SLIDING MODE FOR DETECTION AND ACCOMMODATION OF INPUT-DELAY FAULT IN UNCERTAIN SYSTEM

JOSÉ P. F. GARCIA, LIZETE M. C. F. GARCIA, FERNANDO B. RODRIGUES, GISELE C. APOLINÁRIO

Laboratório de Pesquisa em Controle, Departamento de Engenharia Elétrica, UNESP, Universidade Estadual Paulista, Faculdade de Engenharia de Ilha Solteira, Avenida Brasil, 56, 15385-000, Ilha Solteira, SP.

E-mails: jpaulo@dee.feis.unesp.br, lizetega@mat.feis.unesp.br, f.barrosrodrigues@gmail.com, giapolinario@yahoo.com.br

Abstract—Input-delay in system reduces a robustness and also degrades its performance. In this paper the input-delay in a control system is considered as a fault to be detected and the controllers are adapted to improve the performance of the faulty system. For this purpose, it is presented two sliding mode control designs: a sliding mode control that doesn’t consider input-delay is reviewed and a sliding mode controller which takes into account the input-delay is presented. A strategy for fault detection and controller adaptation is proposed, based in the residual functions generated by sliding mode observers. An example is given to illustrate the design procedure and the effectiveness of the method.

Keywords—Sliding Mode Control, Uncertain Input-delay System, Fault Detection.

Resumo—Atraso na entrada em sistemas reduz a robustez e também degrada a sua performance. Neste trabalho o atraso na entrada em um sistema de controle é tratado como uma falta a ser detectada e os controladores são adaptados para melhorar o desempenho do sistema faltoso. Para este propósito dois projetos de controle com modos deslizantes são apresentados: um controle com modos deslizantes que não leva em consideração o atraso na entrada e um controlador com modos deslizantes que leva em consideração o atraso na entrada. É proposta uma estratégia para detecção de falta, baseada em funções resíduos gerada por observadores com modos deslizantes. Um exemplo é dado para ilustrar o procedimento de projeto e para mostrar a eficácia do método.

Palavras-chave—Controle com Modo Deslizante, Sistema Incerto com Atraso na Entrada, Detecção de Falha.

1 Introdução

Several types of time-delay systems are found such as hydraulic transmission systems, pneumatics/mechanics systems, and thermal systems. In remote control systems are also common the delay due to communication system. In these types of systems, the outputs will not response to input before the time delay occurs.

In general, the closed loop systems with delays are subject to more stability problems than the systems without delays, independent of the control strategy to be used (Choi and Chung, 1995).

Many authors deal with the control problem of the time-delay systems via predictor-based controllers (Kojima; Uchida; Shimemura and Ishijima, 1994; Roh and Oh, 2000). Predictor-based controllers include a predictor to compensate the time delay or, at least, to minimize the effect. For a design of a predictor-based controller, the system with delay can be transformed into a delay-free system in which the delay is compensated from the closed loop system.

In (Garcia and Bennaton, 2002) is used a sliding mode control for the robust stabilization of uncertain input-delay systems with only plant output access. It is used a robust discontinuous input-delay observer. The predictive states are obtained from estimate states. A control law using the predictive estimate states is proposed to ensure the existence of sliding mode.

In this paper the input-delay is treated as a fault to be detected and appropriate controller is used to minimize its effects. To detect the input-delay fault occurrence, robust sliding mode observers are used (Edwards and Spurgeon, 1994) to generated the residues, that are the estimates error with respect the actual output of the controlled plant. To adapt the controller to the fault condition, a logic using comparison of the residuals functions is proposed.

The paper is organized as it follows. In Section 2, it is reviewed a predictor description that turns the input-delay system free of delay (Roh and Oh, 2000). A sliding mode controller, which takes into account the computation time delay, is presented in Section 3 (Garcia and Bennaton, 2002; Ribeiro; Garcia, Jacomeli and Garcia, 2006). In Section 4, it is summarized a robust sliding mode observer (Edwards and Spurgeon, 1994). A fault detection and controller adaptation technique is proposed in Section 5. In Section 6 an example is given to prove the robustness of the proposed controller and the effectiveness of input-time delay fault detection and adaptation scheme. The conclusions are in Section 8.

2 Predictor Description

First, it is considered an uncertain input-delay system with full state access, described by

\[ \dot{x}(t) = Ax(t) + Bu(t-h) + f(t,x(t)) \]

\[ x(0) = x_0, \ u(\theta) = u(t+\theta), \ u_c(\theta) = \Psi(\theta), \ -h \leq \theta \leq 0 \]

where \( x \in \mathbb{R}^n \), \( u \in \mathbb{R}^m \) and \( 0 \leq h < \infty \) are the state vector, the input vector and the delay known, respectively. \( A \in \mathbb{R}^{n \times n} \), \( B \in \mathbb{R}^{n \times m} \) are constants matrices.

The function \( f(t,x(t)) \) models uncertainties and nonlinearities in the system and

\[ f(t,x(t)) = \sum_{i=1}^{n} h_i(x_i(t),x_{i+1}(t)) \]

\[ h_i(x_i(t),x_{i+1}(t)) = x_i(t) x_{i+1}(t) \]

\[ h_i(x_i(t),x_{i+1}(t)) = x_i(t) + x_{i+1}(t) \]

\[ h_i(x_i(t),x_{i+1}(t)) = x_i(t) - x_{i+1}(t) \]

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\[ h_i(x_i(t),x_{i+1}(t)) = x_i(t) - x_{i+1}(t) \]

\[ h_i(x_i(t),x_{i+1}(t)) = x_i(t) x_{i+1}(t) \]
\[\Psi(\theta) \in C([-h,0], \mathbb{R}^n), \] that is, \(\Psi(\theta)\) is a real continuous function of \(\theta \in [-h,0]\).

The predictor of states is given by
\[
x_p(t) = e^{at} x(t) + \int_{t-h}^{t} e^{a(t-\tau)} Bu(t+h)d\tau
\]  \(\tag{2}\)

**Remark 1.** Consider the system (1) with \(f(.,\cdot) : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n\), satisfying the matching condition
\[f(t,x(t)) = B\xi(t,x(t))\]
such that \(|\xi| \leq \rho\), \(\xi \in \mathbb{R}^n\) and \(\rho > 0\). It is also assumed that \(x_p(t)\) is given by (2). Then, the free-delay system is (Garcia and Bennaton, 2002; Roh and Oh, 2000)
\[
\dot{x}_p(t) = Ax_p(t) + Bu(t) + e^{at}B\xi(t,x(t)).
\]  \(\tag{3}\)

**Remark 2.** If a stabilizing controller exists for the original system (1), \(x(t)\) and \(u(t)\) go to zero. Then, \(x_p(t)\) of (2) goes to zero, that means the system (3) is stable. Conversely, if a stabilizing controller exists for the system (3), then \(x_p(t)\) and \(u(t)\) go to zero and thus does \(x(t)\), which states that the original system (1), is stable (Park; Moon and Kwon, 1999).

3 **Design of Sliding Mode Control for Uncertain Input-Delay System with Only Plant Output Access**

Consider the uncertain continuous system
\[
\dot{x}(t) = Ax(t) + Bu(t-h) + D\xi(t,x,u),
\]  \(\tag{4}\)

where \(x(t) \in \mathbb{R}^n\) is the continuous state vector, \(u(t-h) \in \mathbb{R}^m\) is the time-delayed sliding mode control \(A \in \mathbb{R}^n \times \mathbb{R}^n\), \(B \in \mathbb{R}^n \times \mathbb{R}^m\), \(C \in \mathbb{R}^m \times \mathbb{R}^n\), \(D \in \mathbb{R}^m \times \mathbb{R}^n\), \(p \geq q\) and the function \(\xi : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R}^m\) is unknown but bounded so that
\[\|\xi(t,x,u)\| \leq r\|x\| + \alpha(t,y),\]
where \(r_1 \in \mathbb{R}_+\) and \(\alpha : \mathbb{R}_+ \times \mathbb{R}^m \to \mathbb{R}_+\) is a known function.

The proposed design is shown in block diagram in Figure 1.

![Block Diagram](image)

Figure 1. Block diagram of the proposed design.

3.1 **Design of Sliding Mode Input-Delay Observer (SM-h Observer)**

The observer here reviewed was developed by (Edwards and Spurgeon, 1994).

Suppose that there exists a linear change of coordinates \(T_x\) so that Eq. (4) can be written as,
\[
\begin{align*}
\dot{x}_1(t) &= A_1x_1(t) + A_{12}x_2(t) + B_1u(t-h) \\
\dot{y}_1(t) &= A_1y_1(t) + A_{12}y_2(t) + B_1u(t-h) + D_1\xi,
\end{align*}
\]  \(\tag{6}\)

where \(x_1(t) \in \mathbb{R}^{(n-p)}, \ y(t) \in \mathbb{R}^p\) and the matrix \(A_{11}\) have stable eigenvalues.

Consider a sliding mode observer of the form
\[
\begin{align*}
\dot{\hat{x}}_1(t) &= A_1\hat{x}_1(t) + A_{12}\hat{x}_2(t) + B_1u(t-h) - A_1\hat{e}_1(t) \\
\dot{\hat{y}}_1(t) &= A_1\hat{y}_1(t) + A_{12}\hat{y}_2(t) + B_1u(t-h) - (A_{12}A_{12}^T)\hat{e}_1(t) + v,
\end{align*}
\]  \(\tag{7}\)

where \(e_1 = \hat{y}_1 - y\) and \(A_{12}\) is a stable design matrix.

Consider \(P_i \in \mathbb{R}^{p \times p}\) a symmetrical positive Lyapunov matrix for \(A_{12}\). The discontinuous vector \(v\) is defined by
\[
v = \begin{cases} -\rho(t,y,u) & \text{if } e_1 \neq 0 \\ 0 & \text{otherwise} \end{cases},
\]  \(\tag{8}\)

where the function \(\rho : \mathbb{R}_+ \times \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}_+\) satisfies
\[
\rho(t,y,u) \geq r_1\|u\| + \alpha(t,y) + \gamma_0
\]  \(\tag{9}\)

and \(\gamma_0 \in \mathbb{R}_+\).

The state error vector is defined as \(e_1 = \hat{x}_1 - x_1\) and the sliding surface is \(S_0 = \{e_1(t) \in \mathbb{R}^p : Ce_1 = e_1 = 0\}\), where
\[
e_1(t) = [e_1(t) \ e_1(t)]^T.
\]

The dynamic equation of the error is:
\[
\begin{align*}
\dot{e}_1(t) &= A_1e_1(t) \\
\dot{e}_2(t) &= A_{12}e_1(t) + (A_{12}A_{12}^T)e_1(t) + v - D_1\xi,
\end{align*}
\]  \(\tag{10}\)

**Theorem 1.** The dynamic of the error, Eq. (10), is asymptotically stable.

**Proof.** In (Edwards and Spurgeon, 1994) and (Garcia and Bennaton, 2002).

In original coordinates, if \(\dot{\hat{x}}(t)\) represents the states estimate for \(x(t)\) and \(e(t) = \hat{x}(t) - x(t)\), then the robust observer can conveniently be written as:
\[
\begin{align*}
\dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t-h) - G\hat{e}(t) + G_\mu v, \\
\end{align*}
\]  \(\tag{11}\)

where the linear gain
\[
G_\mu = T_0^{-1}\begin{bmatrix} A_1 \\ A_{12} \\ A_{22} - A_{22}^T \end{bmatrix},
\]  \(\tag{12}\)

the nonlinear gain
\[
G_\mu = \|D_1\|_F T_0^{-1}
\]  \(\tag{13}\)

and
\[
v = \begin{cases} -\rho(t,y,u) \|P_1\|C_\mu & \text{if } C_\mu \neq 0 \\ 0 & \text{otherwise} \end{cases}.
\]  \(\tag{14}\)
Here, the robust SM-h Observer (11)-(14) will be used, not only to estimate the states, but also to detection, diagnosis and accommodation of time delay fault, as it will be seen in the Section 5.

3.2 Design of Sliding Mode Controller (SM-h Controller)

Consider a prediction of estimate state, \( \hat{x}_{p} \in \mathbb{R}^{n} \), as

\[
\hat{x}_{p}(t) = e^{At} \hat{x}(t) + \int_{0}^{t} e^{A(t-\tau)} S \hat{x}(\tau) d\tau.
\]  (15)

Differentiating (15) with respect to time gives

\[
\dot{\hat{x}}_{p}(t) = A \hat{x}_{p}(t) + Bu(t) + e^{At} G (\hat{x}(t) - y(t)).
\]  (16)

By writing \( \mathcal{Z}(t, y(t), \hat{y}(t)) = e^{At} \Phi(t, y(t), \hat{y}(t)) \), with \( \Phi(t, y(t), \hat{y}(t)) = \{G \hat{x}(t) - G (\hat{x}(t) - y(t)) \} \) such that \( \| \Phi \| \leq \rho \), \( \rho > 0 \), it follows that

\[
\dot{\hat{x}}_{p}(t) = A \hat{x}_{p}(t) + Bu(t) + \mathcal{Z}(t, y(t), \hat{y}(t))
\]  (17)

where \( \mathcal{Z}(\cdot, \cdot) : \mathbb{R}^{m} \times \mathbb{R}^{n} \times \mathbb{R}^{n} \rightarrow \mathbb{R}^{n} \) represents the uncertainties and non-linearities of the system.

The sliding surface is defined as

\[
\{ \hat{x}_{p}(t) | \sigma(t) = S \hat{x}_{p}(t) = 0 \}
\]  (18)

for \( \sigma \in \mathbb{R}^{m} \) and \( S \in \mathbb{R}^{m \times m} \).

The proposed sliding surface includes the estimate state predictor to compensate the input delay. In this sliding mode control, it is assumed that matrix \( S \) is full rank and matrix \( SB \) is non-singular. Then, the matrix \( S \) is chosen such that the dynamics on the surface sliding has the desired closed loop behaviour.

Consider the following control structure of the form

\[
u(t) = u_{c}(t) + u_{sl}(t)
\]  (19)

where \( u_{c}(t) \) is an equivalent control for the nominal system of (18) and \( u_{sl}(t) \) is the switching control to overcome the uncertainties of the system.

The equivalent control is

\[
u_{c}(t) = - [SB]^{-1} S \hat{x}_{p}(t)
\]  (20)

The switching control \( u_{sl}(t) \) is chosen by

\[
u_{sl}(t) = \begin{cases} 
(SB)^{-1} \| Se^{At} \| \delta \quad \text{if } \| \sigma \| \neq 0 \\
0 \quad \text{otherwise}
\end{cases}
\]  (21)

with \( \delta = \rho + \beta \), \( \beta > 0 \).

In order to derive a sliding mode dynamics, it consider the time derivative of \( \sigma \) along the trajectories of the system (17) as

\[
\dot{\sigma}(t) = S \ddot{x}_{p}(t) = S \left[ A \dot{x}_{p}(t) + Bu(t) + e^{At} \Phi(t, y(t), \hat{y}(t)) \right]
\]  (22)

Substituting (19) and (20) into the equation (22) yields

\[
\dot{\sigma}(t) = SBu_{sl}(t) + Se^{At} \Phi(t, y(t), \hat{y}(t))
\]  (23)

**Proposition 1.** If the control law (19), with \( u_{c}(t) \) and \( u_{sl}(t) \) and the sliding surface given by (20), (21), and (18), respectively, is used for system (4), then the sliding mode always exists, i.e., the dynamics (23) is asymptotically stable.

**Proof:** Consider the Lyapunov function

\[
V(t, \sigma) = \frac{1}{2} \sigma^{T} \sigma
\]  (24)

Differentiating \( V(t, \sigma) \) with respect to time along the trajectories of dynamics (23) gives

\[
\dot{V} = \sigma^{T} \sigma = \sigma^{T} \left[ S \dot{x}_{p}(t) + Bu + e^{At} \Phi \right]
\]  (25)

Substituting equations (19)-(21) into equation (25) yields

\[
\dot{V} \leq - \| \sigma \| || Se^{At} || \| \Phi \|.
\]  (26)

Thus,

\[
\dot{V} \leq - \beta \| \sigma \| || S e^{At} || < 0
\]

for \( \sigma \neq 0 \). Then, \( \sigma \to 0 \) as \( t \to +\infty \).

3.2.1 Dynamics in the Sliding Mode

A non-singular transformation \( T \in \mathbb{R}^{m \times m} \) is introduced such that

\[
TB = \begin{bmatrix} 0 \\ B_{2} \end{bmatrix}
\]

where \( B_{2} \in \mathbb{R}^{m \times m} \) is non-singular and define

\[
\hat{z} = T \hat{x}_{p} = \begin{bmatrix} T_{1} \\ T_{2} \end{bmatrix}, \quad \hat{z}_{p} = \begin{bmatrix} \hat{z}_{1} \\ \hat{z}_{2} \end{bmatrix}
\]  (27)

with \( \hat{z}_{1} \in \mathbb{R}^{m-n} \), \( \hat{z}_{2} \in \mathbb{R}^{n} \) and \( T_{1}, T_{2} \) of compatibly dimension.

Then, the reduced order dynamics in sliding mode is obtained by

\[
\dot{\hat{z}}_{1}(t) = A_{1} \hat{z}_{1}(t) + A_{2} \hat{z}_{2}(t) + \hat{f}(t, \hat{z}_{1}(t), \hat{z}_{2}(t))
\]  (28)

where

\[
T A_{1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}
\]

\[
\hat{f}(t, \hat{z}_{1}(t), \hat{z}_{2}(t)) = T_{2} e^{At} \Phi
\]

The sliding surface can be written as

\[
\{ \hat{z} | \sigma = \hat{S} \hat{z} = 0 \}
\]  (29)

with \( \hat{S} = ST^{-1} = [\hat{S}_{1} \hat{S}_{2}] \) with \( \hat{S}_{1} \in \mathbb{R}^{m \times (m-n)} \) and \( \hat{S}_{2} \in \mathbb{R}^{m \times m} \).

Then, from equation (28) results \( \hat{z}_{2} = -K \hat{z}_{1} \) with \( K = \hat{S}_{2}^{-1} \hat{S}_{1} \), where \( \hat{S}_{1} \) is chosen to be non-singular matrix and

\[
\hat{z}_{1}(t) = (A_{1} - A_{2}K) \hat{z}_{1}(t) + \hat{f}(t, \hat{z}_{1}(t), \hat{z}_{2}(t))
\]  (30)

is the reduced order dynamics in sliding mode. The vector \( \hat{f}(t, \hat{z}_{1}(t), \hat{z}_{2}(t)) \) represents the unmatched uncertainties which will affect the ideal sliding dynamics.
4 Design of Sliding Mode Observer (SM-o) and Controller (SM-o) for Undelayed System with Only Plant Output Access

Consider an uncertain system (4) without delay, i.e., \( h=0 \). The sliding mode control (SM-o) for this system is
\[
   u(t) = u_L(t) + u_{NL}(t)
\]
where
\[
   u_L(t) = - \left[ SB \right]^{-1} S A \ddot{x}(t).
\]
The switching control \( u_{NL}(t) \) is chosen by
\[
   u_{NL}(t) = \begin{cases} 
   - (SB)^{-1} \sigma \dot{\sigma} & \text{if } \|\sigma\| \neq 0 \\
   0 & \text{otherwise}
   \end{cases}
\]
and the sliding surface is defined as
\[
   \{ \ddot{x}(t) | \sigma(t) = S \ddot{x}(t) = 0 \}
\]
with the observer (SMO-o) of the form
\[
   \ddot{x}(t) = A \ddot{x}(t) + Bu(t) - G_I(\dot{x}(t) - y(t)) + G_n v
\]
with \( G_I, G_n \) and \( v \) defined in (12)-(14).

5 Fault Detection and Controller Adaptation

In present proposed scheme it is assumed that a system without input-delay is an unfaulty system and that a system with input-delay is a faulty system, which should be identified and the controllers SM-o and SM-h should be switched in order to improve the performance and the relative stability.

5.1 Residuals

The available continuous-time output of the actual plant \( y(t) \in \mathbb{R}^p \) is compared with the outputs of two sliding mode observers, SM-o and SM-h, for residues generation: in the structure of the first one (SM-o) it is supposed that the controlled system is without input-delay, providing the output \( \dot{\hat{y}}(t) \in \mathbb{R}^p \) and the states \( \ddot{\hat{x}}(t) \in \mathbb{R}^n \); in the other observer (SM-h) it is assumed that the input-delay is present, providing the output \( \dot{\hat{y}}(t) \in \mathbb{R}^p \) and the states \( \ddot{\hat{x}}(t) \in \mathbb{R}^n \).

Here, the residuals functions are defined as
\[
   r^o(t) = \| \dot{\hat{y}}(t) - y(t) \|
\]
and
\[
   r^h(t) = \| \dot{\hat{y}}(t) - y(t) \|.
\]

5.2 Fault Diagnosis and Controller Adaptation

The difference of the residual functions, \( r(t) = r^o(t) - r^h(t) \), is defined as decision function.

The fault diagnosis can be formulated as:

i) If \( r^o(t) < r^h(t) \) then the actual system is operating without input-delay, i.e., under unfault condition.

ii) If \( r^o(t) > r^h(t) \) then the actual system is operating with input-delay, i.e., under fault condition.

The controller adaptation can be switched as the following logic:

i) If condition (35) is hold, use the SM-o controller, Eqs. (30)-(33);

ii) If condition (36) is hold, use the SM-h controller, Eqs. (18)-(21).

Then, the decision function to adapt the controllers to the fault/unfault condition is chosen as
\[
   \begin{cases} 
   r(t) < 0 \Rightarrow \text{controller SM-h should be activated} \\
   r(t) \geq 0 \Rightarrow \text{controller SM-o should be activated}
   \end{cases}
\]

Figure 2 illustrates the proposed scheme.

6 Design Example

Consider the lateral axis model of an L-1011 in cruise flight conditions. The system described originally in (Edwards and Spurgeon, 1994) does not have delay therefore due to this work it was include a delay in airplane control.

The delay-time system is given by
\[
   \ddot{x}(t) = A x(t) + B u(t - h) \\
   y(t) = C x(t)
\]
with \( x \in \mathbb{R}^3, u \in \mathbb{R}^4, y \in \mathbb{R}^3, \xi \in \mathbb{R}^5, 0 \leq h \leq 0.5 \) sec.

For nominal unfaulty system, the input-delay \( h \) is zero.

The state variables are bank angle (rad), yaw rate (rad/s), roll rate (rad/s), sideslip angle (rad), washed
out filter state, respectively. The control inputs are the rudder deflection (rad) and aileron deflection (rad), respectively. The outputs are washed out yaw rate, roll rate (rad/s), sideslip angle (rad) and bank angle (rad), respectively.

The matrices \( A, B \) and \( C \) are given by

\[
A = \begin{bmatrix}
0 & 0 & 1.0000 & 0 & 0 \\
0 & -0.1540 & -0.0042 & 1.5400 & 0 \\
0 & 0.2490 & -1.0000 & -5.2000 & 0 \\
0.0386 & -0.9960 & -0.0003 & -0.1170 & 0 \\
0 & 0.5000 & 0 & 0 & -0.5000
\end{bmatrix},
\]

\[
B = \begin{bmatrix}
0 & 0 \\
-0.7440 & -0.0320 \\
0.3370 & -1.1200 \\
0.0200 & 0 \\
0 & 0
\end{bmatrix},
\]

\[
C = \begin{bmatrix}
0 & 1 & 0 & 0 & -1 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0
\end{bmatrix}.
\]

Consider the problem of designing for this aircraft system a robust input-delay SM-h observer, which is insensitive to matched uncertainty. The used law was approximated by

\[
v = \begin{cases}
-\sigma(t,y,u)\frac{P_dCe}{\|P_dCe\| + \delta} & \text{if } Ce \neq 0 \\
0 & \text{otherwise}
\end{cases},
\]

where \( \rho(t,y,u) = 10 \) and \( \delta = 0.01 \).

For the system (39), a SM-h control law is derived from equations (19) – (21). The sliding surface is given by equation (10), and the matrix \( S \) is obtained as

\[
S = \begin{bmatrix}
0.2197 & -1.3033 & 0.0372 & 0.8889 & -0.1613 \\
-0.9722 & -0.3955 & -0.8816 & 0.1423 & -0.0343
\end{bmatrix}.
\]

The nonlinear control law, equation (21), is approximated by

\[
u_N(t) = -\sigma(S)^{-1} \sigma(S)e^{\delta t} - \sigma(S)^{-1}\delta_1,
\]

with \( \delta_1 = 15 \) and \( \delta = 0.01 \).

In the design for unfault condition was used in (30) - (34) for SM-o observer and SM-o controller.

### 7 Simulation

The inherent operational condition of the system was simulated like it is shown in (Table 1). The unfault case indicates that doesn’t exist input-delay, so that the system should be controlled by SM-o, Eqs. (30)-(33). The fault condition indicates that there exists input-delay, so that the system should be controlled by SM-h, Eqs. (18)-(21). Two values of delay were simulated: 0.2 and 0.5 seconds. The reference signal used for bank angle was a square wave.

<table>
<thead>
<tr>
<th>Time (sec.)</th>
<th>0 ≤ t &lt; 40</th>
<th>40 ≤ t &lt; 80</th>
<th>80 ≤ t &lt; 120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delay 1</td>
<td>0</td>
<td>0.20</td>
<td>0</td>
</tr>
<tr>
<td>Delay 2</td>
<td>0</td>
<td>0.50</td>
<td>0</td>
</tr>
</tbody>
</table>

Figures (3) and (4) show the Bank angle and the performance of the switched controllers for input-delay 0.20 seconds. Figures (5) and (6) give the results for input-delay 0.50 seconds.

Figures (4) and (6) show the active controller at each interval of time: when the system operates with delay, the controller SM-h is active; when the system works without delay, the controller SM-o is active. The SM-o/SM-h controllers were adequately switched by the proposed detection/adaptation scheme for each unfault/fault condition. The proposed scheme provided a good performance, even in the presence of larger value of input-delay.
8 Conclusions

In this paper the input-delay in a control system was treated as a fault to be detected and the controllers were adapted to improve the performance of the faulty system. For this purpose, it was presented two sliding mode control designs: a sliding mode control that doesn’t consider input-delay (SM-o) was reviewed and a sliding mode controller which takes into account the input-delay (SM-h) was presented. A strategy for fault detection and controller adaptation was proposed, based in the residual functions of sliding mode observers. Simulations of lateral axis model of a L-1011 in cruise flight were presented. The obtained results proved the robustness of the proposed SM-h design and the effectiveness of the fault detection and controller adaptation strategy. The proposed scheme had a good performance, even for larger values of input-delay.

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References


