variant matrix entries were independent to each other, the generated set in the parameter space would have 4096 vertices, what would make the problem impracticable. However, this number is much smaller once there are several correlated elements. In order to exemplify such correlations, consider $\varphi_i, i = 5,\ldots,8$ plots shown in Fig. 1. Note that φ_5 and φ_7 as well as φ_6 and φ_8 have their maximum and minimum points at the same value of ω_s. This characteristic and the consideration that φ_5 and φ_7 are independent of φ_6 and φ_8 reduce the total number of vertices to be considered. Fortunately, using the same treatment for the others parameters, it is found that only 16 vertices compose the polyhedral set which describes the whole system and restricts the optimisation problem (26).

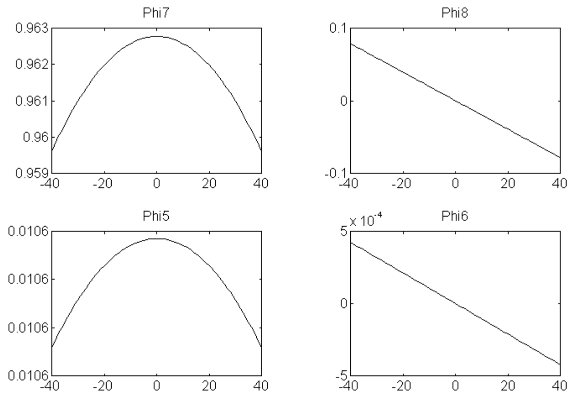

Fig 1 - Parameters changes

The definition of the matrices

$$C=\begin{bmatrix}1&0&0&0&0\\0&0&0&0&0\\0&1&0&0&0\\0&0&0&0&0\\0&0&1&0&0\\0&0&0&0&0\\0&0&0&1&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&1\end{bmatrix},\quad D=\begin{bmatrix}0&0\\1&0\\0&0\\0&1\\0&0\\0&0\\0&0\\0&0\\0&0\\0&0\end{bmatrix}\quad \text{and}$$

$$S=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&-1.0518\end{bmatrix}$$

determines the optimisation cost that leads to the stabilizing gain

$$K=\begin{bmatrix}12.68&-3.81&497.80&-88.03&-8.6\times10^{-4}\\4.37&14.33&122.90&440.34&4.3\times10^{-4}\end{bmatrix}$$

The root locus of the closed loop system (23), considering the computed gain, is shown in the Fig. 2. The robust gain guarantees the stability of the system all over the specified operating range. The poles near to the point (1,0) are due to the speed equation, while the ones near to the origin of the *z*-plane are due to the electrical equations. This pole placement determines a faster dynamics for the electrical variables than that for the mechanical ones. These characteristics are in accordance with the conditions remarked at section 3.

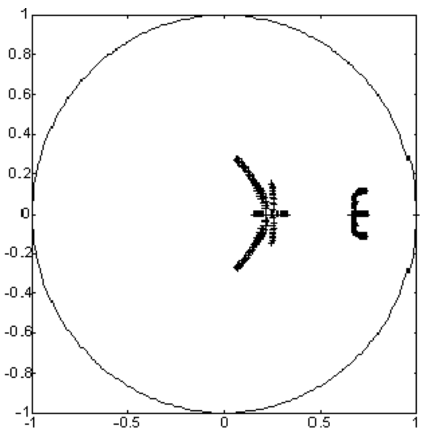

Fig. 2 - Closed loop poles

The performance of the drive system can be evaluated through figures 3 to 7. Initially, the rotor flux reference, $\lambda_{ro}=[0\ \ 1]^T$, is provided to the system, while the speed reference is set to zero. The load torque is composed by two components: one constant, set to 25% of the motor's rated torque while the other one is proporcional to the shaft speed. Figures 3 and 4 show that the system presents a good response to very slow speeds and that vector control is reached once the rated rotor flux is oriented to the direct axis. Still refering to figures 3 and 4, at instant 2 seconds, the speed reference is changed to $\omega_{ro}=300\,rad\,/\,s$. Note that the change in the rotor speed does not cause any consirable transient in the rotor flux orientation and the rotor speed reaches the desired value with no errors.

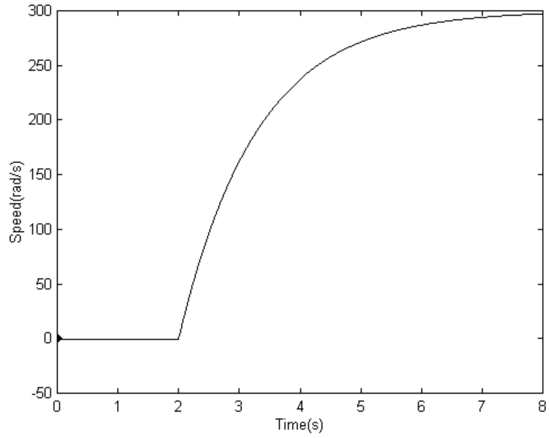

Fig. 3 - Rotor speed

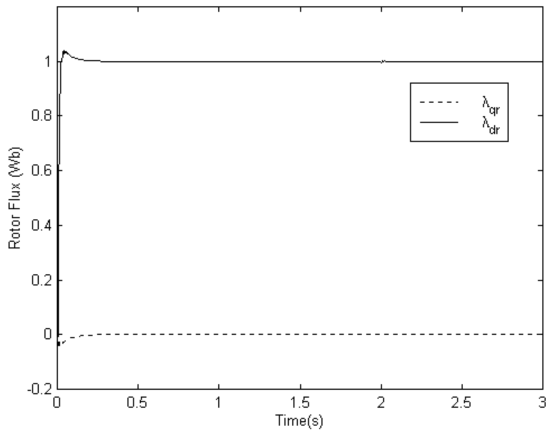

Fig. 4 - Rotor flux

The performance of the system in the presence of load torque perturbation is shown in figures 5 to 7. Initialy, the speed reference is $\omega_{ro}=300\,rad\,/\,s$ and the rotor speed is in steady state. Then, at the instant *t=3s* a 100 % increase in the load torque is applied. As a response to the external perturbation, the generated torque promptly changes to try to keep the speed unaltered (figure 6) and the speed is driven to its reference value (figure 5). It is important to notice that the system dynamics to disturbance rejection depends on the dynamics of the load torque estimation, given by equation (11), which is discussed in remarks 1 and 4. In the present case, the +100 % step in the load torque caused a –2 % deviation in the shaft speed, which is a quite acceptable result. Besides that, the influence of the perturbation in the rotor flux orientation is very low, as shown in the figure 7. These results demonstrate a good disturbances rejection.

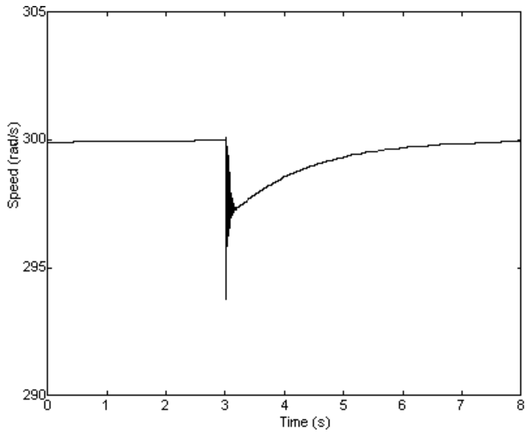

Fig. 5 - Rotor speed - Perturbed system

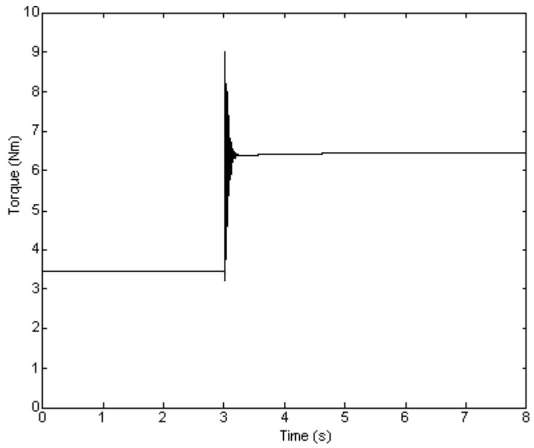

Fig. 6 - Electromagnetic Torque - Perturbed System

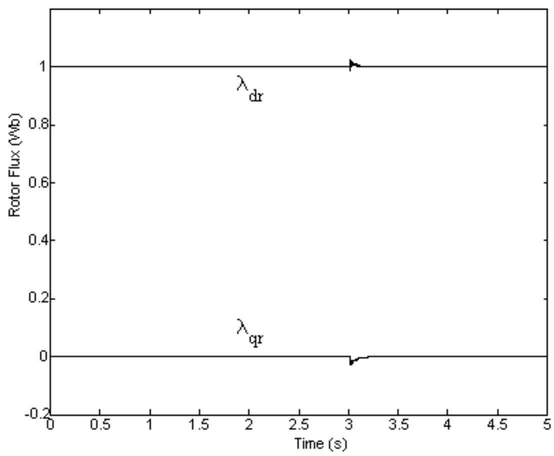

Fig 7 - Rotor Flux - Perturbed system

6 CONCLUSIONS

In this paper, a new proposal for induction machine modelling and robust digital vector control was presented. In this proposal, the non-stationarity and the non-linearity of the system were absorbed by the description of the parameters changes of a linear uncertain system. This approach allowed the employment of a robust controller synthesis by an H_2 optimisation method. The simulation results validate the proposal.

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