CORRECTIVE SOLUTIONS OF STEADY STATE POWER SYSTEM VIA NEWTON OPTIMIZATION METHOD

Luciano V. Barboza* **Roberto Salgado**

GSP – LABSPOT – Department of Electrical Engineering
Federal University of Santa Catarina
88040-900 – Florianópolis – SC – Brazil

* Catholic University of Pelotas, Federal Center for Technological Education of Pelotas – RS – Brazil

Abstract: This paper focuses a methodology to determine corrective adjustments in the electric network when the steady state power system equations have no real solution. Generally, this happens if the system is heavily loaded and/or if a severe contingency occurs. The proposed methodology is a version of the Newton optimization method with step length control. The summation of the squared active and reactive power mismatches is used as the cost function. The efficiency of the proposed strategy is assessed with five power systems, the IEEE 14, 30 and 118 buses and two real networks of 749 and 1916 buses equivalent to the Brazilian South-Southeastern power system.

Keywords: Unsolvable power flow, optimal power flow, Newton optimization method, corrective power flow solutions.

1 INTRODUCTION

Under critical load conditions, the solutions of power flow equations coexist in a saddle node bifurcation (Dobson, 1992; Taylor, 1994). In this point, the system has a trend to both voltage instability and loss of the system steady state stability. If the load level is higher than the critical load, the power flow equations have no real solution, that is, the traditional methods (Newton-Raphson method, for instance) fail to find a power flow solution. This generally happens if the active and reactive loads reach too high levels or if a stressed system is submitted to a contingency. In this situation, the availability of efficient numerical tools to determine the modifications in the load is essential to restore the solution of the power flow equations.

From the point of view of solvability, the set of power flow solutions can be divided into two regions of the parameter space (Wu et alii, 1988; Mercede et alii, 1988; Alvarado et alii, 1991):

- *Solvable region* - is a set of operating points for which the power flow equations have real solutions. If the operation limits are satisfied the system is said to be in the *normal state*. If operation limits are violated the system is considered to be in the *emergency state*. Usually, it is possible to operate the system in the latter state for a short period of time. In the solvable (feasible) region the power flow equations have two solutions, only one of which is usable for power system operation.
- *Unsolvable region* - is a set of points for which the power flow equations have no real solutions. Naturally, it is not possible to operate the power system in this region. Nevertheless, if adequate changes in the controls are performed, the solution of power flow equations can be restored. Since many power systems operating close to the maximum loadability have been subject to these conditions, the development of methodologies to restore the solvability of the power flow equations has become essential in the power system steady state analysis.

The solvable and unsolvable regions are separated by a surface, denoted here by Σ, as in Overbye (1994), which can be interpreted as the boundary of the solvable and unsolvable regions. On this surface, there is only one solution for the power flow equations. This means that the two solutions of the solvable region vanish in a saddle node bifurcation, the system being subject to voltage instability.

Review of the power system literature shows that some attention has been given to this subject. In Overbye (1994) and Overbye (1995) is proposed the computation of a measure of the unsolvability with corresponding determination of the changes in the power system variables to restore the solution of the network equations. This method is based on both the load flow in cartesian coordinates with step length control (Iwamoto and Tamura, 1981) and the use of the left eigenvector associated with zero eigenvalue of the singular Jacobian matrix (Dobson and Lu, 1992). In Granville et alii (1996) a method to restore the solvability of the load flow equations based on optimization techniques is presented. The Primal-Dual Interior Point Method is used, with the minimum load shedding as the cost function. While in Overbye (1994) and Overbye (1995) only the constraints in the reactive power generation are modeled (recall that this approach uses the traditional power

Artigo Submetido em 22/12/1999
1a. Revisão em 27/06/2000
Aceito sob recomendação do Ed. Consultor Prof. Dr. José Luiz Resende Pereira

flow algorithm), in Granville et alii (1996) it is possible to model any operation constraint (active and reactive power generation limits, voltage magnitude limits, line flow limits, etc).

This paper presents an approach to solve the problem of the power flow solvability based on the combination of the simplicity of the steady state network equations with the efficiency of the Newton optimization method. The proposed methodology requires less computational effort as that of the methodologies in which the whole set of operational constraints is modeled (Interior Point based methods, for instance). Similarly to Overbye (1994) and Overbye (1995), in the proposed approach only the reactive power generation constraints are modeled. Once an initial solution is obtained, the violations in the other inequality constraints could be removed through conventional security analysis methods. The main feature of the proposed approach is the use of the Newton optimization method to find the least squares of the summation of the power flow mismatches. From the point of view of formulation of the constraints, this approach has the advantage of allowing the modeling of null injection buses and/or buses where the scheduled load must be rigorously satisfied.

This paper is organized as follows. First, the Newton optimization method is revised. Next, the problem least sum of squares of the power flow mismatches is formulated in terms of the power system variables. Finally, the proposed corrective strategy is assessed through the results obtained with 14, 30 and 118-bus network and two real systems with 749 and 1916 buses, equivalent to the Brazilian South-Southeastern power system.

2 NEWTON OPTIMIZATION METHOD

The optimization problem with equality constraints can be stated as (Sun et alii, 1984)

$$\begin{aligned} & Min \;\; f(\boldsymbol{x}) \\ & s.t. \;\; \boldsymbol{g}(\boldsymbol{x}) = \boldsymbol{0} \end{aligned} \tag{1}$$

where $f(\boldsymbol{x})$ is the objective function;
$\boldsymbol{g}(\boldsymbol{x})$ is the vector of the equality constraints;
$\boldsymbol{x}$ is the vector of the optimization variables.

The Lagrangean function corresponding to the equality constrained problem (1) is (Gill et alii, 1981; Luenberger, 1984)

$$£(\boldsymbol{x}, \boldsymbol{\lambda}) = f(\boldsymbol{x}) + \boldsymbol{\lambda}^T \boldsymbol{g}(\boldsymbol{x}) \tag{2}$$

where $\boldsymbol{\lambda}$ is a vector with the Lagrangean multipliers.

The application of the first order optimality conditions to equation (2) provides

$$\nabla_x £(\boldsymbol{x}, \boldsymbol{\lambda}) = \nabla_x f(\boldsymbol{x}) + [\mathbf{G}(\boldsymbol{x})]^T \boldsymbol{\lambda} = \boldsymbol{0} \tag{3}$$

$$\nabla_{\boldsymbol{\lambda}} £(\boldsymbol{x}, \boldsymbol{\lambda}) = \boldsymbol{g}(\boldsymbol{x}) = \boldsymbol{0} \tag{4}$$

where $\mathbf{G}(\boldsymbol{x}) = \dfrac{\partial \boldsymbol{g}(\boldsymbol{x})}{\partial \boldsymbol{x}}$.

The solution of the nonlinear equations (3) and (4) can be obtained through the Newton-Raphson method (Björck, 1990). At each iteration the following linear system is solved:

$$\mathbf{W}(\boldsymbol{x}, \boldsymbol{\lambda}) \Delta \boldsymbol{x} + [\mathbf{G}(\boldsymbol{x})]^T \Delta \boldsymbol{\lambda} = -\{\nabla_x f(\boldsymbol{x}) + [\mathbf{G}(\boldsymbol{x})]^T \boldsymbol{\lambda}\} \tag{5}$$

$$\mathbf{G}(\boldsymbol{x}) \Delta \boldsymbol{x} = -\boldsymbol{g}(\boldsymbol{x}) \tag{6}$$

where $\mathbf{W}(\boldsymbol{x},\ \boldsymbol{\lambda}) = \nabla_x^2 f(\boldsymbol{x}) + \sum_{i=1}^{m} \lambda_i \nabla_x^2 g_i(\boldsymbol{x})$.

In matrix notation, the linear system of equations (5) and (6) can be written as

$$\begin{bmatrix} \mathbf{W}(\boldsymbol{x}, \boldsymbol{\lambda}) & [\mathbf{G}(\boldsymbol{x})]^T \\ \mathbf{G}(\boldsymbol{x}) & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{x} \\ \Delta \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} -\{\nabla_x f(\boldsymbol{x}) + [\mathbf{G}(\boldsymbol{x})]^T \boldsymbol{\lambda}\} \\ -\boldsymbol{g}(\boldsymbol{x}) \end{bmatrix} \tag{7}$$

The linear system of equations (7) is large but with a high degree of sparsity, such that special techniques must be employed to factorize its coefficient matrix.

The new estimates of the optimization variables are

$$\begin{aligned} \boldsymbol{x}^{(k+1)} &= \boldsymbol{x}^{(k)} + \Delta \boldsymbol{x}^{(k)} \\ \boldsymbol{\lambda}^{(k+1)} &= \boldsymbol{\lambda}^{(k)} + \Delta \boldsymbol{\lambda}^{(k)} \end{aligned} \tag{8}$$

It is well known that the Newton method depends on how close the initial estimate is to the final solution. However, the evaluation of an adequate initial solution rarely is an easy task. Frequently, it is necessary to use other techniques in addition to the Newton method, to obtain fast and secure convergence of the iterative process. In the following section this aspect is discussed in details.

3 PROPOSED METHOD

The problem of restoring the solvability of the power flow equations can be stated as the minimization of the summation of the squares of the power flow mismatches subject to equality constraints, that is

$$\begin{aligned} & Min \;\; \frac{1}{2} [\boldsymbol{f}(\boldsymbol{x})]^T \boldsymbol{f}(\boldsymbol{x}) \\ & s.t. \;\; \boldsymbol{g}(\boldsymbol{x}) = \boldsymbol{0} \end{aligned} \tag{9}$$

where $\boldsymbol{f}(\boldsymbol{x})$ is the vector of the active power mismatches in PQ and PV buses and reactive power mismatches in PQ buses; $\boldsymbol{g}(\boldsymbol{x})$ is the vector of the active and reactive power mismatches in buses whose demand must be rigorously satisfied. Economical and technical criteria can be used to choose the components of the vector $\boldsymbol{g}(\boldsymbol{x})$. For instance, null injection buses compose the $\boldsymbol{g}(\boldsymbol{x})$ vector for technical reasons.

Therefore, the first order optimality conditions for problem (9) are

$$\nabla_x £(\boldsymbol{x}, \boldsymbol{\lambda}) = [\mathbf{F}(\boldsymbol{x})]^T \boldsymbol{f}(\boldsymbol{x}) + [\mathbf{G}(\boldsymbol{x})]^T \boldsymbol{\lambda} = \boldsymbol{0} \tag{10}$$

$$\nabla_{\boldsymbol{\lambda}} £(\boldsymbol{x}, \boldsymbol{\lambda}) = \boldsymbol{g}(\boldsymbol{x}) = \boldsymbol{0} \tag{11}$$

where $\mathbf{F}(\boldsymbol{x}) = \dfrac{\partial \boldsymbol{f}(\boldsymbol{x})}{\partial \boldsymbol{x}}$.

The submatrix $\mathbf{W}(\boldsymbol{x},\ \boldsymbol{\lambda})$ in equation (5) is given by

$$\mathbf{W}(\boldsymbol{x}, \boldsymbol{\lambda}) = [\mathbf{F}(\boldsymbol{x})]^T \mathbf{F}(\boldsymbol{x}) + \sum_{i=1}^{n} f_i(\boldsymbol{x}) \nabla_x^2 f_i(\boldsymbol{x}) + \sum_{j=1}^{m} \boldsymbol{\lambda}_j \nabla_x^2 g_j(\boldsymbol{x}) \tag{12}$$

or

$$\mathbf{W}(\boldsymbol{x}, \boldsymbol{\lambda}) = [\mathbf{F}(\boldsymbol{x})]^T \mathbf{F}(\boldsymbol{x}) + \mathbf{H}(\boldsymbol{x}, \boldsymbol{\lambda}) \tag{13}$$

The convergence of the iterative process is dependent on the initial estimates of the optimization variables. If the initial point is close to the final solution, the convergence is fast. Otherwise, the iterative process can have slow convergence or may even diverge. In order to improve the search for the optimal solution in the beginning of the iterative process, a Gauss-Newton direction is used in the first iterations. This implies in neglecting the second order term $\mathbf{H}(\boldsymbol{x}, \boldsymbol{\lambda})$ in equation (13), that is

$$\mathbf{W}(\boldsymbol{x},\boldsymbol{\lambda}) = [\mathbf{F}(\boldsymbol{x})]^T \mathbf{F}(\boldsymbol{x}) \tag{14}$$

At the subsequent iterations, Newton direction is used until the convergence is obtained. This strategy is based in the following observations:

- If the power flow equations have real solution, the minimum value of the objective function is zero, that is, $f_i(\boldsymbol{x}) = 0$, $i = 1, 2, ..., n$. Since at the optimal point the first order optimal conditions are satisfied, it can be concluded, from equation (10), that all components of vector $\boldsymbol{\lambda}$ are zero. Therefore, in equation (12) the summations become zero, and thus the submatrix $\mathbf{W}(\boldsymbol{x}, \boldsymbol{\lambda})$ is just $[\mathbf{F}(\boldsymbol{x})]^T \mathbf{F}(\boldsymbol{x})$.
- In the optimization problem considered here, the coefficient matrix in equation (7) is not positive definite. This means that the movement in the Newton direction does not necessarily decrease the value of the Lagrangean function.
- Rigorously, the second order terms should be included in $\mathbf{W}(\boldsymbol{x}, \boldsymbol{\lambda})$ during the complete iterative process. Nevertheless, in the case of high load levels, it is not possible to envisage if the power flow equations have real solution. Besides, in the solvable region the use of the Gauss-Newton direction results in a faster convergence to the optimal solution. Therefore, it is reasonable to combine these two directions, to obtain faster convergence.

In the present approach the reactive power generation limits are modeled as in the conventional power flow. At each iteration, these limits are verified. If the limit is violated, the PV bus is converted to PQ bus, and a new coefficient matrix and the gradient vector of the Lagrangean function are computed. Aiming at improving the iterative process, the modeling of reactive power generation limits requires the use of a step length control. Here, a strategy based on linear search is applied to calculate the step length. This technique consists of determining a scalar α, called *step factor*, so that the Lagrangean function value has a reasonable decrease at each iteration of the iterative process. The corrections of the optimization variables and Lagrange multipliers are given by

$$\begin{aligned} \boldsymbol{x}^{(k+1)} &= \boldsymbol{x}^{(k)} + \alpha\Delta \boldsymbol{x}^{(k)} \\ \boldsymbol{\lambda}^{(k+1)} &= \boldsymbol{\lambda}^{(k)} + \alpha\Delta \boldsymbol{\lambda}^{(k)} \end{aligned} \tag{15}$$

where $0 < \alpha \leq 1$.

4 LEFT EINGENVECTOR AND OPTIMAL POINT

The solution of the optimization problem expressed by equation (9) provides the optimal direction in the parameter space, in which the load curtailment must be done to restore the power flow solvability. In this direction, the smallest distance between the scheduled loading and the largest loading that can be supplied by the energy system is found. At convergence, the iterative process provides the left eigenvector associated with the zero eigenvalue of the singular Jacobian matrix of the conventional power flow. This can be seen from the first order optimality condition, equation (10). Let $\boldsymbol{x}^*$ and $\boldsymbol{\lambda}^*$ be the values at optimal solution. Hence

$$\nabla_{\boldsymbol{x}} \pounds(\boldsymbol{x}^*, \boldsymbol{\lambda}^*) = [\mathbf{F}(\boldsymbol{x}^*)]^T \boldsymbol{f}(\boldsymbol{x}^*) + [\mathbf{G}(\boldsymbol{x}^*)]^T \boldsymbol{\lambda}^* = \mathbf{0} \tag{16}$$

This equation can be rewritten as

$$\begin{bmatrix} \mathbf{F}(\boldsymbol{x}^*) \\ \mathbf{G}(\boldsymbol{x}^*) \end{bmatrix}^T \begin{bmatrix} \boldsymbol{f}(\boldsymbol{x}^*) \\ \boldsymbol{\lambda}^* \end{bmatrix} = \mathbf{0} \tag{17}$$

or

$$\begin{bmatrix} \boldsymbol{f}(\boldsymbol{x}^*) \\ \boldsymbol{\lambda}^* \end{bmatrix}^T \begin{bmatrix} \mathbf{F}(\boldsymbol{x}^*) \\ \mathbf{G}(\boldsymbol{x}^*) \end{bmatrix} = \mathbf{0} \tag{18}$$

Note that $[\mathbf{F}(\boldsymbol{x}^*)^T \quad \mathbf{G}(\boldsymbol{x}^*)^T]^T$ is the Jacobian matrix of the conventional power flow at the optimal solution. Therefore, $[\boldsymbol{f}(\boldsymbol{x}^*)^T \quad (\boldsymbol{\lambda}^*)^T]$ is more than the left eigenvector associated to zero eigenvalue of the singular Jacobian matrix. This vector defines the magnitude and direction in which the load must be curtailed to bring the power flow equations to the solvable region. In Dobson and Lu (1993) it is showed that the direction of the left eigenvector of the zero eigenvalue of the singular Jacobian matrix is parallel to the direction of the normal vector to the surface Σ at optimal solution. Figure 1 illustrates this fact for a hypothetical system composed of two buses.

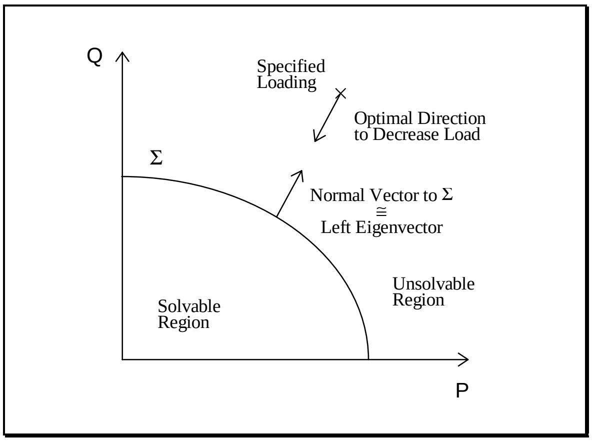

Figure 1. Relation between left eigenvector and normal vector in a hypothetical system.

Note in Figure 1, that the vector normal to the surface Σ at optimal solution points outward at the solvable region. Since the left eigenvector corresponding to zero eigenvalue of the Jacobian matrix has the same direction as the normal vector, the opposite direction can be used to realize load curtailments to restore power flow solvability.

Although the load curtailment based on the left eigenvector is theoretically possible, in realistic cases there are some difficulties to do so because this curtailment involves null injection buses, where any load curtailment is not possible.

5 NUMERICAL RESULTS

To assess the performance of the proposed methodology, tests with power systems of different sizes, ranging from 14 to 1916 buses (including two real networks), were performed. In this section, some general results for all systems and detailed results for a real network are shown. These results were obtained in a Pentium II, 300 MHz computer with 128 Mb of RAM. Three aspects were analyzed: the convergence of Newton's algorithm, the quality of the solutions in terms of load shedding and the performance with respect to the methodology proposed in Overbye (1994). These aspects are discussed in the following subsections.

5.1 Analysis of the Convergence

The study of the convergence of the proposed methodology was based on two points: the use of the Gauss-Newton (G-N) direction at the initial iterations and the application of a step length control. Both strategies aim at increasing the robustness of the algorithm. Tests with five power systems were performed: the IEEE 14, 30 and 118 buses and two reduced power networks of the Brazilian South-Southeastern region. The first of these real networks have 749 buses, 1275 transmission lines and 87 generators, whereas the second has 1916 buses, 2788 transmission lines and 153 generators. Table 1 shows the type of tests realized and Table 2 summarizes the general features of the convergence. In Table 2, 'nc' means non convergent and the number in parenthesis is the number of the G-N iterations.

Table 1. Type of tests.

	Strategy	
Test	G-N direction	Step length
A	no	no
B	yes	no
C	no	yes
D	yes	yes

Table 2. Convergence process for the tested systems.

System	Test A	Test B	Test C	Test D
14	8	5 (2)	8	7 (2)
30	7	6 (3)	7	6 (3)
118	nc	7 (2)	13	9 (2)
749	nc	10 (5)	nc	10 (5)
1916	nc	nc	nc	11 (5)

With respect to Table 2, the following must be remarked:

a) if the G-N direction is not used, convergence is obtained only for the 14 and 30 buses power networks. In the case of the 749 and 1916 bus power systems, this strategy is essential for obtaining convergence;
b) for all tested systems the step length control has influence on convergence. It is observed that, if the convergence is reached without using the step length control, the use of this strategy could result in an increase in the number of iterations (see columns 3 and 5 of Table 2). In case of the 118 buses power system the simple use of the step length control provides the convergence. In case of the 1916 bus power network, the use of the step length control is essential for obtaining convergence;
c) both strategies can improve the convergence performance.

Tables 3 and 4 show in detail the influence of the G-N direction and the step size control on convergence for the 749 bus power system. From these tables, it is observed that the use of the G-N direction can improve the convergence features of the iterative process. However, the increase in the number of G-N iterations does not necessarily imply in improving the convergence characteristic of the complete process. Note that if the number of G-N iterations increases from 5 to 6, the worst case of convergence is obtained.

Table 3. Convergence for test B in 749-bus system.

G-N iterations	1	2	3	4	5	6
Total iterations	nc	11	12	10	10	14
time (sec)		3.41	3.52	2.91	2.86	3.79

Table 4. Convergence for test D in 749-bus system.

G-N iterations	1	2	3	4	5	6
Total iterations	nc	9	8	9	10	11
time (sec)		3.02	2.64	2.91	3.02	3.13

From Table 3, the best feature of convergence corresponds to the cases in which 4 or 5 G-N iterations are used. Although the total number of iterations for convergence is the same in both cases (10 iterations), a small difference is noticed in terms of computing time. This is due to the number of Newton iterations performed. That is, if 4 (or 5) G-N iterations are used, 6 (or 5) Newton iterations are required for reaching the final solution. It must be pointed out that the computational effort required for a Newton iteration is greater than that corresponding to the G-N iteration.

From Tables 3 and 4, it is observed that in this case the use of the step length control generally reduces the number of iterations, and therefore the computing times. It is also observed that from the point of view of convergence, the best result is obtained by using 3 G-N iterations and step size control (Table 4).

5.2 Analysis of the Solutions

The solutions obtained through the proposed methodology were analyzed from two points of view: the amount of satisfied demand (or load shedding) and the number of buses chosen for load curtailment. Two situations were considered: 1) the load of every bus is subjected to curtailment; 2) buses of a specified region are guaranteed to have their load satisfied.

Table 5 presents the results for the 749-bus system, whose total demand is 27,464.50 MW and 10,239.68 MVAr. For this load level there is no real solution for the power flow equations. For technical reasons, this system has 307 buses in which the power injection must be assured. In Table 5, 'ngi' corresponds to the number of buses whose demand is guaranteed and 'npq' is the number of buses subject to load shedding. Besides, P_{satisf} and Q_{satisf} are the satisfied active and reactive power load, respectively, in real values and percentage of total load. From this table, it can be observed that:

a) if the load of all buses is available for curtailment (case 1), the total load shedding has the largest value. This is due to the nature of the objective function. The optimization process tends to attribute load curtailment to every bus, which results in a large amount of load shedding;

b) if the load of the buses of a given region is guaranteed to be satisfied (cases 2 and 3) it is noted that the total amount of satisfied load increases (columns 2 and 3);
c) furthermore, from columns 3 and 4 of this table, it is observed that the greater the number of buses with guaranteed power injection (the smaller the number of buses with demand subject to load shedding) the larger is the total amount of satisfied load.

Table 5. Results for 749-bus system.

Case	P_{satisf} MW and %	Q_{satisf} MVAr and %	ngi	npq
1	26,686.37 97.17%	9,971.44 97.38%	307	355
2	26,791.31 97.55%	9,994.41 97.60%	394	268
3	26,975.89 98.22%	10,058.12 98.23%	566	96

5.3 Accuracy of the Solutions

In order to have an idea about the accuracy of the solutions determined with the proposed methodology, these were compared to those obtained through the approach proposed in Overbye (1994). Two aspects were observed: the total amount of satisfied demand and the index, suggested in Overbye (1994), representing the distance from the best solution to schedule load. This index is defined by

$$d = \sqrt{[\boldsymbol{f}(\boldsymbol{x}^b) - \boldsymbol{S}]^T [\boldsymbol{f}(\boldsymbol{x}^b) - \boldsymbol{S}]} \tag{19}$$

where $\boldsymbol{x}^b$ is the best solution supplied by the algorithm;
$\boldsymbol{S}$ is the scheduled demand.

Table 6 illustrates the results obtained through these two techniques. It can be observed that, in terms of the two amounts considered, the solution determined with the proposed methodology is as accurate as that obtained through the technique proposed in Overbye (1994). A detailed analysis of the results reveals that the load curtailment at each bus suggested by these two methodologies is also similar. This shows the satisfactory level of accuracy of the solutions provided by the proposed approach.

Table 6. Numerical accuracy of the proposed method.

Methodology	P_{satisf} (MW)	Q_{satisf} (MVAr)	d
Proposed	26,343.06	9,817.03	0.7192
Overbye	26,348.83	9,815.30	0.7192

6 CONCLUSIONS

Corrective solutions for the network equations can be obtained through Newton's method, formulating the determination of the load curtailment as a least squares optimization problem. The main advantage of the proposed approach is the combination of the simplicity of the steady state power flow equations with the potentiality of Newton's method.

Numerical results show that Gauss-Newton iterations at the beginning of the iterative process (and in some cases the step length control) are essential for convergence.

The proposed technique allows to select buses for which the power injection must be guaranteed. This procedure shows that the total load shedding can be reduced if these buses are suitably chosen.

7 REFERENCES

Alvarado, F. L., Y. Hu, D. Ray, R. Stevenson & E. Cashman (1991). Engineering Foundations for the Determination of Security Costs. *IEEE Transactions on Power Systems*, Vol. 6, No. 3, pp. 1175-1182.

Björck, Å. (1990). *Least Squares Methods*. In Handbook of Numerical Analysis, Vol. 1, Ed. P. G. Ciarlet and J. L. Lions, Elsevier-North Holland.

Dobson, I. (1992). Observations on the Geometry of Saddle Node Bifurcation and Voltage Collapse in Electrical Power Systems. *IEEE Transactions on Circuits and Systems - I: Fundamental Theory and Applications*, Vol. 39, No. 3, pp. 240-243.

Dobson, I. & L. Lu (1992). Computing an Optimum Direction in Control Space to Avoid Saddle Node Bifurcation and Voltage Collapse in Electric Power Systems. *IEEE Transactions on Automatic Control*, Vol. 37, No. 10, pp. 1616-1620.

Dobson, I. & L. Lu (1993). New Methods for Computing a Closest Saddle Node Bifurcation and Worst Case Load Power Margin for Voltage Collapse. *IEEE Transactions on Power Systems*, Vol. 8, No. 3, pp. 905-913.

Gill, P. E., W. Murray & M. Wright (1981). *Practical Optimization*. Academic Press, London.

Granville, S., J. C. O. Mello & A. C. G. Melo (1996). Application of Interior Point Methods to Power Flow Unsolvability. *IEEE Transactions on Power Systems*, Vol. 11, No. 2, pp. 1096-1103.

Iwamoto, S. & Y. Tamura (1981). A Load Flow Calculation Method for Ill-Conditioned Power Systems. *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-100, No. 4, pp. 1736-1743.

Luenberger, D. G. (1984). *Linear and Nonlinear Programming*. Addison-Wesley Publishing Company, Massachusetts.

Mercede, F., J. C. Chow, H. Yan & R. Fischl (1988). A Framework to Predict Voltage Collapse in Power Systems. *IEEE Transactions on Power Systems*, Vol. 3, No. 4, pp. 1807-1813.

Overbye, T. J. (1994). A Power Flow Measure for Unsolvable Cases. *IEEE Transactions on Power Systems*, Vol. 9, No. 3, pp. 1359-1365.

Overbye, T. J. (1995). Computation of a Practical Method to Restore Power Flow Solvability. *IEEE Transactions on Power Systems*, Vol. 10, No. 1, pp. 280-287.

Sun, D. I., B. Ashley, B. Brewer, A. Hughes & W. Tinney (1984). Optimal Power Flow Solutions by Newton Approach. *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-103, No. 10, pp. 2864-2880.

Taylor, C. W. (1994). *Power System Voltage Stability*. McGraw-Hill LTD, New York.

Wu, F. F., Y. K. Tsai & Y. X. Yu (1968). Probabilistic Steady-State and Dynamic Security Assessment. *IEEE Transactions on Power Systems*, Vol. 3, No. 1, pp. 1-9.