A LARGEST-STEP CENTRAL-PATH ALGORITHM APPLIED TO THE OPTIMAL POWER FLOW PROBLEM

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Resumo: Os métodos não-lineares Primal-Duais de Pontos Interiores têm sido reconhecidos como ferramentas numéricas de grande potencial para resolver problemas de otimização restritos em sistemas de potência elétrica. Atualmente, novas versões deste algoritmo têm sido propostas na área da programação matemática. Neste trabalho, é analisada a aplicação do algoritmo de Máximo Passo na Trajetória Central ao problema de Fluxo de Potência Ótimo. O algoritmo utilizado propõe uma trajetória de busca próxima ao caminho central para atingir a solução ótima. Testes com sistemas reais são utilizados para analisar o comportamento da metodologia proposta.

Abstract*:* Nonlinear Primal-Dual Interior Point methods have been recognized as a numerical tool of great potential to solve constrained optimization problems in electric power systems. Recently, a number of versions of the primal-dual Interior Point algorithm have been proposed in the area of mathematical programming. In this work, the application of the Largest-Step Central-Path algorithm to the Optimal Power Flow problem is analyzed. This algorithm is based on following a central trajectory to reach the optimal solution. Tests with real systems are used to assess the performance of the proposed methodology.

Keywords: Optimization, Nonlinear Programming, Optimal Power Flow, Primal-Dual Interior Point method.

1 INTRODUCTION

The real time Optimal Power Flow (OPF) aims at optimizing the power system operation in steady state, preventing violations in the operational limits (Ramanathan, 1996). The OPF is a nonlinear problem of static optimization, with both equality and inequality restrictions. In the last three decades, many nonlinear programming methods have been used in the solution of OPF problems, resulting three classes of approach: a) extensions of the conventional power flow method (for example, Dommel and Tinney, 1968). In this type of approach, a sequence of optimization problems is alternated with solutions of the conventional power flow; b) direct solution of the optimality conditions for Newton's method (Sun *et alii*, 1984). In this type methodology, the approximation of the Lagrangian function by a quadratic form is used, the inequality constraints being handled through penalty functions. In spite of the good theoretical characteristics of convergence, the manipulation of the inequalities via penalty functions is not efficient; and c) by Interior Point algorithms, approach used in the present work, which is described as follows.

The Interior Point method (El-Bakry *et alii*, 1996) has been extensively used in both linear and nonlinear programming. Many research works have been devoted to the study of its characteristics and applications in optimization problems. In a recent past, the solution of the OPF problem through Interior Point methodologies was proposed. These approaches (Granville, 1994; Wu *et alii*, 1994; Wei *et alii*, 1998) show the potential of the Interior Point method to deal with the inequality constraints in large-scale optimal power flow problems.

With respect to the optimization algorithm, some alternative versions of the primal-dual Interior Point algorithm have been developed. The main modifications exploit the characteristics of the path to the optimal solution, to improve the robustness and/or the convergence speed. One of the versions more frequently used in the OPF is the Predictor-Corrector Interior Point method, proposed for linear programming by Mehrotra (1992). This algorithm aims at reducing the number of iterations to the convergence, estimating the “need” of the centralization in the trajectory to the optimal solution (Wu *et alii*, 1994; Wei *et alii*, 1998, Castronuovo *et alii*, 2000).

In the present work, the Largest-Step Central-Path algorithm is applied to solve the OPF. This method is based on centralizing the trajectory to the optimal solution, to increase the robustness of the methodology. This optimization algorithm was proposed in references (Gonzaga and Bonnans, 1996) and (Gonzaga, 1997) for linear complementary problems. The application of this strategy to the nonlinear OPF is analyzed in the following sections.

Artigo Submetido em 30/11/1999
1a. Revisão em 27/06/2000
Aceito sob recomendação do Ed. Consultor Prof. Dr. José Luiz Resende Pereira

The analysis of the performance of the Largest-Step Central-Path algorithm is based on the results obtained for three test systems (IEEE 30, 57 and 118 buses) and two real networks, equivalent to the South-Southeastern Brazilian (SSB) power system (176 and 352 buses).

2 MATHEMATICAL FORMULATION

Suppose a general optimization problem represented by

$$\text{Min.} \quad f(\underline{x}) \tag{1.a}$$

$$\text{s. t.} \quad \underline{h}(\underline{x})=\underline{0} \tag{1.b}$$

$$\underline{g}(\underline{x}) \le \underline{0} \tag{1.c}$$

where, $\underline{x}$ is the vector of optimization variables; $f(\underline{x})$ represents the performance index to be optimized; $\underline{h}(\underline{x})$ and $\underline{g}(\underline{x})$ are the equality and inequality constraints, respectively.

The optimality conditions of Karush-Kuhn-Tucker (KKT) for problem (1) are expressed in equations (2).

$$\underline{\nabla_x L}(\underline{x},\underline{\lambda_h},\underline{\lambda_g}) = \underline{0} \tag{2.a}$$

$$\underline{h}(\underline{x}) = \underline{0} \tag{2.b}$$

$$\underline{g}(\underline{x}) \le \underline{0} \tag{2.c}$$

$$[\underline{\lambda_g}]\ \underline{g}(\underline{x}) = \underline{0} \tag{2.d}$$

$$\underline{\lambda_g} \ge \underline{0} \tag{2.e}$$

where, $\underline{\nabla_x L}(\underline{x},\underline{\lambda_h},\underline{\lambda_g}) = \nabla_x f(\underline{x}) + \nabla_x \underline{h}(\underline{x})\ \underline{\lambda_h} + \nabla_x \underline{g}(\underline{x})\ \underline{\lambda_g}$ is the vector of first derivatives of the Lagrangian function with respect to the variables $\underline{x}$; $\underline{\lambda_h}$ and $\underline{\lambda_g}$ are the Lagrange multipliers of the equality and inequality constraints, respectively; and *[..]* denotes a diagonal matrix with the variables considered.

In the iterative process based on Interior Points methods, two modifications of the KKT conditions are proposed (El-Bakry *et alii*, 1996): the conversion of the inequalities (2.b) in equalities by using the slack variables $\underline{s} > \underline{0}$, and the perturbation of the complementarily equations (2.d) through the parameter μ. The new set of equations is

$$\underline{\nabla_x L}(\underline{x},\underline{\lambda_h},\underline{\lambda_g}) = \underline{0} \tag{3.a}$$

$$\underline{h}(\underline{x}) = \underline{0} \tag{3.b}$$

$$\underline{g}(\underline{x}) + \underline{s} = \underline{0} \tag{3.c}$$

$$[\underline{\lambda_g}]\ \underline{s} - \mu\, \underline{e} = \underline{0} \tag{3.d}$$

$$(\underline{s},\ \underline{\lambda_g}\ ,\ \mu) \ge 0 \tag{3.e}$$

where, μ is called the *parameter of perturbation*; and $\underline{e}$ is the unitary vector $[1,...,1]^t$.

The solution of the nonlinear equations (3) is obtained through Newton's method. At each iteration, the increments in the primal and dual variables are determined in two steps (Granville, 1994). In the first, the corrections $\Delta\underline{x}$ and $\Delta\underline{\lambda_h}$ are evaluated by solving the reduced set of linear equations

$$\begin{bmatrix} H & J \\ J^t & 0 \end{bmatrix} * \begin{bmatrix} \Delta\underline{x} \\ \Delta\underline{\lambda}_h \end{bmatrix} = \begin{bmatrix} \underline{t_1} \\ \underline{t_2} \end{bmatrix} \tag{4}$$

where

$$H = \nabla_{xx} L + \nabla \underline{g}(\underline{x})^t \begin{bmatrix} \underline{\lambda_g} \\ \underline{s} \end{bmatrix} \nabla \underline{g}(\underline{x}) \tag{5.a}$$

$$J = \nabla \underline{h}(\underline{x}) \tag{5.b}$$

$$\underline{t_1} = -\underline{\nabla_x L}(\underline{x},\underline{\lambda_h},\underline{\lambda_g}) - \nabla \underline{g}(\underline{x})^t \left[\underline{s^{-1}}\right] \left(\mu \underline{e} + [\underline{\lambda_g}]\underline{g}(\underline{x})\right) \tag{5.c}$$

$$\underline{t_2} = -\underline{h}(\underline{x}) \tag{5.d}$$

$$\nabla_{xx} L = \nabla_{xx} f(x) + \nabla_{xx} h(x) + \nabla_{xx} g(x) \tag{5.e}$$

In the second step, the increments in the slack variables and corresponding multipliers are obtained by

$$\Delta\underline{s} = -\underline{g}(\underline{x}) - \underline{s} - \nabla \underline{g}(\underline{x})\ \Delta\underline{x} \tag{6.a}$$

$$\Delta\underline{\lambda}_g = -\underline{\lambda}_g + \left[\underline{s^{-1}}\right] * \{\mu \underline{e} - [\underline{\lambda_g}]\Delta\underline{s}\} \tag{6.b}$$

In order to assure the non-negativity of $\underline{s}$ and $\underline{\lambda_g}$, two step sizes are used to update the primal and the dual variables respectively (equations 7) (Granville, 1994).

$$\alpha_p = min\left[\min_{\Delta s_j < 0} \frac{s_j}{|\Delta s_j|}, 0.99995\right] \tag{7.a}$$

$$\alpha_d = min\left[\min_{\Delta\lambda_{gj} < 0} \frac{\lambda_{gj}}{|\Delta\lambda_{gj}|}, 0.99995\right] \tag{7.b}$$

where the factor *0.99995* assures that constraint (3.e) is satisfied.

The primal and dual variables for the following iteration are

$$\underline{x}^{k+1} = \underline{x} + \alpha_p\, \Delta\underline{x} \tag{8.a}$$

$$\underline{s}^{k+1} = \underline{s} + \alpha_p\, \Delta\underline{s} \tag{8.b}$$

$$\underline{\lambda_h}^{k+1} = \underline{\lambda_h} + \alpha_d\, \Delta\underline{\lambda_h} \tag{8.c}$$

$$\underline{\lambda_g}^{k+1} = \underline{\lambda_g} + \alpha_d\, \Delta\underline{\lambda_g} \tag{8.d}$$

At each iteration, the parameter of perturbation (μ) is gradually reduced to zero, such that at the final solution the KKT conditions expressed by equations (2) and (3) are equivalent. In the present work, the parameter of perturbation μ is evaluated as suggest in (Gonzaga, 1994), through of the primal-dual distances to the optimal point, measured in the complementarily restriction. This value is dependent on a parameter σ, as expressed in equation (9).

$$\mu = \sigma\ \hat{\mu} = \sigma \frac{\underline{\lambda_g}^t \underline{s}}{n_{iq}} \tag{9}$$

where, σ is the so-called *parameter of the direction combination*; $\hat{\mu}$ is the *complementarily condition average distance*; and n_{iq} is the number of inequality constraints.

3 DETERMINATION OF THE DIRECTION COMBINATION PARAMETER

The distance between a point of the search direction and the current solution (measured in the complementarily equations) is unique ($\hat{\mu}$). Therefore, σ is the parameter that actually determines the perturbation used in the complementary equation (3). Thus, the shape of the trajectory followed to find the optimum is defined by the parameter σ.

The meaning of the parameter σ can be understood, by observing that the right side of equations (4) and (6) are affected by the parameter of perturbation μ, which is in turn dependent on the parameter σ. With respect to the value of σ, two cases must be analyzed:

- $\sigma = 0$, which corresponds to the so-called *affine-scaling direction*. If only the affine-scaling direction is used along the iterative process, the search for the optimal point can be interpreted as the simple non-perturbed solution of the KKT conditions;

- $\sigma = 1$, which results in the so-called *centralization direction*. In this case, a perturbed set of nonlinear equations witch does not correspond to the KKT conditions of the original problem is solved. Therefore, if only the centralization direction is used in the iterative process, a non-optimal solution is founded, without appreciable diminution in the initial value of μ.

The value of σ between these limits represents a linear combination of the affine-scaling and the centralization directions. To decrease the value of μ during the iterative process, it is necessary that $0 < \sigma < 1$, as suggested in El-Bakry *et alii* (1996). Note that in the conventional Interior Points method, σ is a value pre-determined by the user, and constant during the iterative process. That is, the linear combination of the direction is constant along the search.

Based on the principle of the superposition, it follows that the Newton steps (4) and (6) satisfies equation (10).

$$(\Delta\underline{x}, \Delta\underline{s}, \Delta\underline{\lambda}_h, \Delta\underline{\lambda}_g) = \sigma\,(\Delta\underline{x}^c, \Delta\underline{s}^c, \Delta\underline{\lambda}_h{}^c, \Delta\underline{\lambda}_g{}^c) + (1-\sigma)\,(\Delta\underline{x}^{a\text{-}s}, \Delta\underline{s}^{a\text{-}s}, \Delta\underline{\lambda}_h{}^{a\text{-}s}, \Delta\underline{\lambda}_g{}^{a\text{-}s}) \qquad (10)$$

where $(\Delta\underline{x}^{a\text{-}s}, \Delta\underline{s}^{a\text{-}s}, \Delta\underline{\lambda}_h{}^{a\text{-}s}, \Delta\underline{\lambda}_g{}^{a\text{-}s})$ and $(\Delta\underline{x}^c, \Delta\underline{s}^c, \Delta\underline{\lambda}_h{}^c, \Delta\underline{\lambda}_g{}^c)$ are the solution of equations (4) and (6) in the present iteration, with $\sigma=0$ and $\sigma=1$, respectively.

It can be noted from (10) that, depending on the value of σ at each iteration, there is an infinity of search paths that leads to the optimal point. A point $(\underline{x}, \underline{s}, \underline{\lambda g})$ rely in the *central path* if it satisfies the equations (3.c), (3.e) and $[\underline{\lambda}_g]\ \underline{s} = \hat{\mu}\ \underline{e}$. However, this point is at a distance δ from the central path if it satisfies equation (11).

$$\delta(\underline{\lambda}_g, \underline{s}, \mu) = \left\| \frac{\underline{\lambda}_g{}^t\ \underline{s}}{\mu} - \underline{e} \right\| \qquad (11)$$

where $\|..\|$ represents the Euclidean norm.

The Largest-Step Central-Path algorithm allows searching the optimal solution through a trajectory close to the central path. In the next iteration, the point must be at a pre-specified distance δ of the central path (Gonzaga, 1997). For this purpose, σ is calculated in order to satisfy the equation (12).

$$\delta(\underline{\lambda}_g{}^{k+1}, \underline{s}^{k+1}, \mu^{k+1}) = \left\| \frac{\left(\underline{\lambda}_g{}^{k+1}\right)\ \underline{s}^{k+1}}{\sigma\ \hat{\mu}} - \underline{e} \right\| \qquad (12)$$

The steps α_p and α_d are evaluated after the determination of the search direction. Therefore, in the σ calculation, equation (12) can be approximated by expression (13).

$$(\underline{x}^{k+1}, \underline{s}^{k+1}, \underline{\lambda}_h{}^{k+1}, \underline{\lambda}_g{}^{k+1}) \cong (\underline{x}, \underline{s}, \underline{\lambda}_h, \underline{\lambda}_g) + (\Delta\underline{x}, \Delta\underline{s}, \Delta\underline{\lambda}_h, \Delta\underline{\lambda}_g) \qquad (13)$$

Consequently, the primal-dual product is expressed as

$$(\underline{\lambda}_g{}^{k+1})^t\ \underline{s}^{k+1} = (\underline{\lambda}_g)^t\ \underline{s} + (\underline{\lambda}_g)^t\ \Delta\underline{s} + (\Delta\underline{\lambda}_g)^t\ \underline{s} + (\Delta\underline{\lambda}_g)^t\ \Delta\underline{s} \qquad (14)$$

The substitution of equation (6.b) in (14) and (12), results in

$$\delta(\sigma) = \left\| \frac{\left(\Delta\underline{\lambda}_g\right)^t\ \Delta\underline{s}}{\sigma\ \hat{\mu}} \right\| \qquad (15)$$

The replacement of the values of $\Delta\underline{s}$ and $\Delta\underline{\lambda}_g$ by those obtained from the equation (10) provides

$$\delta(\sigma) = \left\| \frac{(\sigma\Delta\underline{\lambda}_g{}^c + (1-\sigma)\Delta\underline{\lambda}_g{}^{a-s})^t(\sigma\Delta\underline{s}^c + (1-\sigma)\Delta\underline{s}^{a-s})}{\sigma\ \hat{\mu}} \right\| = \varepsilon \qquad (16)$$

where $\varepsilon > 0$ is the pre-specified value of distance to the central path in the next iteration, in units of μ.

The control over the value of ε allows modifying the proximity of the search trajectory to the central path. In optimization problems with low degree of convexity, a trajectory closer to the central path is generally required. This implies in a compromise between the convergence speed and the robustness.

For a pre-specified value of ε, the only variable of (16) is σ. Therefore, σ is obtained by solving a quartic equation. The value of σ can be determined through a bisection scheme. This provides the largest root of (16) in the interval $0 < \sigma < 1$. In some circumstances, there are not real roots in this interval. In these cases, the value $\sigma = 0.1$, similarly to the conventional primal-dual Interior Point Method, can be taken.

The calculation of two directions (affine-scaling and centralization) at each iteration does not request a high additional computational effort. Since the value of σ only affects the right side of the linear system, each direction is obtained through a backward-forward substitution after the linear system factorization. High Performance Computation Techniques can be efficiently used in order to reduce the cpu-times corresponding to the solution of the linear system, as showed in Castronuovo *et alii*, 1998 and 1999.

The Largest-Step Central-Path algorithm can be summarized in the following algorithm.

LARGEST-STEP CENTRAL-PATH ALGORITHM

Data: $\varepsilon > 0$, $(x^0, \underline{\lambda}_h^0)$, $(\underline{s}^0, \underline{\lambda}_g^0) > \underline{0}$
DO

Affine-Scaling Direction: Solve equations (4) and (6) for $\sigma = 0$, obtaining $(\Delta\underline{x}^{a\text{-}s}, \Delta\underline{s}^{a\text{-}s}, \Delta\underline{\lambda}_h^{a\text{-}s}, \Delta\underline{\lambda}_g^{a\text{-}s})$.

Centralization Direction: Solve equations (4) and (6) for $\sigma = 1$, obtaining $(\Delta\underline{x}^{c}, \Delta\underline{s}^{c}, \Delta\underline{\lambda}_h^{c}, \Delta\underline{\lambda}_g^{c})$.

Bisection Scheme: Find the largest root in the $0 < \sigma < 1$ interval, that satisfies equation (16).

Direction Combination: Solve equation (10).

Actualization of the Variables: Solve equations (7), (8) and (9).

Convergence Test: Evaluate the convergence tolerances in $(\mu, \nabla_x \underline{L}(\underline{x}, \underline{\lambda}_h, \underline{\lambda}_g), \underline{h}(\underline{x}))$.

END DO

4 RESULTS

In this section, the results of the application of the Largest-Step Central-Path algorithm to the OPF problem are shown. In order to observe the robustness of the proposed methodology with respect to the convexity of the objective function, two performance indexes were considered. In the first, the minimum price of the active power generation is sought, with the generation prices represented by quadratic curves. In the second, the minimization of a reference bus active power injection with the power injection of the other buses fixed in a pre-specified value is proposed. In both cases, the equality constraints are the power balance equations and the inequality constraints are the limits in the active and reactive power generation and the bus voltage magnitude. The optimization variables are the bus voltages and the active power generation.

The tolerance for the power mismatches, the right-side vector and the parameter of perturbation (μ) for convergence is 10^{-6}. The pre-specified distance to the central path is $\varepsilon = 3.0$. The algorithm was implemented in Fortran 90 and the numerical results were obtained in a computer Pentium II 400Mhz with 128Mb of Ram memory. To assess the performance of the proposed algorithm, three IEEE test systems (30, 57 and 118 buses) and two Brazilian South-southeast (SSB) real systems (of 176 and 352 buses) were used. The main characteristics of these networks are show in the Table 1.

Table 1
Characteristics of the Test Systems

	IEEE 30	IEEE 57	IEEE 118	SSB 176	SSB 352
Branches	41	78	179	196	385
Generators	6	7	34	30	30

In the Table 2, the results of the applications of the Largest-Step Central-Path and the Interior Point algorithms are presented. The latter algorithm was implemented in its conventional form, as formulated in the section 2 (with $\sigma = 0.1$).

Table 2
Minimum Price of the Active Power Generation.
Number of Iterations and Computational Times (seconds).

Test System	Interior Point		Largest-Step	
	Iter.	Time	Iter.	Time
IEEE 30	10	0.21	10	0.27
IEEE 57	12	0.59	12	0.71
IEEE 118	16	2.18	18	2.75
SSB 176	16	1.63	18	2.03
SSB 352	16	3.20	18	4.48

From Table 2, the solution of the optimization problem by the Largest-Step Central-Path algorithm requires cpu-times larger (average of 26%) than to those obtained with the conventional Interior Point algorithm. There are two main reasons for this: the increase in the number of iterations for the largest systems and additional operations at each iteration. The increase in the number of operations fundamentally results from the additional backward-forward substitution requested by the Largest-Step Central-Path algorithm in the calculation of σ. The computational times to the convergence of the Largest-Step Central-Path algorithm are slightly larger than those obtained with the conventional Interior Point algorithm.

In Fig. 1, the some indexes related to the iterative process of the Largest-Step Central-Path algorithm are shown, for the system SSB 352 buses.

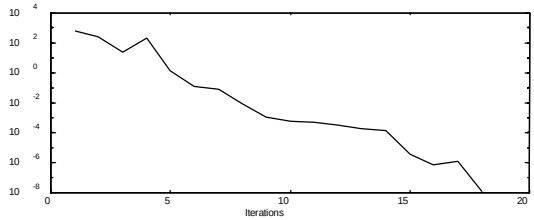


Fig. 1.a: μ Variation. - SSB 352
(Semi-logarithmic scale)

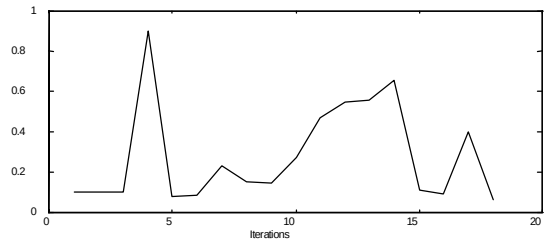
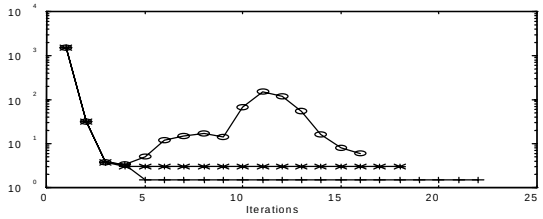


Fig. 1.b: σ Variation. - SSB 352

The values of μ are shown in Figure 1.a. The sudden changes in the value of μ are due to variations in the value of σ (Fig. 1.b). The Largest-Step Central-Path algorithm searches for a direction close to the central trajectory, determining the value of σ such as to ensure the expected proximity ε. This algorithm combines the centralization and the affine-scale directions, calculating an appropriate value of σ in terms of the proximity at the central path. From Figure 1.b, it can be observed that at the first iterations the algorithm does not obtain the previously defined distance, and thus uses a pre-specified value of σ (in this work, 0.1). In the following iterations, the value of σ that assures the expected proximity to the central path is obtained.



-o- Conventional -*- Long-Step (ε=3.0) -+- Long-Step (ε=1.5)

Fig. 2: Distances to the Central Path, in units of μ. SSB 352 - (semi-logarithmic scale)

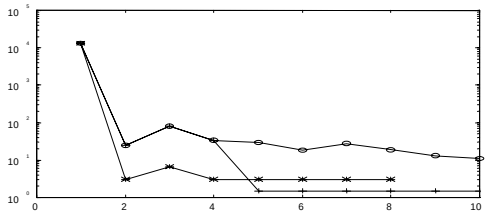
Figure 2 shows the distances between the search trajectory and the central path for both the Largest-Step Central-Path and the conventional Interior Point algorithms. The distances are expressed in terms of the Euclidean norms. It is observed in this figure, the trajectory to the optimal solution obtained with the conventional Interior Point algorithm does not result in minimum distances to the central path. In reality, this trajectory, at any iteration, is considerably far from the central trajectory. On the other hand, the Largest-Step Central-Path algorithm is effective to guarantee the pre-specified distance. At the first iterations, the algorithm does not find an appropriate σ and thus, follows the path of the conventional Interior Point method. At the subsequent iterations, the algorithm follows a trajectory as central as previously specified. In Fig. 2, measures of the distances for two values of ε (3.0 and 1.5) are shown. The centralization characteristic of the algorithm results in a number of iterations, which is dependent on the specified value of ε.

To test the Largest-Step Central-Path algorithm in problems with low convexity characteristics, a loss reduction problem is proposed. The objective is the minimization of the active power generation in a reference bus, with pre-specified active powers in the other generation buses. The solution space of the problem is modified by reducing the intervals of variation of the active powers around pre-specified values. In the present case, the pre-specified active generation is the optimal solution of the OPF, and the adopted variations are ±0.1 p.u. In Table 3, the number of iterations and the cpu-times for this problem are shown.

Table 3
Minimization of the Active Power Losses.
Number of Iterations and Computational Times (seconds).

Test System	Interior Point		Largest-Step	
	Iter.	Time	Iter.	Time
IEEE 30	11	0.22	10	0.22
IEEE 57	11	0.55	9	0.52
IEEE 118	10	1.32	8	1.30
SSB 176	11	1.05	9	0.98
SSB 352	10	2.12	8	1.98

The number of iterations of the conventional Interior Point method in this case are generally lower than those obtained in the minimum cost problem. This is due to the reduction in the number of the variables of the first problem. In this second case only the active power generation in the reference bus can be effectively changed. The small interval of variation of the other active power generation buses reduces the convexity of the solution space, emphasizing the efficiency of the Largest-Step Central-Path algorithm. From Table 3, it is observed that the centralization ability of this algorithm results in a number of iterations and cpu-times lower than those obtained with the conventional Interior Point method. In Fig. 3, the distances at the central path for both algorithms are shown, for the SSB 352 network.



-o- Conventional -*- Long-Step (ε=3.0) -+- Long-Step (ε=1.5)

Fig. 3: Distances to the Central Path, in units of μ. SSB 352 - (semi-logarithmic scale)

Figure 3 shows that the Largest-Step Central-Path algorithm has the ability of maintaining a pre-specified distance of the central path. For $\varepsilon = 1.5$, the proposed algorithm request five iterations to find an adequate combination between the affine-scale and the centralization directions. Afterwards, the algorithm follows a pre-specified distance to the central path. For $\varepsilon = 3.0$, only in the first and in the third iterations a direction combination witch provides the pre-specified distance is not obtained. This gives an idea about the efficiency of the Largest-Step Central-Path algorithm in low convexity problems.

5 CONCLUSIONS

The conventional nonlinear Interior Point method search for the optimal solution maintaining a variable distance between the current solution and the central path. A new version of this method, the Largest-Step Central-Path algorithm, allows the monitoring of this distance, providing the possibility of

increasing the robustness of the Interior Point method. Although in some cases the speed of the convergence becomes compromised, the search along the central trajectory can be more suitable if optimization problems with low degree of convexity are considered.

The application of the Largest-Step Central-Path algorithm to a nonlinear OPF problem shows the efficiency of this method. Tests performed with real systems, shown the potential of the proposed algorithm.

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7 ACKNOWLEDGMENT

The authors wish to thank to the CNPq and RECOPE/FINEP for the financial support.