CONTROLLER FOR DISTURBANCE REJECTION IN A FIXED WING UAV

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Abstract— This paper presents a nonlinear $H_\infty$ controller for the rejection of input disturbances in a fixed wing Unmanned Aerial Vehicle (UAV). The controller acts in all states of a complete nonlinear dynamic model. Simulation results shows that the aircraft is successfully maintained in the desired stable condition after a finite pulse disturbance.

Keywords— Nonlinear $H_\infty$ Control, Fixed Wing UAV.

Resumo— Este trabalho apresenta um controlador $H_\infty$ não linear para rejeição de perturbação em um Véuículo Aéreo Não Tripulado (VANT) de asa fixa. O controlador atua em todos os estados de um modelo completo não linear. Resultados em simulação mostram que a aeronave é mantida no regime desejado após uma perturbação em pulso finito.

Palavras-chave— Controle $H_\infty$ Não Linear, VANT de Asa Fixa.

1 Introduction

Unmanned Aerial Vehicles (UAVs) have a wide range of applications including surveillance, search and rescue, target tracking, detection and exploration, both in military and civilian areas (Low, 2010), (Jiao et al., 2010). Trajectory tracking is one of the UAV’s most important functions which will account for flight safety and task survivability (Jiao et al., 2010). For this task autonomous vehicles usually require a control system composed of an inner loop for the flight control system and an outer loop for the navigation control (Stevens and Lewis, 1992), (Low, 2010), (Jiao et al., 2010).

An aircraft’s dynamic model is highly nonlinear. The interaction of the airflow generated by the propeller contribute to complex aerodynamic forces that affect the vehicle’s motion, and in turn makes the motion control of a UAV a challenging task. The system’s dynamics is not only coupled and nonlinear, but also difficult to be characterized due to the complexity of the system’s aerodynamic properties (Zarafshan et al., 2010).

Many nonlinear control techniques have been applied to flight control design, including dynamic inversion, feedback linearization, variable structure system (VSS), and nonlinear predictive control (Zarafshan et al., 2010). Another potential tool for solving nonlinear flight control problems is the $H_\infty$ control approach. There are some examples of the nonlinear $H_\infty$ control applied to aerospace problems such as spacecraft attitude control, missile guidance and control, aerospace airplane ascend, and jet engine control (Zarafshan et al., 2010). Most application of nonlinear flight control are restricted to the lateral or longitudinal motion alone, the design of a nonlinear flight controller for the complete six degrees-of-freedom (6-DOF), including both lateral and longitudinal motion, is still a challenge for all nonlinear control design methods (Zarafshan et al., 2010).

The main difficulty in applying nonlinear $H_\infty$ control theory to flight dynamics is to solve the associated Hamilton-Jacobi partial differential inequalities (HJPI). A solution for this problem was proposed by (Yang and Kung, 2000) for a general 6-DOF motion which can be applied to various types of vehicles such as airplanes, missiles or helicopters. The drawback of this work is that the designing of the controller is based only on the external forces acting on the vehicle and the aerodynamics characteristics of the aircraft are neglected. In this case it is unclear how the commands generated by the controller is translated to the aircraft’s actuators. A nonlinear $H_\infty$ controller combined with a control surface inverse algorithm is presented by (Kung, 2008) to determine the control action and angles of control surface deflection. However this solution still relies on the combination of two separate problems and leads to a two level control structure.

This paper presents a nonlinear $H_\infty$ controller using state feedback for the inner loop for the flight control of a fixed wing model aircraft. A complete nonlinear model is used with the input control signals modeled as the angles of control surface deflection and propeller angular speed. This way, the controller is designed to act directly in the aircraft’s actuators. External disturbances are modeled as additive noise or disturbance to the control inputs.

This paper is organized as follows: Section 2 presents the fundamentals of the nonlinear
Consider the following nonlinear system

\[ \dot{x}(t) = f(x) + g_1(x)w + g_2(x)u, \quad (1) \]
\[ z(t) = h_1(x) + k_{12}(x)u, \quad (2) \]

where \( x \in \mathbb{R}^n, z \in \mathbb{R}^p, w \in \mathbb{R}^{m_1} \) and \( u \in \mathbb{R}^{m_2} \) are the system state, regulated output, exogenous input and controlled input, respectively. We assume that \( f(x), g_1(x), g_2(x), h_1(x) \) and \( k_{12}(x) \) are smooth and defined in a neighborhood of the origin in \( \mathbb{R}^n \) and have zero values at the origin. We also make the following simplifying assumption on the plant

\[ h_1^T(x)k_{12}(x) = 0, \quad k_{12}^T(x)k_{12}(x) = I. \]

The nonlinear state feedback problem is formally stated as follows.

**The nonlinear \( H_\infty \) state feedback problem:**

Find a control law \( u = l(x) \) where \( l(x) \) is locally defined sufficiently smooth function with \( l(0) = 0 \) such that

1. The closed-loop system is locally asymptotically stable when \( w = 0 \);
2. The closed-loop response \( z : [0, T] \rightarrow \mathbb{R}^p \) of system (1)–(2) with \( u = l(x) \) satisfies

\[ \int_0^T z^T(\tau)z(\tau)d\tau \leq \gamma^2 \int_0^Tw^T(\tau)w(\tau)d\tau \]

for some \( \gamma \) and all \( T \).

The Hamiltonian function associated with the above problem is

\[ \mathcal{H}(x, p, w, u) = p^Tf(x) + g_1(x)w + g_2(x)u + \frac{1}{2}\|h_1(x) + k_{12}(x)u\|^2 - \gamma^2\|w\|^2. \]

The Hamilton-Jacobi-Isaacs equation is given by (van der Schaft, 1992)

\[ \mathcal{H}(x, p) = \mathcal{H}(x, p, \alpha_1(x, p), \alpha_2(x, p)) = 0, \]

where

\[ \alpha_1(x, p) = \frac{1}{T}g_1^Tp, \quad \alpha_2(x, p) = -g_2^Tp. \]

It was shown in (van der Schaft, 1992) that if there exists a positive-definite \( C^1 \) function \( \Pi(x) \) locally defined in a neighborhood of the origin in \( \mathbb{R}^n \) that satisfies

\[ \mathcal{H}(x, \Pi^T_x, \alpha_1(x, \Pi^T_x), \alpha_2(x, \Pi^T_x)) = 0, \]

or, more explicitly,

\[ \Pi_xf(x) + \frac{1}{2}h_1^Th_1 + \frac{1}{2}\Pi_x \left( \frac{1}{\gamma}g_1g_1^T - g_2g_2^T \right) \Pi_x = 0, \]

where \( \Pi_x \) denotes the Jacobian of \( \Pi(x) \), then the control law

\[ u = -g_2^T(x)\Pi_x^T \]

solves the nonlinear, infinite horizon \((T = \infty), H_\infty \) state feedback problem.

**3 UAV Equations of Motion and Controller Design**

**3.1 UAV Equations of Motion**

The \( H_\infty \) controller will be designed for the a complete nonlinear model of a fixed wing UAV. The complete nonlinear model for a fixed wing UAV developed by (Paw and Balas, 2011) has the state vector

\[ x = [U \ V \ W \ P \ Q \ R \ \phi \ \theta \ \psi \ h \ \omega]^T, \]

where \( U, V, W \) are the velocities in m/s, \( P, Q, R \) are the angular speeds in rad/s, \( \phi, \theta, \psi \) the Euler angles in rad, \( h \) the height in m and \( \omega \) the angular speed of the propeller in rad/s according to fig. 1 from (Paw and Balas, 2011). This type of aircraft has four control surfaces: elevator, two ailerons and the control input for the propeller’s motor \( \delta_T \) (throttle, in percentage). The two ailerons acts always in a coordinated matter, therefore constitutes one control input. The control input vector is defined in Eq. (6)

\[ u = [\delta_e \ \delta_a \ \delta_r \ \delta_T]^T. \]

In this work the control input vector represents the throttle control input \( (\delta_T) \) as (Paw and Balas, 2011) and the angle of deflection of the control surfaces \( (\delta_e, \delta_a, \delta_r) \).

The space state model is built using the force equations, the kinematics equations, the moment equations, the Z-axis navigation equation and the propeller’s dynamic. All equations in this section were taken from (Stevens and Lewis, 1992), (Paw and Balas, 2011) and (Nelson, 1989).

**3.2 Force equations**

The force equations used in this work is the classical force equations

\[ \dot{U} = RV - QW - g \sin(\theta) + \frac{F_u}{m}, \]
\[ \dot{V} = -RU + PW + g \sin(\phi) \cos(\theta) + \frac{F_v}{m}, \]
\[ \dot{W} = QU - PV + g \cos(\phi) \cos(\theta) + \frac{F_w}{m}, \]
The kinematics model is given by

\[
\begin{align*}
    \dot{P} &= (c_1 R + c_2 P) Q + c_3 L + c_4 N, \\
    \dot{Q} &= c_5 P R - c_6 (P^2 - R^2) + c_7 M, \\
    \dot{R} &= (c_8 P - c_2 R) Q + c_4 L + c_9 N.
\end{align*}
\]

with

\[
\begin{align*}
    L &= q S b c_1 - T_m, \\
    M &= q S b c_m, \\
    N &= q S b c_n,
\end{align*}
\]

\[
c_i = c_{i \alpha} \beta + c_{i \alpha} \delta_{\alpha} + c_{i \delta} \delta_{\beta} + \frac{b}{2 V_a} (c_{i P} P + c_{i R} R),
\]

\[
c_m = c_{m \alpha} + c_{m \alpha} \alpha + c_{m \delta} \delta_{\alpha} + \frac{c}{2 V_a} (c_{m \alpha} \alpha + c_{m Q} Q),
\]

\[
c_n = c_{n \alpha} \beta + c_{n \alpha} \delta_{\alpha} + c_{n \delta} \delta_{\beta} + \frac{b}{2 V_a} (c_{n P} P + c_{n R} R),
\]

and the coefficients

\[
\begin{align*}
    \Gamma &= I_{xx} I_{zz} - I_{xz}^2, \\
    c_1 &= \frac{(I_{yy} - I_{zz}) I_{zz} - I_{xx}^2}{\Gamma}, \\
    c_2 &= \frac{(I_{xx} - I_{yy} + I_{zz}) I_{zz}}{\Gamma}, \\
    c_3 &= \frac{I_{zz}}{I_{yy}}, \\
    c_4 &= \frac{I_{zz}}{I_{yy}}, \\
    c_5 &= \frac{I_{zz} - I_{xx}}{I_{yy}}, \\
    c_6 &= \frac{I_{zz}}{I_{yy}}, \\
    c_7 &= \frac{1}{I_{yy}}, \\
    c_8 &= \frac{I_{xx} (I_{xx} - I_{yy}) + I_{xx}^2}{I_{yy}}, \\
    c_9 &= \frac{I_{xx}}{I_{yy}}.
\end{align*}
\]

3.4 Moment equations

The moment equations are

\[
\begin{align*}
    \dot{\phi} &= P + \tan(\theta) (Q \sin(\phi) + R \cos(\phi)), \\
    \dot{\theta} &= Q \cos(\phi) - R \sin(\phi), \\
    \dot{\psi} &= (Q \sin(\phi) + R \cos(\phi)) / \cos(\theta).
\end{align*}
\]

3.5 Navigation equations

In this model the only component of the classical navigation equations used is the altitude, denoted by the variable \( h \).

\[
\dot{h} = -U \sin(\theta) + V \sin(\phi) \cos(\theta) + W \cos(\phi) \cos(\theta).
\]

3.6 Aircraft propulsion

An extra variable, the propeller angular velocity \( \omega \), is added to the nonlinear dynamical model, the aircraft’s propulsion dynamics. This variable, used to monitor and control the thrust generated by the propeller, is described by

\[
\dot{\omega} = \frac{T_m - T_p}{T_m + T_p}.
\]
where $T_m$ is the output torque at motor shaft, $T_p$ is the torque generated by the propeller, $I_m$ is the moment of inertia of the rotating motor body, and $I_p$ is the moment of inertia of the propeller.

4 Numerical Simulation

A numerical simulation was created in order to show the applicability and performance of the proposed controller for an UAV. In this work the 6-DOF model presented in the previous section, along with the developed controller was implemented via MATLAB SIMULINK. The nonlinear model’s parameters used in this simulation where taken from (Paw, 2009). First a trim condition (states and inputs values such that the aircraft holds a steady state) is obtained. The trim condition used in here is

$$x = \begin{bmatrix} 17 & -0.0546 & 0.3593 & 0 & 0.0211 & 0 \\ 0 & 0 & -100 & 537.122 \end{bmatrix}^T,$$

$$u = \begin{bmatrix} 0.0918 & 0.0308 & -0.0139 & 0.593 \end{bmatrix}^T.$$

The purpose of this simulation was to compare the effectiveness of the $H_\infty$ controller in rejecting an input disturbance, which in this case consisted of a pulse for a period of 1 s and amplitude of $\pi/40$ rad added to the elevator channel at time 5 s as shown in fig. 7(a).

The first run was carried out in an open loop. Although the fixed wing UAV is in a stable condition around this operation point, a small disturbance in the control input will render the system go to another point of operation as shown in figs. 2(a), 3(a), 4(a), 5(a) and 6(a).

The objective of the controller is to reject noise and disturbances on the input channels and keep the aircraft in the desired stable state. Figs. 7(b) and 8(b) show the control actions performed by the inputs in order to compensate a disturbance. The resulting dynamics of the output variables can be seen in Figs. 2(b), 3(b), 4(b), 5(b) and 6(b). In this case the aircraft stabilizes in the desired initial condition after the transient. It should be noticed that due to the coupling in the complete nonlinear model, that the controller acts both the elevator and the throttle inputs (see figs. 7(b) and 8(b)).

5 Conclusion

This paper presented a $H_\infty$ controller for the rejection of input disturbances in a fixed wing UAV. The controller acted in all states of a complete nonlinear dynamic model. Simulation results showed that the aircraft is successfully maintained in the desired stable condition after a finite pulse disturbance.
Figure 4: $\phi, \theta, \psi$ dynamics

Figure 5: $z$ dynamics

Figure 6: $\omega_p$ dynamics

Figure 7: Elevator, aileron and rudder inputs


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**References**


