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Opinion Dynamics over a Finite Set in Cooperative Multi-robot Systems: An Asynchronous Gossip-Based Consensus Approach ⋆

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Abstract: The focus of this paper is to present an algorithm that allows robotic teams to make decisions between a finite set of choices. The approach used was based on models that represent the way groups of humans evolve their opinions through time. Numerous works have explored models that consider the opinion as continuous values, while the literature less frequently considers groups trying to reach an agreement when only a finite set of possible opinions is given. The main contribution of this paper is to present a consensus algorithm that can be applied in those scenarios. For this purpose, it is briefly reviewed some crucial concepts for the definition of the proposed algorithm, which is based on asynchronous gossip. Due to the stochasticity of this approach, it is not possible to precisely predict the behavior of the network. However, the results from both computational and laboratory experiments indicate the eigenvector centrality score as a valuable metric to predict the probability of an initial opinion to become the prevailing one for the group when they reach consensus. Also, the asynchrony of the proposed algorithm made it possible to reach consensus in scenarios where synchronous approaches could not.

Keywords: Multi-Robot Systems, Agent-Based Systems, Cooperating Robots, Consensus, Opinion Dynamics.

1. INTRODUCTION

There are many situations in robotics where a team needs to decide over a finite set of options. For example, for a distributed group decision making on resource allocation under conditions of uncertainty that disallow formal optimization (Friedkin et al., 2019); an effective scheme to address influence maximization, which aims to select most influential nodes and obtains the maximal propagation of the most ideal information (He et al., 2019); a distributed sensor selection architecture for a class of networked systems, dependent on the quality of the measurements to select the most suitable sensors (Tedesco et al., 2018); distributed strategy selection based on the individual opinions and the relative credibility of each agent (Ekenberg et al., 1994); and in many situations, like multi-robot formation, it is necessary to choose a leader.

As an example scenario, it is possible to think of a group of aerial robots that covers a specific region attempting to identify forest fires. Consider that the agents find several spots with high temperatures but their ability to extinguish the fires depends on the group acting together. In this case, the first step for the robotic team will be to decide which of the detected fires they will fight. Since the consensus algorithms considered in the literature are like convex combinations of the available options, they do not apply to situations such as this example.

When analyzing humans interacting in the process of social influence, uncountable situations rise where that kind of decision must be taken. As examples, it can be mentioned political elections, where it is not possible to elect some weighted average of the candidates; Another scenario to illustrate that is the popular juries, in which case consensus is needed among all participants over a binary value (i.e., guilty or not).

The growing interest in social network analysis, especially in processes of opinion dynamics (Sîrbu et al., 2017), (Chamley et al., 2013), (Proskurnikov and Tempo, 2017), (Proskurnikov and Tempo, 2018), brought to sight a question over the applicability of consensus models for representing groups of agents that evolve their opinions through time. As mentioned previously, there are scenarios where the robotic teams will also have to come to a consensus between a finite set of options instead of computing intermediaries values, which is the case on the models exploited on the literature.

These situations, in which just a finite set of possible choices are available to the agents to choose, will be referenced in this paper as consensus with discrete opinions. The proposed algorithm was derived from the asynchronous

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gossip consensus (Boyd et al., 2006). The modifications were made to achieve the feature of reaching consensus over a finite set of possible opinions.

A large number of computational simulations were carried out to test if the proposed algorithm is capable to reach consensus in the same scenarios that others found in the literature are. These simulations also explored situations where other algorithms are not able to succeed. The simulation is over when the group reaches consensus in one of the initial opinions, we call consensus opinion the one that prevailed at the end of the simulation. The results corroborate the hypothesis proposed by the authors that the eigenvector centrality (Bonacich, 1972) of the network indicates the probabilities of each initial opinion become the consensus opinion for the group.

In order to analyze if the algorithm works under real-world adversities and limitations, experiments were done using a group of five robots acting as communication nodes. It was chosen a topology with one node that have its centrality score hugely discrepant to help in identifying a tendency, even over a small number of iterations.

The rest of the paper is organized as follows. Section II features some necessary information such as the DeGroot model, the eigenvector centrality, and the gossip consensus algorithm; then, an asynchronous asymmetric gossip-based algorithm that aims to reach consensus over a finite set of opinions is presented. Section III is organized in three subsections in which the overall scenarios proposed and the computational simulations are presented; then some exceptional cases in comparison with the classical consensus algorithms are highlighted; at last, the analysis and results for the twelve scenarios performed with numerical simulations are presented. After all, in section IV a laboratory experiment with low-cost robots, known as kilobots, is presented. Finally, section V contains our conclusions and future works.

2. ALGORITHM FORMULATION

This section presents an algorithm for handling consensus over a finite set of opinions. For this purpose, it is necessary to first present some backgrounds, such as the DeGroot model, the eigenvector centrality, and the gossip consensus algorithm. These matters grounded the proposal presented here.

2.1 The DeGroot Model

The available empirical evidence is consistent with the assumption that individuals update their opinions as convex combinations of their own and the displayed opinions by others. These updates are based on weights that are automatically generated by individuals in their responses to the displayed opinions of others (Jia et al., 2015).

The DeGroot model (DeGroot, 1974) is an early approach for describing the evolution of the opinions in a group of individuals trying to reach a consensus. A common way of representing the connections between agents comes from graph theory.

Graphs are abstractions for information flow between agents in a network, without concerning what kind of protocol used, which exchanged information, and the process implemented on it (Mesbahi and Egerstedt, 2010). In algebraic form, a graph is usually represented as an adjacency matrix $\boldsymbol{A} = [a_{ij}] \in \mathbb{R}^{n \times n}$, where $a_{ij} > 0$ if there is a connection from j to i, otherwise $a_{ij} = 0$.

Further, the DeGroot model is defined as follows, consider a group of n agents with real-valued opinions, $\boldsymbol{x}(k) \in \mathbb{R}^n$, where k is the iteration step. The social influence network formed by them can be described as a weight matrix $\boldsymbol{W} = [w_{ij}] \in \mathbb{R}^{n \times n}$ satisfying $w_{ij} \in [0, 1]$ for all i and j and $\sum_{j=1}^{n} w_{ij} = 1$ for all i (that is, $\boldsymbol{W}$ is *row-stochastic*). Hence, the DeGroot model states that the opinion dynamics for this group will be given by

$$\boldsymbol{x}(k+1) = \boldsymbol{W}\boldsymbol{x}(k), \quad k = 1, 2, \ldots \tag{1}$$

Each edge of this network, $j \xrightarrow{w_{ij}} i$, represents the weight that individual i accords to the opinion from individual j, i.e., the influence of j over i. This also applies for self-loops, $i \xrightarrow{w_{ii}} i$, what could be interpreted as a measure of self-confidence. Also, a trivial way to generate matrix $\boldsymbol{W}$ is considering that all agents assign the same weight to each of its neighbors.

In this model, the condition to ensure that consensus is reached is given by the eigenvalues of $\boldsymbol{W}$. Most precisely, it is dependent on the algebraic multiplicity of its largest eigenvalue. Let $\boldsymbol{\lambda} = [\lambda_1 \ldots \lambda_n]^\top \in \mathbb{C}^n$ be a vector containing the ascending ordered eigenvalues of $\boldsymbol{W}$. Then, Equation (1) converges if and only if $\lambda_n = 1$ is unique in modulus, i.e., the following inequality holds

$$|\lambda_1| \leq |\lambda_2| \leq \ldots \leq |\lambda_{n-1}| < \lambda_n = 1.$$

2.2 Eigenvector Centrality

Eigenvector centrality score is a metric that quantifies the influence of a node over the network, in other words, it measures how many connections a node has and how much its neighbors appreciate its opinion. This measurement was first proposed by Bonacich (1972), and since then, it is widely adopted to define the relative importance of an individual in a social network.

There are many centrality measures such as Katz centrality (Katz, 1953), and the Page Rank centrality (Gleich, 2015), to say some. This work is mainly interested in the eigenvector centrality because of its relation with some network algebraic properties, which will be shown in simulations and experiments.

For a matrix $\boldsymbol{W} \in \mathbb{R}^{n \times n}$, a left eigenvector corresponding to the eigenvalue λ_i is a vector $\boldsymbol{v}_i \in \mathbb{C}^n$ satisfying $\boldsymbol{v}_i^\top W = \lambda_i \boldsymbol{v}_i^\top$.

The eigenvector centrality scores are calculated as

$$\boldsymbol{\sigma} = \boldsymbol{v}_{max} / \|\boldsymbol{v}_{max}\|_1 \tag{2}$$

where $\boldsymbol{v}_{max} \in \mathbb{R}^n$ is the dominant left eigenvector, i.e., associated with the maximum eigenvalue (for a row-stochastic matrix, $|\lambda_n| = 1$). The i-th element of $\boldsymbol{\sigma} = [\sigma_1 \ldots \sigma_n]^\top$ is called the centrality score of agent i.

2.3 Gossip-based Consensus

The gossip-based consensus assumes that two randomly selected agents interact at each time step, in which one (asymmetric) or both (symmetric) of their opinions can be changed. In this paper, we assume an asymmetric and asynchronous gossip algorithm, since it is unreal to consider that groups of agents will act synchronously when arguing about a given issue (Proskurnikov and Tempo, 2018).

The gossip model adopted is grounded at the formulation presented at (Proskurnikov and Tempo, 2018). This model assumes, at each step k, one agent $i = i(k)$ is randomly activated; the sequence $i(k)$ is independent and identically distributed and uniformly distributed in $1, \ldots, n$. It is assumed that agent i interacts with agent j with probability p_{ij}. These probabilities are arranged in the matrix $\boldsymbol{P} = [p_{ij}] \in \mathbb{R}^{n\times n}$. Hence, whenever an interaction occurs, the opinion of agent i is updated as follows

$$y_i(k+1) = (1-\tau_i)y_i(k) + \tau_i y_j(k) \tag{3}$$

where $\tau_i \in (0,1)$ is a constant, describing the "trust" of agent i in its neighbors. The opinions of the other agents (including j) remains unchanged

$$y_l(k+1) = y_l(k) \quad \forall l \neq i(k) \tag{4}$$

2.4 The Proposed Algorithm

The model shown in (3) is not suitable to deal with discrete opinions, considering it calculates a weighted average between the opinions of agents i and j. Hence, the proposed model considers $\tau_i = 1, \forall i$. This way, for allowing agent i to keep its opinion between iterations the probability p_{ii} must be non-zero. That said, it is possible to rewrite equation (3) as

$$y_i(k+1) = y_j(k) \tag{5}$$

This modification guarantees that the final consensus opinion will belong to the set of initial opinions, being that finite set the only possibles opinion states that the agents can assume.

Therefore, to connect the proposed model of consensus over discrete opinions with the applications in social and influence networks the matrix of interactions probabilities, $\boldsymbol{P} = [p_{ij}] \in \mathbb{R}^{n\times n}$, from the gossip-based consensus it is equal to the matrix of influence weights, $\boldsymbol{W}$, from the DeGroot model, i.e.,

$$\boldsymbol{P} \triangleq \boldsymbol{W} \tag{6}$$

By this definition and the agents trust in its neighbors ($\tau_i = 1,\ \forall i$), it is possible to state that the proposed algorithm have its final consensus opinion influenced by the connections between agents and not by the content of the opinions.

Also, we define a matrix, $\boldsymbol{R} = [r_{ij}] \in \mathbb{R}^{n\times n}$, containing the interaction ranges of each agent. These ranges are useful in the computational simulations to randomly select the interactions obeying the distribution defined by $\boldsymbol{P}$. It is used for the specific way in which the algorithm were designed. Hence, each element of $\boldsymbol{R}$ is computed as

$$r_{ij} = \sum_{q=1}^{j} p_{iq},\ i, j = 1, \ldots, n \tag{7}$$

3. COMPUTATIONAL SIMULATIONS

In this section, the results of numerical simulations to support our observations presented previously are shown. The simulations are executed on different networks to evaluate the effect of the topology on opinion diffusion, and how it influences the eigenvector centrality and the spreading probability for each opinion. Algorithm 1 represents the pseudo-code to carry out the simulations.

Algorithm 1 Steps of the computational experiment

```
Set the parameters of the experiment:
define number of agents n and number of simulations n_s.

create the row-stochastic matrix W = [w_ij] from
an arbitrary unweighted adjacency matrix A. The
computation of each element of W is done by normalizing
each row of A.

compute the interaction ranges matrix R = [r_ij] as in
(7)
--------------------------------------------------------
Simulation iterations:
for s = 1, ..., n_s do
    Consensus iterations:
    k = 1
    while ∃(i, j) : y_i(k) ≠ y_j(k) do
        randomly select the active agent i(k) ∈ [1, ..., n]
        following an uniform distribution
        i = i(k)
        generate a single uniformly distributed random
        number, ρ, in the interval (0, 1) for picking the
        agent j for interacting with i in the following loop
        for j = 2, ..., n do
            if ρ > r_i(j−1) and ρ < r_ij then
                y_i(k + 1) = y_j(k)
                break
            else if ρ < r_i1 then
                y_i(k + 1) = y_1(k)
                break
            end if
        end for
        k = k + 1
    end while
    Store the occurrence of each consensus opinion in
    vector e ∈ N^n
end for

compute the eigenvector centrality scores, σ ∈ R^n, for
matrix W and compare to the occurrence ratios, e/n_s.
```

3.1 Reaching Consensus

Twelve different topologies were chosen for testing the algorithm. In each scenario, 100,000 simulations were run to get some statistic knowledge about the algorithm. The initial opinions were the same in all simulations using the same topology. Each agent started with a unique letter from the alphabet as its opinion. The algorithm led the group to a consensus in every simulation run.

The scenarios used can be observed in Figure 1, where the nodes are presented, with their initial opinion, and the connections between them. The appraisal that the agents have to its neighbors, in other words, the values of each connection respect the restriction of the row-stochastic matrix. The components of this weighted adjacency matrix are the probabilities of interaction between the agents. This means each agent assigns equal weights for the opinion of every neighbor, including their own.

3.2 Special cases

Some of the topologies presented in the previous topic can be highlighted here, as some of them have different behavior when it is considered the classic gossip algorithms or even the DeGroot model. As can be verified, topologies (b), (e) and (l) are unable to achieve consensus using these models.

This occurs because the weighted adjacency matrix has its largest eigenvalue with algebraic multiplicity greater than one. The behaviour observed in these scenarios are the interchange of two remaining opinions in the network in an oscillatory way. It is relevant to say that this happens even when the algorithms are averaging over real-valued opinions.

The algorithm presented here, as stated before, is able to reach consensus in those cases. This happens due to asynchrony, a feature usually undesired but that enables the network to break out of those oscillatory states.

3.3 Probability of the Consensus Opinion

As claimed previously, it is desired to test if the centrality of a given agent can be a way of predicting the probability of its initial opinion becoming the consensus opinion for the group. In the carried simulations it was also computed an occurrence ratio, that is, how many times each letter was chosen as the consensus opinion over the total number of simulations.

It is possible to observe in Figure 2 that the occurrence ratio is very close to the eigenvector centrality scores. The error between these two values goes to zero as the number of simulations increases. This is an important result for the proposed algorithm, giving some sense of predictability in its behavior. Following, it will be presented a more detailed analysis of the simulations results.

First, consider the linear topology (Figure 1 (a)) in which all nodes have the same number of neighbors with the exception of the both ends. The nodes with two neighbors have centrality score $\sigma_i = 0.1125$ and the nodes with just one neighbor have $\sigma_i = 0.05$. As expected the percentage of occurrences were very close to their respective centrality scores.

The topologies showed at Figure 1 (b), (d), (e), (h), (i), (j), (k) have the same centrality scores for all nodes in their topology $[0.100, 0.100, 0.100, 0.200, 0.166, 0.125, 0.090]$, respectively. Thus, for these systems all nodes have the same amount of importance in the network, this way the $\boldsymbol{\sigma} = \mathbf{1}_n/n$, where n is the number of nodes in the network. Again all percentage of occurrences were near the expected value.

The topology (c) is the one with more variability in the centrality scores, in which nodes H and J have the greatest influence and the nodes A and G are the least influential. One more time the probabilities approached the centrality scores.

The topologies (f) and (g) have a source and a sink respectively. The sink have $\sigma_i = 0$ and it is not possible to reach consensus in this opinion. Also, the neighbors of the sink node will have lower influence than the others, since the sink node has no influence in the network. And the case with a source node is quite the opposite with the source node with $\sigma_i = 1$ and all the others with $\sigma_i = 0$, indicating that the consensus will always converge for the opinion of the source node.

Finally the topology (l) is the example in which one node has much more influence than all the others, as the central node communicates with everybody and all the others communicates only with the central node. Thus in this case the σ_i for the most influential node is equal to 0.5 and for all the others is equal to 0.1. As expected the probabilities were nearly the centrality scores.

Therefore, the simulations corroborated with the hypothesis that the eigenvector centrality scores can be interpreted as the probability of a certain opinion to occur in the end of the consensus process.

4. LABORATORY EXPERIMENTS

In this section, we perform experiments intending to validate our numerical observations in a practical scenario, where there are communication issues and other physical limitations that may interfere in the information diffusion. The experiments were performed with Kilobots, a low-cost robot platform designed originally by researchers at Harvard University to make the test of algorithms for collective robots accessible to researchers worldwide (Rubenstein et al., 2012).

4.1 Scenario Description

The Kilobots used in the experiments are a personalized version, developed by professors and students from the Department of Automation and Systems at the Federal University of Santa Catarina [1]. The experiments involved five robots arranged in a star topology with self-loops, as described by Figure 3. All border robots can communicate only with the central node, which can reach everyone else.

The robots are all static in a fixed position (they do not move) which allows them to have the communication topology as described by Figure 3(b). Under this configuration, the eigenvector centrality becomes $\boldsymbol{\sigma} = [0.385\ 0.154\ 0.154\ 0.154\ 0.154]$, indicating node 1 as the more influential over the network and the other nodes as equally influential.

[1] `http://kilobots.paginas.ufsc.br/`

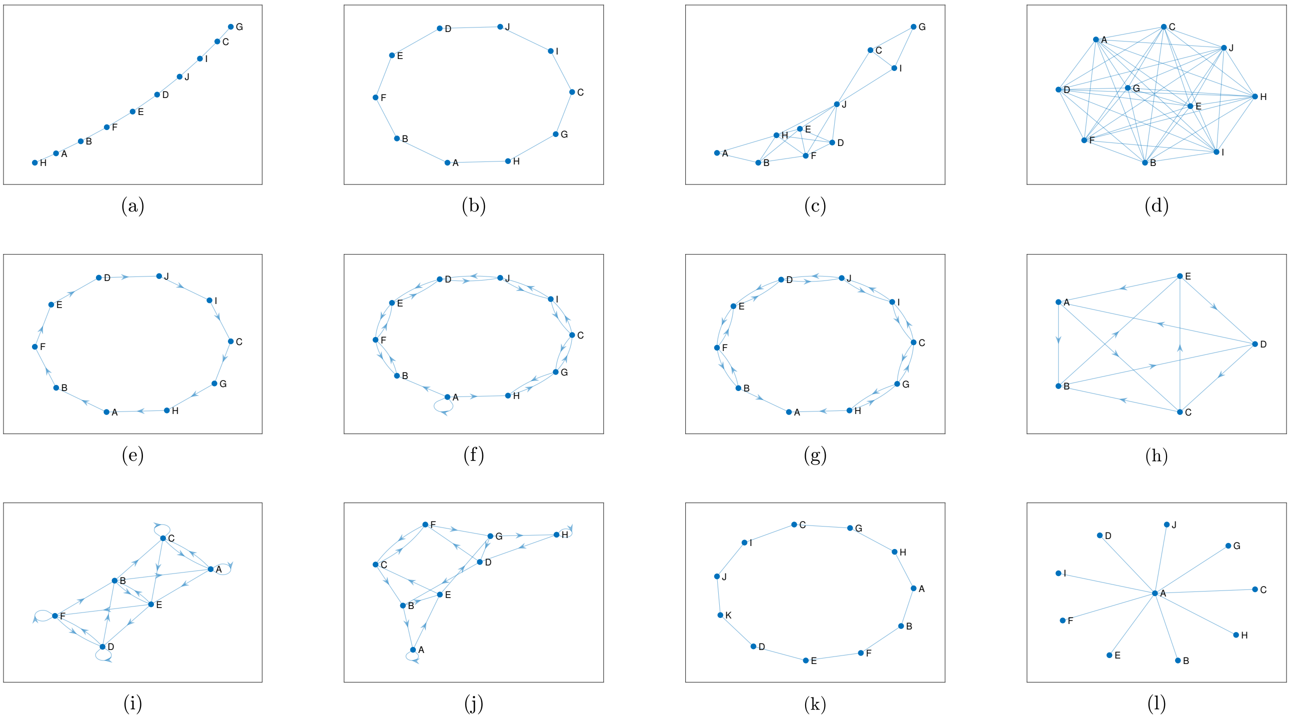


Figure 1. Topologies: (a) Linear graph (b) Circular graph with even number of nodes (c) Arbitrary graph (d) Full connected graph (e) Circular digraph with clockwise direction (f) Circular digraph with source (g) Circular digraph with sink (h) Five nodes, each with two connections (i) Six nodes, each with three connections (j) Eight nodes, each with two connections (k) Circular digraph with odd number of nodes (l) Star graph

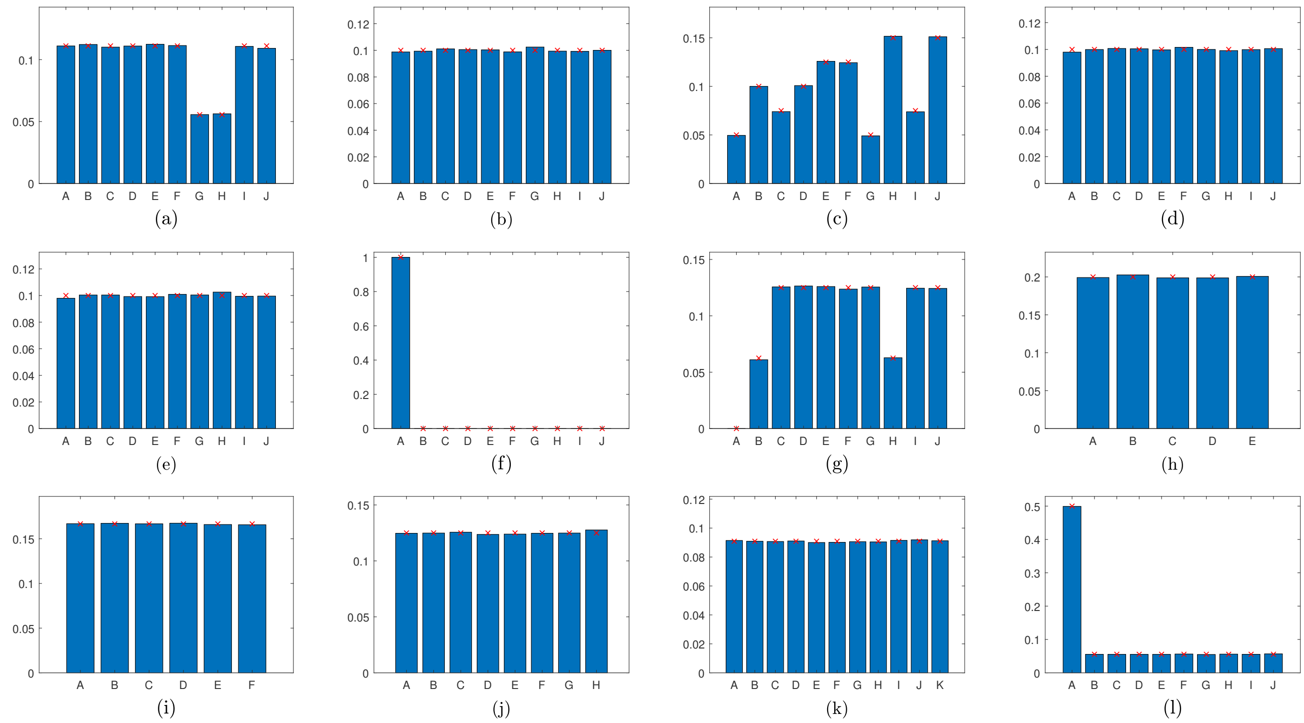
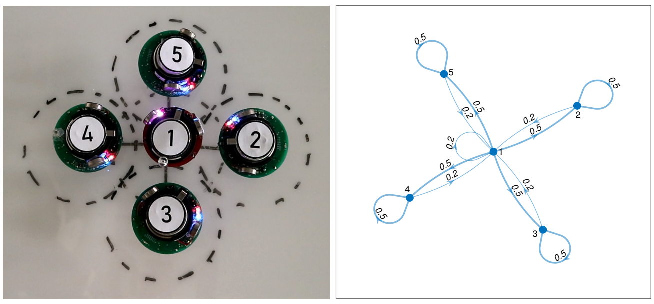


Figure 2. Comparison between the occurrence ratio of each initial opinion and the centrality score (red crosses) of the agents in the respective topologies from Figure 1

The communication issues change the consensus probability for each initial opinion, which differs from the eigenvector centrality score, since, in our case, the transition matrix does not take into account those issues. However, the expected communication issues are limited and relatively small: according to Rubenstein et al. (2012), in an experiment with 25 robots, the communication channel could support on average 32% of usage for five-byte packages



(a) (b)

Figure 3. Adopted topology: (a) top view of the robots, (b) underlying topology graph. Dashed circles represent an estimation of Kilobots communication range, continuous lines are logical connections between them, and the edge weights represent the importance each robot attributes to its neighbors.

without packet loss due to collision. In our case, there are only five robots, and their communication packages have 9 bytes. Despite the influence of communication issues, we expect that the results will not deviate significantly from those obtained in the simulations.

Each robot has its opinion at the beginning of the experiment set as $\mathbf{y}_0 = [\mathrm{A\ B\ C\ D\ E}]$, i.e., all robots have distinct opinions. Such opinions are represented through five different colors that are displayed by the robots through a led: A is purple, B is red, C is green, D is blue, and E is black (light off).

4.2 Results

To verify our hypothesis, we perform three tests [2] composed by sets of observations looking for the prevailing opinion after the consensus be reached. Each test has exactly 50 executions of the asynchronous gossip algorithm under the same initial conditions. By the law of large numbers, we expect that as the number of execution increases, the error between the expected probability of consensus over a particular opinion and the fraction of occurrence of each opinion decreases satisfactorily.

Table 1 contains the occurrence of each opinion at the consensus and the expected values given by the eigenvector centrality. As one can see, consensus in opinion A had a greater occurrence ratio than the other opinions for all tests.

Table 1. Results from the experiments with kilobots

Opinion	Predicted	Test 1	Test 2	Test 3
A (purple)	38.46%	38%	34%	48%
B (red)	15.38%	22%	22%	20%
C (green)	15.38%	14%	12%	10%
D (blue)	15.38%	18%	18%	10%
E (black)	15.38%	8%	14%	12%

By the prediction of eigenvector centrality, opinions B, C, D, and E should have the same probability of occurrence, since the robots that start with these states have the same importance to robot 1 (the center of the star). There is an error between the prediction and occurrence for each opinion: the most significant errors happen at opinion E (7.38%) in Test 1, opinion C (3.38%) in Test 2, and opinions C and D (5.38%) in Test 3. Note that, the increase in the prevalence of opinion B may indicate that link from robot 1 to robot 2 was weaker (due to packet loss or other physical factors) than other links in the network. The same interpretation can be used to explain why in Test 3, opinion A was more frequent than what it is supposed to be, which is probably the result of communication issues.

There may be some variations from one test to another, e.g., lower battery levels. However, the three tests are essentially sessions of the same experiment, allowing us to analyze their outcomes together. Hence, considering the 150 executions of the asynchronous gossip algorithm, we get an aggregate occurrence ratio for each information. In Figure 4, we can see the absolute errors of these outcomes in relation to the values predicted by the eigenvector centrality.

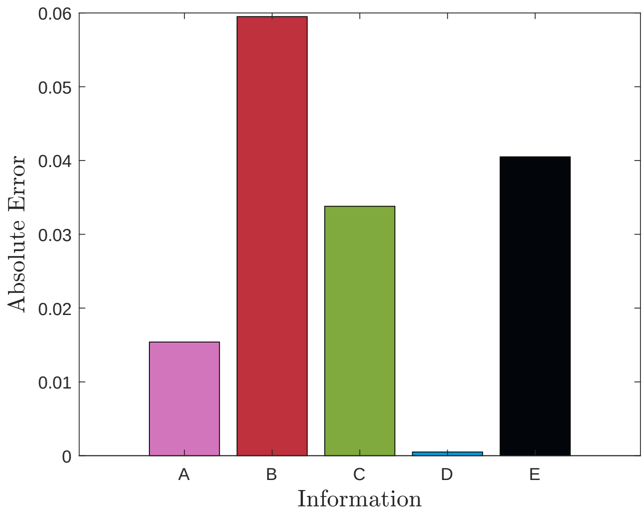


Figure 4. Absolute errors between the eigenvector centrality scores and the aggregate outcomes from all three tests.

Although these errors, the prediction is pretty accurate, given the number of simulations. If it were possible to increase substantially such value, and include in the probability transition matrix the fault probability for each link, it would be expected lower errors in the observations and a more precise validation of our hypothesis.

After all, the simulations and the experiments with Kilobots confirm the hypothesis that eigenvector centrality is indicative of the occurrence probability of each node's initial opinion when the group reaches consensus. For the best of our knowledge, it is the very first time such relation is pointed out in the literature.

5. FINAL REMARKS

We show, through experiments, evidences the proposed algorithm can achieve consensus over the same conditions that previous works did. Also, it can handle discrete opinions and reach consensus in scenarios where synchronous approaches present oscillatory responses. At last, it was confirmed that there is a relation between the eigenvector

[2] Videos available at `http://bit.ly/CBAutomatica2020`

centrality scores and the probability of reaching consensus on a particular opinion.

For future works, we intend to achieve formal mathematical proofs for the convergence of the algorithm and the relationship between algebraic graph theory properties and the probabilities to reach consensus on a certain opinion.

Also, as mentioned in the paper, the presented algorithm has the weights of the connections as the only impacting factor in the final consensus opinion for the network. Hence, there is room for investigation in ways of considering the content of the opinions and not only the influence that agents have in each other. For applications in robotics, factors like the precision of sensors from each robot may be embedded in the weighted adjacency matrix.

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