NUMERICAL STUDY OF THE THREE-DIMENSIONAL CLOSE-APPROACH

Gislaine de Felipe and Antonio Fernando Bertachini de Almeida Prado

Instituto Nacional de Pesquisas Espaciais - São José dos Campos - SP - 12227-010 – Brazil - Phone 55-(12)345-6201 - Fax 55-(12)345-6226 – gislaine@dem.inpe.br; prado@dem.inpe.br

Abstract : In the present paper the swing-by maneuvers are studied under the model given by the three-dimensional circular restricted three-body problem. A numerical algorithm to study this problem is build and used to generate several results. The main goal is to study the variation of the inclination in the trajectory of a spacecraft that performs this maneuver. The results shows that: i) for the planar maneuvers the variation in inclination can assume only the values $\pm 180^o$ and 0º; ii) for the polar maneuver, or for maneuvers with angle of approach α = 0º or 180º, the variation in inclination is zero. The effects of an out-of-plane component for the velocity at periapsis in the variation of the inclination, energy and angular momentum of the spacecraft are also described in details. This research has applications to design interplanetary missions.

Resumo : No presente trabalho a manobra de "Swing-By" é estudada com o modelo dado pelo problema restrito de três corpos circular e tri-dimensional. Um algoritmo numérico é construído para estudar esse problema e ele é utilizado para gerar diversos resultados. O objetivo principal é estudar a variação da inclinação na trajetória de um veículo espacial que executa uma manobra desse tipo. Os resultados mostram que: i) para a manobra plana a variação na inclinação pode assumir somente os valores 180º e 0º; ii) para a manobra polar, ou para manobras com ângulo de aproximação a = 0º ou 180º, a variação na inclinação é zero. Os efeitos de uma componente fora do plano para a velocidade do periapse na variação da inclinação, energia e momento angular do veículo especial são descritos em detalhes. Esta pesquisa tem aplicações para planejar missões interplanetarias.

Keywords: Swing-By,Restricted problem,Orbital maneuvers, Astrodynamics.

Palavras Chave: Passagem próxima,Problema restrito, Manobrasorbitais, Astrodinâmica.

1 INTRODUCTION

The swing-by maneuver is a very popular technique used to decrease fuel expenditure in space missions. The most usual approach to study this problem is to divide the problem in three phases dominated by the "two-body" celestial mechanics. Other models used to study this problem are the circular restricted three-body problem Broucke (1988 e 1993), Prado (1993) and the elliptic restricted three-body problem (Prado, 1997). In particular, references (Broucke 1988) and Brouche (1993) have numerical simulations for the planar maneuvers, classifying the orbits according to the effects caused by the close-approach maneuvers. A very complete set of numerical simulations of the planar restricted three-body can be found in (Winter, 1994a) and (Winter, 1994b). Another important problem that can be studied with numerical simulations of the restricted three-body problem in astronautics is the problem of gravitational capture. Some results in this problem are available in (Yamakawa, 1992) and (Vieira Neto, and Prado, 1998). Regarding applications of the swing-by maneuver, some examples are: the study of missions to the satellites of the giant planets (D'Amario, Byrnes, and Stanford, 1982); new missions to Neptune (Swenson, 1992) and Pluto (Weinstein, 1992); the study of the Earth's environment (Farquhar, and Dunham, 1981; Farquhar, Muhonen, and Church, 1985); fast reconnaissance missions of the solar system (Flandro, 1966; Carvell, 1986); and transfers between hyperbolic asymptotes (Gobetz 1963; Walton 1975).

The present paper comes in the sequence of the literature and numerical simulations are made in the three-dimensional restricted three-body problem, with the primary goal of studying the behavior of the inclination in this maneuver. The assumptions made here are: i) the system is formed by two main bodies that are in circular orbits around their center of mass; ii) a massless third body (the spacecraft) is moving in the three-dimensional space under the gravitational attraction of the two primaries; iii) the swing-by is performed around the secondary body of the system; iv) when the spacecraft is far from the secondary body, the system primary-spacecraft can be considered a two-body system.

The main contribution of this paper is to simulate a complete set of initial conditions for those orbits and measure the effects caused by the close approach in the orbit of the spacecraft. To perform this task, the equations of motion are integrated numerically forward and backward in time, until the spacecraft is at a distance that can be considered far enough from M_2. It is necessary to integrate in both directions of time because the set of initial conditions used gives information about the spacecraft exactly at the moment of the closest approach. At the two points where the spacecraft is considered far from M_2, the effect of M_2 can be neglected and the system formed by M_1 and the spacecraft can be considered a two-body system. So, the

Artigo Submetido em 05/05/00
1a. Revisão em 29/08/00; 2a. Revisão em 03/09/01
Aceito sob recomendação do Ed. Consultor Prof. Dr. Liu Hsu

two-body celestial mechanics formulas are valid to compute the energy, angular momentum and inclination. From those simulations the effects of the swing-by maneuver are calculated, analyzed and explained based in the physical model.

2. THE SWING-BY IN THREE DIMENSIONS

The swing-by maneuvers in three dimensions consists in studying the motion of a massless spacecraft passing close to o celestial body M_2, that is the smaller body of a system of primaries M_1-M_2, as shown in Fig.1.

It is assumed that the system has three bodies: a primary (M_1) and a secondary (M_2) body with finite masses that are in circular orbits around their common center of mass and a third body with negligible mass (the spacecraft) that has its motion governed by the two other bodies. The spacecraft leaves the point A, passes by the point P (the periapsis of the trajectory of the spacecraft in its orbit around M_2) and goes to the point B. The points A and B are chosen in such a way that the influence of M_2 at those two points can be neglected and, consequently, the energy can be assumed to remain constant after B and before A (the system follows the two-body celestial mechanics). Among the several sets of initial conditions that can be used to identify uniquely one swing-by trajectory, the following five variables are used (see Fig. 1): V_p, the velocity of the spacecraft at periapsis of the orbit around the secondary body; two angles (α and β) that specify the direction of the periapsis of the trajectory of the spacecraft around M_2 in a three-dimensional space; r_p the distance from the spacecraft to the center of M_2 in the moment of the closest approach to M_2 (periapsis distance); γ, the angle between the velocity vector at periapsis and the intersection between the horizontal plane (a plane parallel to the x-y plane) that passes by the periapsis and the plane perpendicular to the periapsis that holds $\vec{V}_p$.The distance r_p is not to scale, to make the figure easier to understand. The result of this maneuver is a change in velocity, energy, angular momentum and inclination in the Keplerian orbit of the spacecraft around the central body. Using the "patched conic" approximation, the equations that quantify those changes are available in the literature (Broucke, 1988). Under this approximation the maneuver is considered as composed of three parts, where each of those systems are governed by the two-body celestial mechanics. The first system describes the motion of the spacecraft around the primary body before the close encounter (the secondary body is neglected). When the spacecraft comes close to the secondary body, the primary is neglected and a second two-body system is formed by the spacecraft and the secondary body. After the close encounter the spacecraft leaves the secondary body, and it goes to an orbit around the primary body again. Then, the secondary is neglected one more time. The most important equations for the planar maneuver under this model are:

$$\delta = \sin^{-1}\left(1\Big/\left(1+\frac{r_p V_{inf}^2}{\mu_2}\right)\right), \tag{1}$$

$$\Delta V = 2V_{inf}\sin\delta\,, \tag{2}$$

$$\Delta E = \omega\Delta C = -2V_2 V_{inf}\sin\delta\sin\alpha\,, \tag{3}$$

where δ is half of the total deflection angle of the trajectory of the spacecraft, V_2 is the linear velocity of M_2 in its motion around the center of mass of the system M_1-M_2, V_{inf} is the velocity of the spacecraft with respect to M_2 when the approach

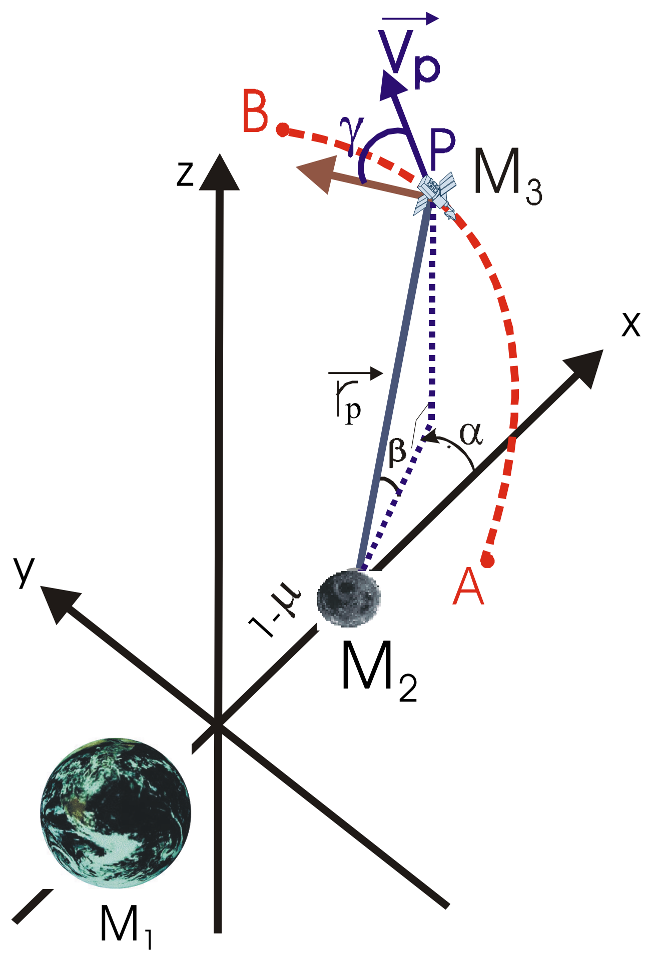


Fig. 1 - The Swing-By in Three Dimensions

starts, and μ_2 is the gravitational parameter of M_2. From Equations 1-3 it is possible to get the fundamental well-known results: a) the variation in energy (ΔE) is equal to the variation in angular momentum multiplied by the angular velocity of the primaries ($\omega\Delta C$) (Eq. 3); b) if the Fly-By is in front of the secondary body, there is a loss of energy, and this loss has a maximum at $\alpha = 90°$; c) if the Fly-By is behind the secondary body, there is a gain of energy, and this gain has a maximum at $\alpha = 270°$.

Equations (1) to (3) use V_{inf} as an independent parameter. Later in this paper the variable V_p will be used. The fact is that both parameters are equivalent, since the orbit around M_2 is considered Keplerian (Hyperbolic) in the approximation used to derive those equations ("patched-conics"). They are related by the expression

$$V_{inf}^2 = V_p^2 - (2\mu/r_p).$$

3. THE THREE-DIMENSIONAL CIRCULAR RESTRICTED PROBLEM

For the research performed in this paper, the equations of motion for the spacecraft are assumed to be the ones valid for the well-known three-dimensional restricted circular three-body problem. The standard dimensionless canonical system of units is used, which implies that: the unit of distance is the distance between M_1 and M_2; the mean angular velocity (ω) of the motion of M_1 and M_2 is assumed to be one; the mass of the smaller primary (M_2) is given by $\mu = m_2/(m_1+m_2)$ (where m_1 and m_2 are the real masses of M_1 and M_2, respectively) and the mass of M_1 is (1-μ); the unit of time is defined such that the period of the motion of the two primaries is 2π and the gravitational constant is one.

There are several systems of reference that can be used to describe the three-dimensional restricted three-body problem (Szebehely, 1967). In this paper the rotating system is used. In the rotating system of reference, the origin is the center of mass of the two massive primaries. The horizontal axis (x) is the line that connects the two primaries at any time. It rotates with a variable angular velocity in a such way that the two massive primaries are always on this axis. The vertical axis (y) is perpendicular to the (x) axis. In this system, the positions of the primaries are: $x_1 = -\mu$, $x_2 = 1-\mu$, $y_1 = y_2 = 0$. In this system, the equations of motion for the massless particle are (Szebehely, 1967):

$$\ddot{x} - 2\dot{y} = x - (1-\mu)\frac{x+\mu}{r_1^3} - \mu\frac{x-1+\mu}{r_2^3} \tag{4}$$

$$\ddot{y} + 2\dot{x} = y - (1-\mu)\frac{y}{r_1^3} - \mu\frac{y}{r_2^3}, \tag{5}$$

$$\ddot{z} = -(1-\mu)\frac{z}{r_1^3} - \mu\frac{z}{r_2^3}, \tag{6}$$

where r_1 and r_2 are the distances from M_1 and M_2.

4. ALGORITHM TO SOLVE THE PROBLEM

A numerical algorithm to solve the problem has the following steps: 1) Arbitrary values for the parameters r_p, V_p, α, β and γ are given; 2) With these values the initial conditions in the rotating system are computed. The initial position is the point (X_i, Y_i, Z_i) and the initial velocity is (V_{xi}, V_{yi}, V_{zi}), where:

$$X_i = 1 - \mu + r_p \cos(\beta)\cos(\alpha), \tag{7}$$

$$Y_i = r_p \cos(\beta)\sin(\alpha), \tag{8}$$

$$Z_i = r_p \sin(\beta), \tag{9}$$

$$V_{Xi} = -V_p \sin(\gamma)\sin(\beta)\cos(\alpha) - V_p \cos(\gamma)\sin(\alpha) + r_p \cos(\beta)\sin(\alpha), \tag{10}$$

$$V_{Yi} = -V_p \sin(\gamma)\sin(\beta)\sin(\alpha) + V_p \cos(\gamma)\cos(\alpha) - r_p \cos(\beta)\cos(\alpha), \tag{11}$$

$$V_{Zi} = V_p \cos(\beta)\sin(\gamma) \tag{12}$$

3) With these initial conditions, the equations of motion are integrated forward in time until the distance between M_2 and the spacecraft is larger than a specified limit d. At this point the numerical integration is stopped and the energy (E_+) and the angular momentum (C_+) after the encounter are calculated; 4) Then, the particle goes back to its initial conditions at the point P, and the equations of motion are integrated backward in time, until the distance d is reached again. Then the energy (E_-) and the angular momentum (C_-) before the encounter are calculated.

For all of the simulations shown, a fourth-order Runge-Kutta method with step size control and a Runge-Kutta of 8-th order was used for numerical integration. The result of this comparison is that there is no distinction in the plots obtained. The constant value for the Jacobian constant also is a proof that both numerical integration methods worked very well. The criteria to stop numerical integration is the distance between the spacecraft and M_2. When this distance reaches the value d = 0.5 (half of the semimajor axis of the two primaries) the numerical integration is stopped. The value 0.5 is a lot larger than the sphere of influence of M_2 for the Earth-Moon system, that is used here (which is, 0.00077 in canonical units), which avoids any important effects of M_2 at these points. Simulations using larger values for this distance were performed, and it increased the integration time, but did not significantly change the results. To study the effects of numerical accuracy, several cases were simulated using different integration methods and/or different values for the accuracy required with no effects in the results. All of the calculations were performed with an IBM-PC computer (Pentium 233Mhz) using the Microsoft FORTRAN Power Station 4.0 Compiler.

5. NUMERICAL SIMULATIONS

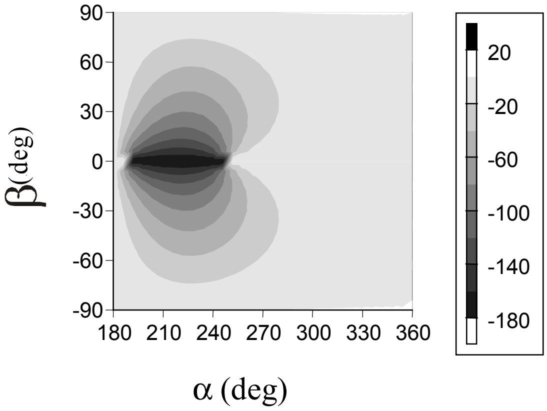
5.1 Effects on the inclination for γ = 0

An interesting question that appears in this problem is what happens to the inclination of the spacecraft due to the close approach. To investigate this fact the inclination of the trajectories were calculated before and after the closest approach. To obtain the inclinations the equation $\cos(i) = Cz/C$ is used, where C_z is the Z-component of the angular momentum and C is the total angular momentum. Fig. 2 shows results for a series of initial conditions, considering the case γ = 0. This constraint is assumed, because it is the most usual situation in interplanetary research, since the planets have orbits that are almost coplanar. The horizontal axis represents the angle α, and the vertical axis represents the angle β. The variation in inclination is shown in the contour plots. All the angles are expressed in degrees.

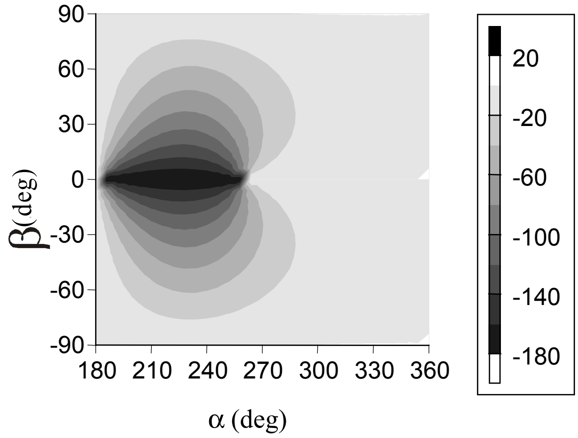
Several conclusions come from those results. The most interesting ones are: i) when β = 0º (planar maneuver) the variation in inclination can have only three possible values: $\pm 180^o$, for a maneuver that reverse the sense of its motion, or 0º for a maneuver that does not reverse its motion. Those numerical results agree with the physical-model, since the fact that β = 0º implies in a planar maneuver that does not allow values for the inclination other than 0º or 180º. This is clearly shown in the figures, following the line β = 0º. The plots are divided in two parts: one with $\Delta i = \pm 180^o$ and the other one with $\Delta i = 0^o$; ii) Looking at any vertical line (a line of constant α) it is clear that the change in inclination goes to zero at the poles $(\beta = \pm 90^o)$. Then, in the case where $\Delta i = \pm 180^o$, the change in inclination starts at zero in β = -90º, increases in magnitude until β = 0º and then it starts decreasing again until zero when β = 90º is reached. When Δi = 0º for β = 0º the behavior of Δi oscillates, with two maximum for the magnitude (one in the interval –90º < β < 0º and the other in the interval 0º < β < 90º) and three zeros at β = -90º, 0º, 90º. It is also clear that the variation in inclination is symmetric with respect to the angle β (+β and –β generate the same Δi); iii) when $\beta = \pm 90^o$ the variation in inclination is very close to zero. It means that a passage by the poles with the velocity parallel to the X-Y keeps the inclination of the trajectory unchanged; iv) when α = 0º or α = 180º there is no change in the inclination. This is in agreement with the fact that a maneuver with this geometry does not change the trajectory at all. Looking at any horizontal line (a line of constant β) it is visible that this curve has a maximum in the magnitude of Δi somewhere between the two fixed zeroes at α = 0º and α = 180º; v) when the periapsis distance or the velocity at periapsis increases, the effects of the swing-by in the maneuver are reduced. In the plots shown, this can be verified by the fact that the area of the regions where the variation in inclination is close to zero increases.

5.2 Effects of the out-of-plane velocity at periapsis

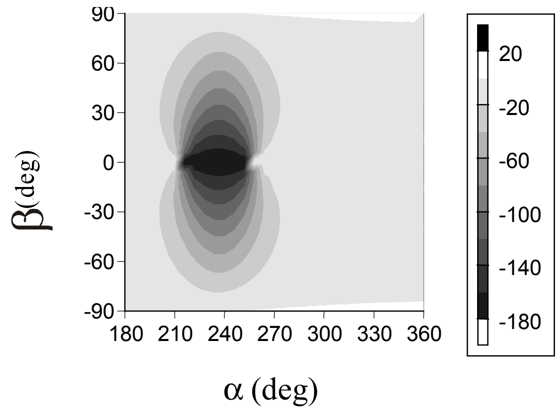
In this section, the study of the swing-by maneuver is extended to consider non-zero values for the out-of-plane component of the velocity at periapsis. It means that the angle γ shown in Fig.1 is no longer zero. To perform this task the variation in energy was calculated and plotted in Fig. 3 as a function of γ. It is possible to see that the effects of the variation in γ cause a sinusoidal periodic oscillation. The amplitude of this oscilation depends on the initial conditions, but it is never greater than 0.04 canonical units of energy. The maximums and minimums of those oscillations are also dependent on the initial conditions. The variation in inclination is shown in Fig 4. The results show that this angle plays a very important rule in the



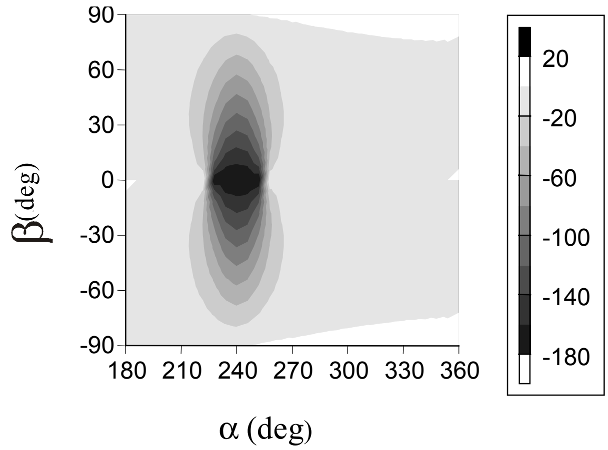
$R_p = 0.00675$ e $V_p = 2.2$



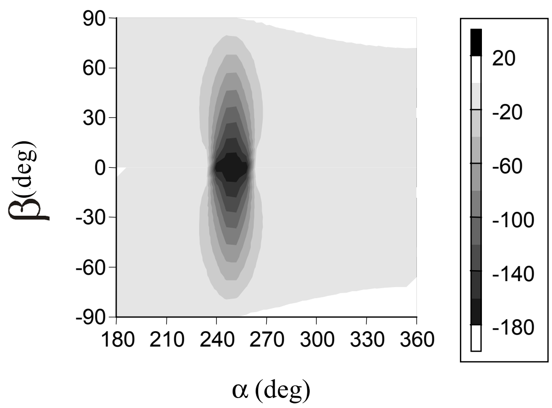
$R_p = 0.00476$ e $V_p = 2.6$



$R_p = 0.00675$ e $V_p = 2.6$



$R_p = 0.009$ e $V_p = 2.6$



$R_p = 0.00675$ e $V_p = 3.4$

Fig. 2 – Inclination chance resulting from a close approach.

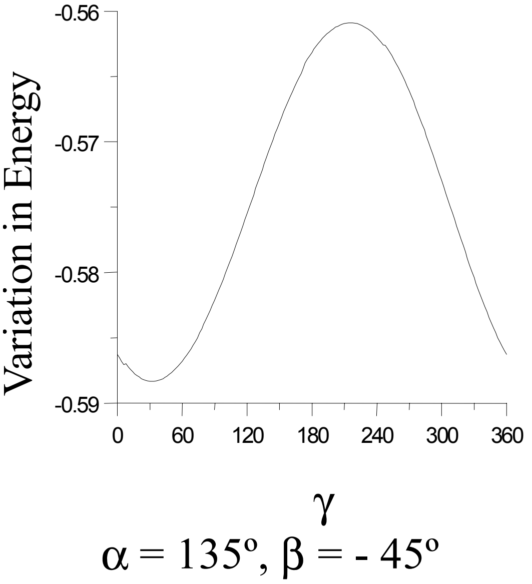


Fig. 3 – Variation in Energy vs. γ.

maneuver. Simulations were made for the cases α = 180º, β = 0º; α = 180º, β = 90º; α = 360º, β = 0º, but the figures are omitted here because the variation in inclination was zero for all the the values of γ. Fig. 4 shows some results. The characteristics of this problem, are: i) The variation in inclination is very small (less then 3º for any value of γ) when the passage occurs at the poles (β = ± 90º); ii) Looking at intermadiate values, like β = ± 45º, it is visible the symmetry that occurs both for the values α = 180º and 360º. The values for the variation in inclination for γ = 180º + Δ (0º < Δ < 180º) and 180º - Δ have the same magnitude and opposite signs; iii) For β = ± 45º, it can be seen that the variation in inclination for γ and - γ (= 360º - γ) have the same magnitude and opposite signs between the two figures for α = 135º and 225º; iv) For β = 0º, there is a symmetry with respect to γ = 180º; v) For α = 270º and β = 45º and β = – 45º there is a symmetry where the values for the variation in inclination for the range 0º ≤ γ ≤ 180º are the same ones for the range 180º ≤ γ ≤ 360º between the two figures for β = 45º and β = - 45º.

6. CONCLUSIONS

In this paper the three-dimensional restricted three-body problem is described and used to study the swing-by maneuver. The effects of the close approach in the inclination of the spacecraft is studied and the results show several particularities, like: β = 0º allows only $\pm 180^o$ and 0º for Δi, $\beta = \pm 90^o$ or α = 0º or 180º implies in Δi = 0º, etc. The effects of an out-of-plane component for the velocity at periapsis were also studied and simulations showed its importance, changing the values for the variation in inclination, energy and angular momentum, as described in the plots. In this way, this research can be used by mission designers to obtain specific mission goals.

7. ACKNOWLEDGMENT

The authors are grateful to the National Council for Scientific and Technological Development (CNPq), Brazil, for the research grants received under Contract 300221/95-9 and to the Foundation to Support Research in the São Paulo State (FAPESP) for the research grants received under Contracts 1995/9290-1 and 97/13739-0.

8. REFERENCES

Broucke, R.A. and Prado, A.F.B.A., 1993, "Jupiter Swing-By Trajectories Passing Near the Earth," Advances in the Astronautical Sciences, Vol. 82, No 2, pp. 1159-1176.

Broucke, R.A., Aug. 1988. "The Celestial Mechanics of Gravity Assist," AIAA Paper 88-4220.

Carvell, R. ,1986, "Ulysses-The Sun From Above and Below," Space, Vol. 1, pp. 18-55.

D'Amario, L.A., Byrnes, D.V. and Stanford, R.H., "Interplanetary Trajectory Optimization with Application to Galileo," Journal of Guidance, Control, and Dynamics, Vol. 5, No. 5, 1982, pp. 465-471.

Farquhar, R., Muhonen, D. and Church, L.C., 1985, "Trajectories and Orbital Maneuvers for the ISEE-3/ICE Comet Mission," Journal of the Astronautical Sciences, Vol. 33, No. 3, pp. 235-254.

Farquhar, R.W. and Dunham, D.W., 1981,"A New Trajectory Concept for Exploring the Earth's Geomagnetic Tail," Journal of Guidance, Control and Dynamics, Vol. 4, No. 2, pp. 192-196.

Flandro, G., 1966, "Fast Reconnaissance Missions to the Outer Solar System, Utilizing Energy Derived from the Gravitational Field of Jupiter," Astronautical Acta, Vol. 12, No. 4, pp. 329-337.

Gobetz, F.W., 1963, "Optimum Transfers between Hyperbolic Asymptotes," AIAA Journal, Vol. 1, No. 9, pp. 2034-2041.

Prado, A.F.B.A. , 1997, “Close-approach Trajectories in the Elliptic Restricted Problem”, Journal of Guidance, Control, and Dynamics, Vol. 20, No. 4, pp. 797-802.

Prado, A.F.B.A., 1993. "Optimal Transfer and Swing-By Orbits in the Two- and Three-Body Problems," Ph.D. Dissertation, Dept. of Aerospace Engineering and Engineering Mechanics, Univ. of Texas, Austin, TX, Dec.

Swenson, B.L., Aug. 1992, "Neptune Atmospheric Probe Mission," AIAA Paper 92-4371.

Szebehely, 1967, V., Theory of Orbits, Academic, New York, Chap. 10.

Vieira Neto, E. and Prado, A.F.B.A., (Jan-Feb/98), “Time-of-Flight Analyses for the Gravitational Capture Maneuver”. Journal of Guidance, Control and Dynamics, Vol. 21, No. 1, pp. 122-126.

Walton, J. M., Marchal, C., and Culp, R.D. , 1975, "Synthesis of the Types of Optimal Transfers between Hyperbolic Asymptotes," AIAA Journal, Vol.13, No. 8, pp. 980-988.

Weinstein, S.S. , Aug. 1992., "Pluto Flyby Mission Design Concepts for Very Small and Moderate Spacecraft," AIAA Paper 92-4372

Winter, O. C. and C. D. Murray (1994a). Atlas of the Planar, Circular, Restricted Three-Body Problem. I. Internal Orbits. QMW Maths Notes No. 16, Queen Mary and

Westfield College, Mile End Road, London E1 4NS, U. K.

Winter, O. C. and C. D. Murray (1994b). Atlas of the Planar, Circular, Restricted Three-Body Problem. II. External Orbits. QMW Maths Notes No. 17, Queen Mary and Westfield College, Mile End Road, London E1 4NS, U. K.

Yamakawa, H., 1992. On Earth-Moon Transfer Trajectory with Gravitational Capture. PhD. Dissertation, (University of Tokyo)

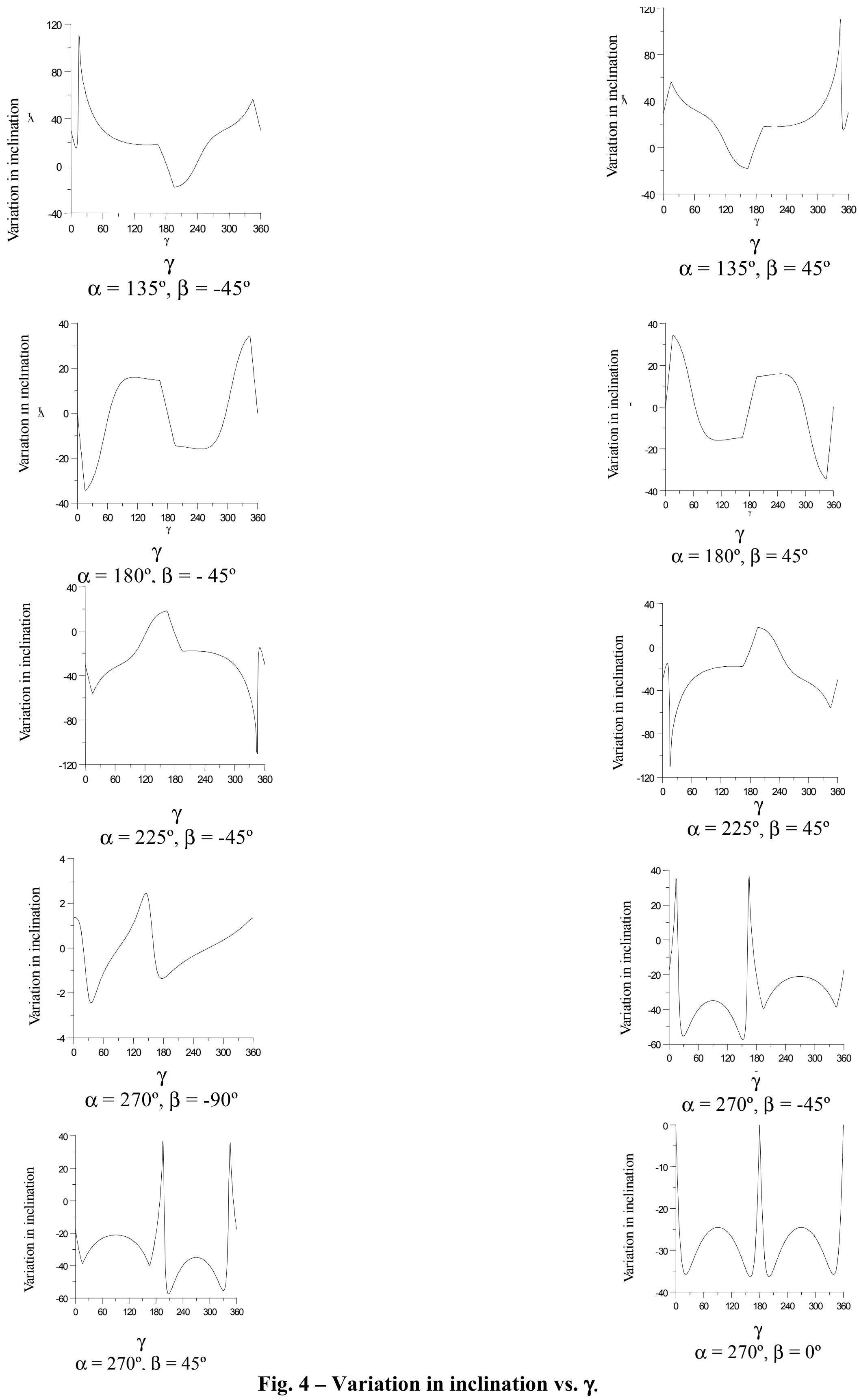


Fig. 4 – Variation in inclination vs. γ.