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Dynamic matrix control with input saturation constrained

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Abstract: It is proposed, in this paper a dynamic matrix control (DMC) with an anti-windup (AW) based on linear matrix inequalities (LMI). The DMC-AW control is applied in the ball-beam system, in which the main purpose is to control the ball position on a rotating beam. System modeling was performed, which presented two degrees of freedom. For the mechanical system implementation a microcontroller, an ultrasonic sensor, and a servo motor were used. The proposed control was implemented both numerically with the software MATLAB and with the microcontroller ATmega328Pu. The simulation results validated the efficiency of the proposed DMC-AW and showed that the approach improves the response of the system under input saturation.

Keywords: Ball-Beam; Dynamic Matrix Control; Anti-Windup

1. INTRODUCTION

Over the years, there had been many classical control problems that have been studied to help explain the general mechanical systems concepts. One of these classic systems is ball-beam (Yu e Ortiz, 2005). In this system, the main objective is to control a ball position on a rotating beam (Sathiyavathi e Krishnamurthy, 2013; Schvarcz e Diniz, 2010). The ball movement happens when the beam slides and the weight of the ball placed on the bar creates a torque in the center of rotation, causing the ball to rotate on the beam (Hirsch, 1999). The ball position is previously entered via the keyboard by the user, and the beam will have to rotate so that the ball moves to the desired position. For this to be possible, it is necessary to design a controller.

The ball-beam system can represent many typical real systems, such as horizontally stabilizing an airplane (Wang, 2007). In this way, this paper develops a dynamic matrix control (DMC) with an anti-windup (AW) technique to control the ball-beam system.

The DMC is a model predictive control (MPC) type. The true birth of MPC happened in the industry in the mid-1970s. Presented in the study predictive control based on the heuristic model (MHRC) of Testud et al. (1978) and Cutler e Ramaker (1980) who proposed the DMC technique. Then, MPC's strategy has become popular in the petrochemical industry. During this period, there were a lot of new MPC variants Löfberg (2003); Kwong (2005); Camacho e Alba (2013). The DMC technique is based on the impulse response, and it usually deals with restrictions. However, for systems with unexpected saturation, the control response often recovers slowly, deteriorating system performance and exhibiting what is traditionally known as the windup (Rego et al., 2018; Zaccarian e Teel, 2011; Qi et al., 2018; Turner et al., 2003; Rego et al., 2018).

In this way, the windup is an unwanted effect. Hence, in the last two decades, the problem of designing the anti-windup compensator that guarantees closed-loop stability and satisfies certain performance criteria has been extensively explored (De Doná et al., 2000; Turner et al., 2003; Ran et al., 2016; Adegbege e Levenson, 2017; Wada e Saeki, 2016; Lamrabet et al., 2018; Fang et al., 2018; Rego et al., 2018; Errouissi e Al-Durra, 2018; Rego e Costa, 2020). De Doná et al. (2000) discussed the relationship between the AW technique and the model predictive control, in which the AW control considered in his work, is based on the law of closed-loop control with state saturation applied to the LTV system. Wada e Saeki (2016) proposed a method of designing an anti-windup compensator with the MPC control for a system with input restrictions. In the same way, Ran et al. (2016) extended the application of AW to uncertain systems. And Fang et al. (2018) presented results on a dynamic anti-windup compensator for the

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flexible AC transmission system-based wide-area damping controller.

Thus, in this work, it is proposed the use of the dynamic matrix control with AW technique of Herrmann et al. (2003) based on procedures of Kwong (2005). The control proposed is applied in the ball-beam system, and it is implemented numerically with the software MATLAB, and in a microcontroller.

The paper is organized as follows. First, in Section 2, it is presented the mathematical modeling of the ball-beam system. In Section 3, it is presented the problem and the control strategy, and also some basic concepts concerning dynamic matrix control are recalled. In Section 4, it is presented the numerical example with a comparative analysis between the DMC with AW and DMC without AW. In Section 5, the experimental results are presented. Finally, in Section 6, it is discussed the conclusions of the study.

2. BALL AND BEAM DYNAMICS

In the ball-beam system, the positioning of the ball (at a predefined setpoint) on a horizontal rod must be ensured. The ball-beam system problem is generally compared to actual control problems, such as the horizontal stability of an airplane at the moment of landing or during turbulence (Wang, 2007).

It is shown, in Figure 1 the system free-body diagram. This system has two degrees of freedom. The first is the ball rolling right and left over the bar, and the other is the bar rotating around the axis. The equation that describes

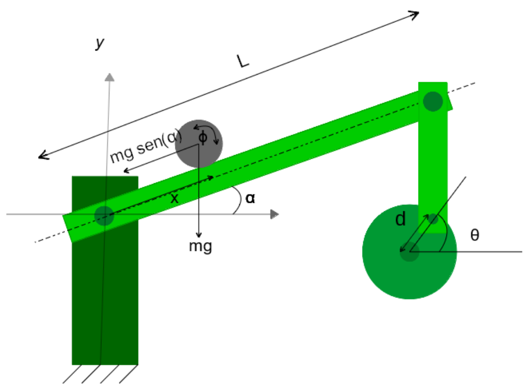


Figure 1. System free-body diagram.

the system model is represented by a set of nonlinear differential equations, which represents a great difficulty for the implementation of a linear controller designed with classical methods. For simplification, modeling and simulation are performed only for the ball and beam. For simplicity reason, it is assumed that all friction in the system is negligible.

By Newton's second law one has to,

$$\sum F = m\ddot{x}, \tag{1}$$

where m is the mass and $\ddot{x}$ is the acceleration.

It is obtained from this law and the Lagrange equations of motion the differential equation that governs the ball-beam system. So, the total kinetic energy of the system is given by,

$$\mathcal{K} = \frac{1}{2}(mv^2 + I_{beam}\dot{\alpha}^2 + I_{ball}\omega^2), \tag{2}$$

where, v is the velocity, I_{beam} and I_{ball} are the moment of inertia of the beam and ball respectively. The total potential energy of the system and the gravitational potential energy acting on the ball given by,

$$\mathcal{U} = -mgxsen(\alpha). \tag{3}$$

The distance traveled by the ball is given by $x = r\phi$, where ϕ is the rotation angle of the ball. The rotational speed of the ball, therefore, is given by,

$$\omega = \dot{\phi} + \dot{\alpha} = \frac{\dot{x}}{r} + \dot{\alpha}. \tag{4}$$

The speed of translation of the ball can be written as

$$v = \sqrt{\dot{x}^2 + (x\dot{\alpha}^2)}. \tag{5}$$

Substituting the equations (4) and (5) in the equation (2),

$$\mathcal{K} = \frac{1}{2}[m(\dot{x}^2 + (x\dot{\alpha}^2)) + I_{ball}(\frac{\dot{x}}{r} + \dot{\alpha})^2 + I_{beam}\dot{\alpha}^2]. \tag{6}$$

It will be used the Lagrangian, which is given by $\mathcal{L} = \mathcal{K} - \mathcal{U}$, so the following differential equation that governs the ball-beam system is obtained,

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) - \frac{\partial \mathcal{L}}{\partial x} = (\frac{I_{ball}}{r^2} + m)\ddot{x} + mgsen(\alpha) - mx(\dot{\alpha})^2. \tag{7}$$

The equation (7) is not linear. Applying a simple linearization process, the transfer function will be given by,

$$T(s) = \frac{X(s)}{\Theta(s)} = \frac{-mg\frac{d}{L}}{(\frac{I_{ball}}{r^2} + m)s^2} \quad [\frac{m}{rad}]. \tag{8}$$

3. CONTROL STRATEGY

3.1 Dynamic Matrix Control

The idea of predictive control by a dynamic matrix is to generate a system action to predict a certain effect on the system response and to avoid or diminish it (Kwong, 2005).

Thus, the prediction model is defined by

$$y_k = y_{k-1} + \sum_{i=1}^{N} h_i \Delta u_k, \tag{9}$$

such that, $\Delta u_k = u_k - u_{k-1}$ is the input, y_k is the output, and h_i are the coefficients of the unit step response. N is the number of terms of the impulse response sequence that has been retained and corresponds to the horizon of the model.

At each sampling instant k, the prediction model is used to predict the output trajectory over a finite future time interval given in terms of the prediction horizon R.

Minimizing the equation (9) to get Δu,

$$\Delta u = inv(\mathcal{A}^T Q^T Q A + \lambda)\mathcal{A}^T Q^T Q E = KE, \tag{10}$$

where, $\mathcal{A}$ is the toeplitz matrix that stores system step responses, λ is the control motion suppression, Q is the output weight matrix, E is the predicted errors vector, and K is the control matrix defined by,

$$K = inv(\mathcal{A}^T Q^T Q \mathcal{A} + \lambda)\mathcal{A}^T Q^T Q. \tag{11}$$

For DMC implementation it is necessary to define a sequence of L (control horizon) control movements such that the difference between the desired value and the predicted value over the optimization interval is minimized.

3.2 Anti-windup

The suggested anti-windup with DMC control structure to deal with both linear time-invariant and linear time-varying systems with saturating actuators is depicted in Figure 2. Saturation are values outside of the actuator's amplitude limits are mapped into the range of capabilities (Zaccarian e Teel, 2011).

Letting u_{max} correspond to the maximal and u_{min} to the minimal attainable actuator value, the saturation function is described mathematically by,

$$sat(u) = \begin{cases} u_{max}, & if\ u > u_{max} \\ u, & if\ u_{min} \le u \le u_{max} \\ u_{min}, & if\ u < u_{min} \end{cases} \tag{12}$$

Figure 2 shows the proposed closed-loop diagram with anti-windup. Where $T(z)$ is the system transfer function and K is the gain. $M(z)$ and $G(z)$ are coprime matrices.

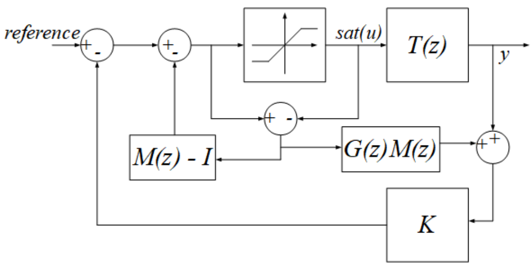


Figure 2. Proposed closed-loop with anti-windup $sat(u(k))$.

The coprime matrices are given by,

$$\Theta(z) = \begin{bmatrix} M(z) - I \\ G(z)M(z) \end{bmatrix} \sim \left[\begin{array}{c|c} A + BF_{AW} & B \\ \hline F_{AW} & 0 \\ C + DF_{AW} & D \end{array}\right], \tag{13}$$

where F_{AW} is the anti-windup gain, and A, B, C and D are the discretized matrices. The expressions of the anti-windup actuator are defined by,

$$x_d(k+1) = (A + BF_{AW})x_d(k) + B\tilde{u}(k), \tag{14}$$
$$u_d(k) = F_{AW}x_d(k), \tag{15}$$
$$y_d(k) = Cx_d(k). \tag{16}$$

where $\tilde{u}(k)$ is defined by,

$$\tilde{u}(k) = Dz(sat(u(k)) - u(k)), \tag{17}$$

where Dz the designation of dead-zone according to modeling defined in (Turner et al., 2003; Herrmann et al., 2003). And $sat(u(k)) = u_m(k)$ is the control signal limited by saturation. In the nominal case, $u(k) = u_m(k)$. When saturation events end, the output of the dead-zone $\tilde{u}(k)$ becomes null.

The signal $y_{lin}(k)$ is given by,

$$y_{lin}(k) = y(k) + y_d(k). \tag{18}$$

So, as $y(k) = y_{lin}(k) - y_d(k)$ the size of y_d is a direct measure of the saturated system's deviation from the nominal linear performance in response to u_{lin}.

The signals generated by the anti-windup compensator are fed into the controller output and the controller input, as can be seen in Figure 2. To calculate the anti-windup gain Turner et al. (2003) proposes the Theorem 1.

Theorem 1. There exists a dynamic compensator $\Theta(z)$ of order n_p which solves strongly the anti-windup problem if there exist matrices $Q_a > 0$, $U_a = diag(\mu_1, ..., \mu_m)$, $L_a \in R^{(m+q)\times m}$ and a scalar $\mu_a > 0$, such that the following linear matrix inequality is satisfied,

$$\min \gamma_a$$
$$\begin{bmatrix} -Q_a & * & * & * & * \\ -L_a & -2U_a & * & * & * \\ 0 & I & -\mu_a I & * & * \\ (C_jQ_a + D_jL_a) & D_jU_a & 0 & -I & * \\ (A_jQ_a + B_jL_a) & B_jU_a & 0 & 0 & -Q_a \end{bmatrix} < 0. \tag{19}$$

Where $F_{AW} = L_a Q_a^{-1}$ is anti-windup action gain, based on coprime factorization. And $\gamma_a = \sqrt{\mu_a}$.

Proof. See (Turner et al., 2003).

4. NUMERICAL SIMULATION

The ball-beam model described in section 2 was used to test and compare the effectiveness of the DMC with the anti-windup technique. The YALMIP with solved SeDuMi was used to compute the LMI (19). The code is available on `https://github.com/roscibely/DMC-with-input-saturation-constrained`. The system parameters are shown in Table 1.

Table 1. System Parameters

Parameters	Meaning
m=3.88g	Ball mass
r=1.5cm	Ball radius
L=29cm	Beam length
$I=\frac{2mr^2}{3}$	Moment of inertia
x=20-15cm	Ball position
θ=73 grad	Motor angle
α=0 grad	Beam angle
d=6.5cm	Motor arm
g=9.8	Gravity

The initial states of the system is assumed as $x = [0\ \ 0]^T$. The set reference was $r_{t\le 20} = 20cm$ and $r_{t>20} = 15cm$. The maximum value of the control signal was $u_{max} = 1.5 \times 10^6$ and the sample time $T_s = 2ms$. Thus, the discrete-time LTI state-space model is,

$$A = \begin{bmatrix} 1 & 0 \\ 0.0020 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0020 \\ 1 \end{bmatrix}, \tag{20}$$
$$C = [\,0\ \ 1.3180\,],\ D = 0.$$

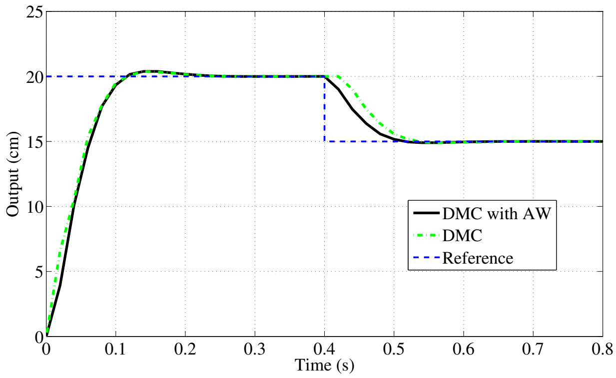
The controller gain obtained was,

$$K = [\,4.4471 \;\; 9.4387 \;\; 6.6252 \;\; -3.9933\,] \times 10^4. \qquad (21)$$

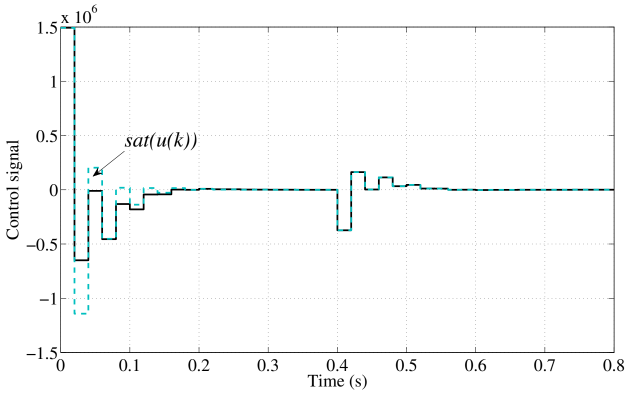
The anti-windup gain was,

$$F_{AW} = [\,-4.8328 \;\; -1.9606\,] \times 10^2. \qquad (22)$$

It is shown, in Figure 3 the output response of the ball-beam system by employing DMC and DMC-AW without input saturation. The closed-loop output is following the desired reference input. It is observed that the system with actuator AW has a regimen recovery faster than the circuit operating only with the DMC controller. The AW compensator produces a signal based on the difference between the controller output and the saturated actuator output, and then augment the signal to the control to deal with the windup phenomenon caused by actuator saturation.



(a) Output $y(k)$.

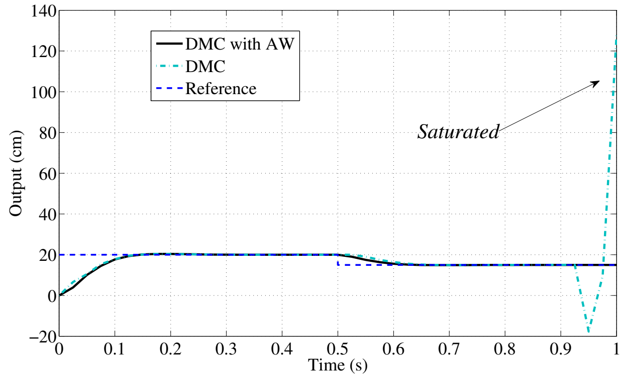


(b) Control signal $u(k)$.

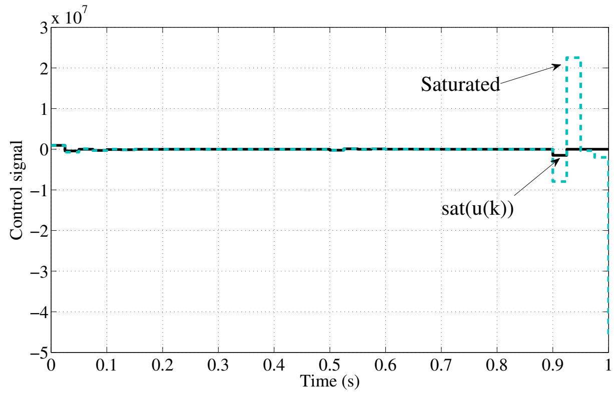
Figure 3. Closed-loop response of position control without saturation.

It is shown, in Figure 4 the simulation results of the control signal limited by saturation $sat(u(k))$ and $u(k)$. It is noted that the control signal with actuator AW has a faster recovery than the circuit operating only with the DMC controller. In this way, the DMC-AW improved performance when the system operated in the saturated mode. The control signal of the DMC controller is longer under saturation effect, impacting the responses of the output.

It is shown, in Figure 5 the input saturated $u(k)$ with DMC and DMC-AW $sat(u(k))$. Scheme recovery of the system with AW is faster than the model without AW. And note how quickly the signal returns to the linear region and how fast the loop recovers from saturation avoiding that the saturation damages its performance in a permanent regime.



(a) Output $y(k)$ with saturation.



(b) Control signal $u(k)$ with saturation.

Figure 4. Closed-loop response of position control with input saturation.

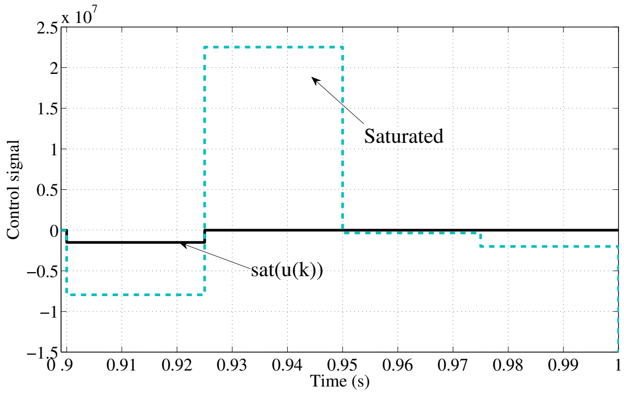
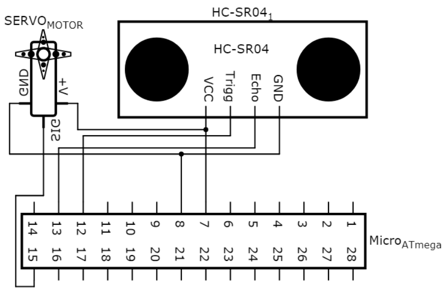


Figure 5. Input saturation $u(k)$ with anti-windup $sat(u(k))$.

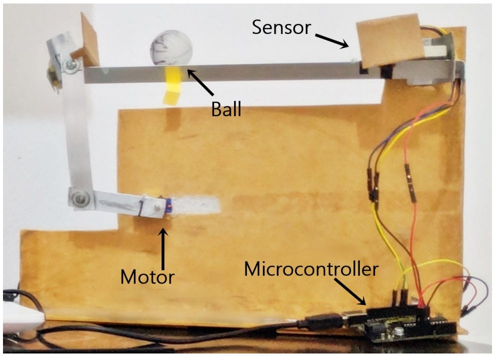
5. EXPERIMENTAL RESULTS

To implement the physical system, a servo motor, an ultrasonic sensor to detect the ball position, and an Atmega328Pu microcontroller were used. For real simulation it was set the reference $r(k) = 20cm$. The software MAT-

LAB was used to send and receive the data. Figure 6 shows the circuit diagram implemented.



(a) Circuit diagram.



(b) Ball-beam system

Figure 6. Ball-beam physical system: a) the circuit diagram with sensor, motor and microcontroller. b) implemented physical system.

It is shown, in Figure 7 the output $y(k)$ for both technique DMC and DMC-AW. Results show that the control with AW presents a better result when compared with the control without AW. The saturated signal causes instability and affects closed-loop system stability margins degrading system performance over as shown in Figure 7.

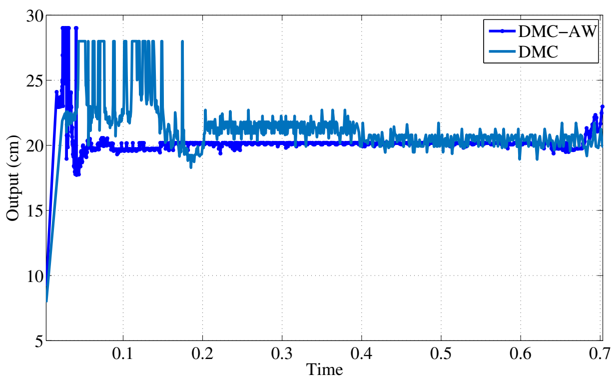


Figure 7. Sensor output $y(k)$ with DMC and DMC-AW.

6. CONCLUSION

The obtained results can validate the good performance of a DMC-AW implemented in the ball-beam system. Despite the instability and oscillatory tendency of the system, the implemented control was successful in controlling the system with rapid response in the simulation and experimental results. Also, it was noted that the AW strategy avoids that the saturation damages the system performance in a permanent regime.

In this work, it was only considered to control the model of the ball position with respect to the beam angle. As future work, the authors intends to control also the nonlinear model of the angle process with respect to the motor voltage.

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