

Using Prognostics and Health Monitoring Data in Load Distribution Optimization Problems

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Abstract: The use of Remaining Useful Life (RUL) predictions as a decision support tool has increased in recent years. The RUL predictions can be obtained from Prognostics and Health Management (PHM) systems that monitor the health status and estimate the failure instant of components and systems. An example of a decision-making problem that can benefit from RUL predictions is the load distribution problem, which is a common problem that appears in many industrial applications. It consists in defining how to distribute a task among a set of components. In this paper, a model to solve load distribution optimization problems is proposed. The proposed model considers the RUL prediction of each component in its formulation. Also, the proposed model assumes that the predicted RUL of each component is a function of the load assigned to that component. Thus, it is possible to distribute the load to avoid multiple components to fail in a short interval. An approach based on the MMKP (Multiple-choice Multidimensional Knapsack Problem) is adopted. The proposed model finds a load distribution that minimizes the operational cost subject to a maintenance personnel capacity constraint, i.e. there is a maximum number of components that can be simultaneously on repair. A numerical case study considering a gas compressor station is presented to illustrate the application of the proposed model.

Keywords: Prognostics; Health Monitoring; Optimization; Load Distribution; Maintenance.

1. INTRODUCTION

A decision-making problem that appears in many industrial applications is the load distribution problem, which consists in defining how to distribute a specific task among a set of components (Mohammad et al., 2013). Most models available in the literature to solve the load distribution problem based on a cost criterion only. In these models the goal is to minimize total power consumption (Paparella et al., 2013), fuel consumption (Kumar and Cortinovis, 2017), or the number of components that must be turned on to accomplish the desired task (Mohammad et al., 2013).

Maintenance activities may have a huge impact on operational cost and system availability. Therefore, a load distribution model should consider how a load distribution solution affects maintenance activities. For this purpose, Prognostics and Health Management (PHM) systems must provide a Remaining Useful Life (RUL) prediction for each component based on the load assigned to that component. A reliability approach has been proposed to compute the degradation rate of each component based on the load distribution solution (Mohammad et al., 2013).

Each possible solution for a load distribution problem has an associated set of RUL predictions that depends on the load assigned to each component. If these predictions are not considered, the solution may cause multiple failure events in a short interval, and the demand for the maintenance personnel may exceed its capacity. Also, if multiple components are failed, the remaining components may not be able to execute the task accordingly.

In this paper, a load distribution model based on RUL predictions is proposed. The proposed model uses RUL predictions based on the load assigned to each component. An approach based on the MMKP (Multiple-choice Multidimensional Knapsack Problem) is adopted. The proposed model finds a load distribution to minimize total power consumption subject to a maintenance personnel constraint that limits the number of components that can be simultaneously failed due to the capacity of maintenance personnel. A numerical case study considering a gas compressor station is presented to illustrate the application of the proposed model. The results show that, with a small increase in power consumption, the proposed model finds a solution that results in a failure distribution that does not violate the maintenance capacity constraint.

The rest of this paper is organized as follows. Section 2 describes the load distribution problem. Section 3 provides a basic theoretical background on Prognostics and Health Management (PHM) techniques and the Multiple-choice Multidimensional Knapsack Problem (MMKP). The proposed model is presented in section 4. Section 5 shows a case study considering a gas compressor station to illustrate the application of the proposed model. Concluding remarks are presented in section 6.

2. PROBLEM DESCRIPTION

Consider a system composed of n components connected in parallel that are used to execute a task. A maintenance team with a limited repair capacity is responsible for executing the maintenance interventions required by the set of components. It is desired that the number of failed components at any time does not exceed the maintenance team capacity, denoted by k .

A PHM system provides a RUL prediction for each component C_i , with $i = \{1, \dots, n\}$. The RUL prediction for each component C_i , denoted by RUL_i , is a function of the load L_i assigned to the component. The power consumption of each component C_i , denoted by $P_C^{(i)}$, can also be expressed as a function of load L_i .

Figure 1 shows the estimated maintenance demand obtained with two different load distribution solutions. A system with 3 components is considered. The maintenance capacity k is 1. In Figure 1(a), the load distribution caused components C_2 and C_3 to have the same RUL predictions. In this case, maintenance demand would exceed the maintenance capacity $k = 1$.

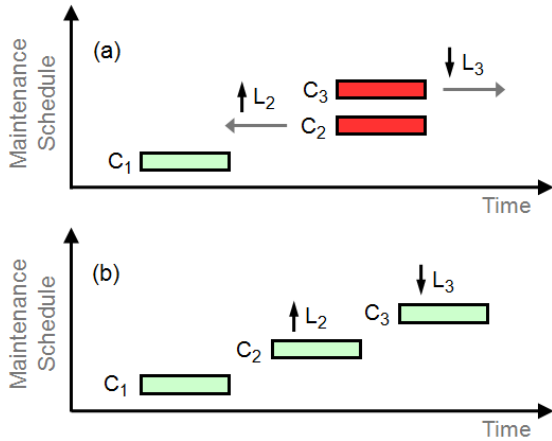


Fig. 1. Impact of load distribution on maintenance demand. (a) Load distribution that causes a simultaneous failure of components C_2 and C_3 . (b) New distribution in which only one component needs a maintenance intervention at a time.

Figure 1(b) shows the estimated maintenance demand obtained with another load distribution. In this new solution, a higher load L_2 is assigned to component C_2 , which reduces its predicted RUL. A lower load L_3 is assigned to component C_3 , which increases its predicted RUL. In the constrained load distribution problem addressed in this paper, the solution used in Figure 1(a) is unfeasible because

it violates the maintenance capacity constraint. Thus, the problem addressed in this paper consists in finding a load distribution that minimizes power consumption subject to a maintenance capacity constraint. Formally, the optimization problem can be defined according to Equations (1) to (4).

$$\min J = \sum_{i=1}^n P_C^{(i)} \quad (1)$$

$$\text{s.t.} \sum_{i=1}^n L_i \geq L_T \quad (2)$$

$$L_i \in \{L_{lb}^{(i)}, L_{ub}^{(i)}\}, \forall i \in \{1, \dots, n\} \quad (3)$$

$$m(t) \leq k, \forall t \in \{t_0, \dots, t_H\} \quad (4)$$

where $m(t)$ is the expected number of failed components in time t , t_H is the maintenance planning horizon, L_T is the total load, and $L_{lb}^{(i)}$ and $L_{ub}^{(i)}$ are the minimum and the maximum load that can be assigned to each component C_i , respectively.

3. THEORETICAL BACKGROUND

3.1 Prognostics and Health Management

Prognostics and Health Management (PHM) techniques are algorithms and methods used to assess the health status and the degradation level of a monitored system. PHM techniques are also used to estimate the Remaining Useful Life (RUL) of monitored systems (Goebel et al., 2007), (Rodrigues, 2018), (El Mejdoubi et al., 2018). The health status is assessed based on a set of sensor measurements that describes the operational behavior of the component. Low degradation levels are associated with new components, while high degradation levels are associated with degraded components. A failure event occurs as soon as the maximum allowed degradation value is reached. By extrapolating the degradation level evolution curve, the RUL prediction is obtained (Eleftheroglou et al., 2019).

Due to measurement uncertainties, RUL predictions are commonly given as a probability distribution. Different distributions can be used to represent RUL probabilities such as Gaussian (Celaya et al., 2012), (Boškoski and Juričić, 2013) or Weibull (Cassity et al., 2012), (Louen et al., 2013).

Figure 2 illustrates the RUL prediction process. Each symbol (+) in Figure 2 represents the degradation index computed at a certain time. The current time is denoted by t_0 . The failure threshold represents the maximum allowed value for the degradation index. i.e. a failure event occurs whenever the degradation level reaches the threshold value. The predicted RUL distribution is obtained by estimating when the degradation index will reach the failure threshold. The expected value for the failure instant is denoted by t_f . The difference $t_f - t_0$ is the predicted RUL.

The literature on PHM techniques contains algorithms for monitoring the health status and predicting the RUL of a wide variety of components such as engines (Babbar et al., 2009), hydraulic pumps (Gomes et al., 2012), and batteries

(Penna et al., 2012). In the last decade, the use of RUL predictions obtained from PHM systems as a decision support tool has been investigated by many researchers. Solutions that incorporate RUL predictions have been proposed for different optimization problems such as preventive maintenance scheduling (Gebrael, 2010), spare parts inventory management (Lin et al., 2017), and resource assignment optimization problems (Rodrigues et al., 2018).

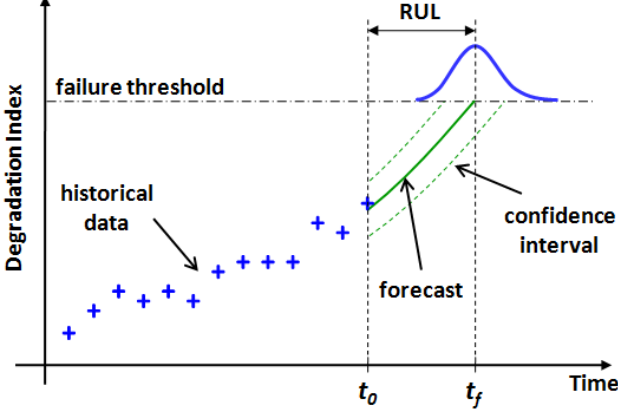


Fig. 2. RUL prediction process.

3.2 MMKP (Multiple-choice Multidimensional Knapsack Problem)

The Multiple-choice Multidimensional Knapsack Problem (MMKP) is a generalization of the well-studied single knapsack problem (Moser et al., 1997). In the MMKP, a group of items is divided into n distinct classes $\{K_1, \dots, K_n\}$. Each class K_i contains m_i items. The j -th item in class K_i has a non-negative value v_{ij} and a weight vector $W_{ij} = (w_{ij}^1, \dots, w_{ij}^u)$. Each element w_{ij}^z of vector W_{ij} , with $z = \{1, \dots, u\}$, is also a non-negative value. The capacity of the knapsack is defined by the capacity vector $C = (c_1, \dots, c_u)$. The goal of the MMKP is to choose exactly one item from each class K_i to maximize the total value of the selected items, subject to the knapsack capacity constraints. The MMKP can be formally defined according to Equations (5) to (8) (Sbihi, 2007).

$$\max J = \sum_{i=1}^n \sum_{j=1}^{m_i} v_{ij} x_{ij} \quad (5)$$

$$\text{s.t. } \sum_{i=1}^n \sum_{j=1}^{m_i} w_{ij}^z x_{ij} \leq c_z, \forall z \in \{1, \dots, u\} \quad (6)$$

$$\sum_{j=1}^{m_i} x_{ij} = 1, \forall i \in \{1, \dots, n\} \quad (7)$$

$$\begin{aligned} x_{ij} &\in \{0, 1\}, \\ \forall i &\in \{1, \dots, n\}, \\ \forall j &\in \{1, \dots, m_i\} \end{aligned} \quad (8)$$

where x_{ij} , with $i = \{1, \dots, n\}$ and $j = \{1, \dots, m_i\}$, are binary decision variables that assume value 1 if the j -th item of class K_i is selected and zero otherwise.

Different algorithms to solve the MMKP have been proposed. Any algorithm can be used in our proposed model. When computational time is restricted, an approximated algorithm can be used. Approximated algorithms provide fast response, however, optimality is not guaranteed (Akbar et al., 2006). In this paper, we use an exact algorithm to solve the MMKP (Sbihi, 2007).

4. PROPOSED MODEL

In this section, the proposed model to solve the constrained version of the load distribution problem is presented. Figure 3 shows a block diagram describing the proposed solution.

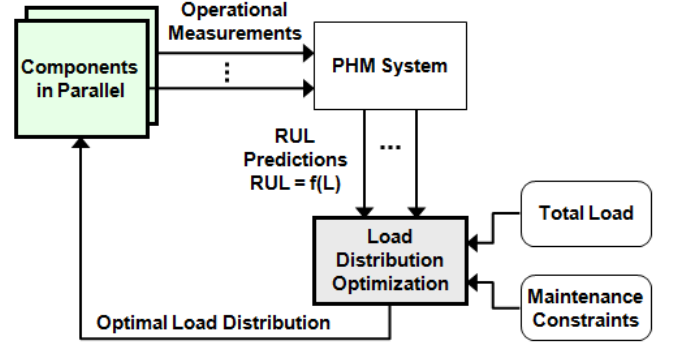


Fig. 3. Block diagram of the proposed model.

As mentioned earlier, the RUL prediction for each component C_i is a function of load L_i . An approach based on the MMKP is used to model the constrained load distribution optimization problem. This approach transforms the problem into a combinatorial optimization problem that has a known procedure to find the optimal solution.

In the proposed model, each component C_i corresponds to a class of the MMKP, and each possible load that can be assigned to component C_i corresponds to an item belonging to class K_i . The cost of operating component C_i with load L_i is associated with the value of the item, and the RUL predictions are associated with the capacity of the knapsack in the MMKP.

Operational measurements are collected from each monitored component and sent to the PHM system. The PHM system computes the predicted RUL for the components as a function of the load assigned to them. The load distribution optimization block in Figure 3 receives the RUL predictions from the PHM system. It also receives the total load that must be distributed among the components and the maintenance personnel constraint. Based on these data, the set of candidate solutions is created and the MMKP is solved to define the best load distribution.

Although the optimal solution for the MMKP can be found, the quality of the final solution depends on the set of items generated in the load distribution optimization block. Some components and systems have a limited number of discrete operation modes such as an EAF (Electric Arc Furnace) in a steel plant (Dalle Ave et al., 2019), or a processor in a computational system (Kong et al., 2010). For these components, it is possible to list all possible operation modes (and the corresponding loads) during

the candidate solution generation process for the MMKP. Other components and systems, however, allow their load to vary continuously within a range. For these components, it is impossible to list all possible operation modes. In order to generate the list of items (candidate loads) for the MMKP, it is necessary to choose a set of possible loads. The higher the number of candidate loads, the higher the probability of finding better solutions. However, the computational time required to solve the MMKP increases with the number of items. The set of items must cover the whole range of load values that can be assigned to the component.

5. CASE STUDY

This section presents a numerical case study to illustrate the application of the proposed model in a gas compressor station.

5.1 System Description

The gas compressor station considered in this example consists of a suction tank and a set of variable speed compressors in parallel. Figure 4 shows a schematic of a gas compressor station with two compressors (Zagorowska et al., 2018).

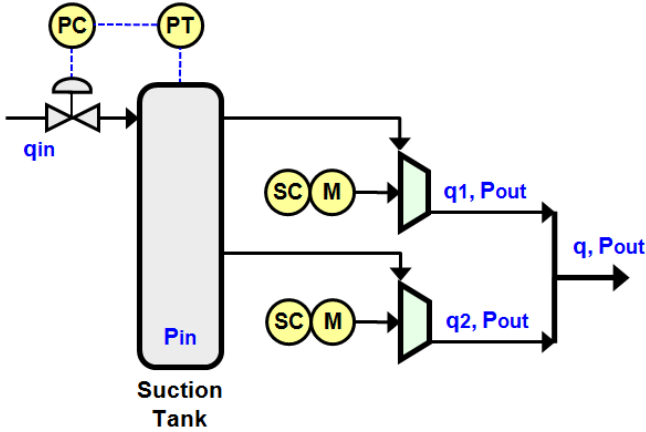


Fig. 4. Schematic of a gas compressor station with two compressors.

The pressure in the suction tank, P_{in} , is controlled by the inlet valve and it is assumed to be constant. A detailed description of the dynamic behavior of the system can be found in the literature (Paparella et al., 2013), (Cortinovis et al., 2016). The station must provide a mass flow rate q at pressure P_{out} . The goal is to assign a mass flow rate to each compressor to provide the desired output with minimum power consumption. A PHM system monitors the health status of compressors and provides a RUL prediction for each compressor as a function of the mass flow rate assigned to it.

Let q_i be the mass flow rate assigned to the i -th compressor. The power consumption of the i -th compressor, denoted by $P_C^{(i)}$, is computed according to Equation (9).

$$P_C^{(i)} = \frac{k \cdot q_i}{\eta_i} \quad (9)$$

where k is a compressor coefficient, and η_i is the compressor efficiency that can be approximated by Equation (10) (Cortinovis et al., 2016).

$$\eta_i(q, \rho) = \alpha_1 q^2 + \alpha_2 \rho^2 + \alpha_3 q \rho + \alpha_4 q + \alpha_5 \rho + \alpha_6 \quad (10)$$

where ρ is the pressure ratio P_{out}/P_{in} , and α_1 to α_6 are model coefficients.

5.2 Simulation Data

In this case study, a gas compressor station with seven compressors in parallel is considered. Table 1 shows the parameters used in the simulations.

Table 1. Simulation Data

Parameter	Description	Value
q	Total flow rate [kg/s]	1.1
k	Model coefficient	30
α_1	Model coefficient	-0.050
α_2	Model coefficient	-0.200
α_3	Model coefficient	0.168
α_4	Model coefficient	-0.166
α_5	Model coefficient	0.580
α_6	Model coefficient	0.180
ρ	Pressure ratio	2.5

Figure 5 presents the efficiency map of each compressor that shows the compressor efficiency η as a function of pressure ratio ρ and mass flow rate q . The maximum mass flow rate of each compressor is 0.2 kg/s. Also, Figure 5 shows the operating envelope of the compressors that limits the range of operating conditions. The operating envelope is limited by the surge line (SL), which defines the minimum flow that avoids instability for a specific pressure ratio. The operating envelope is also limited by the choking line (CL), which is related to the maximum allowed flow. Finally, the operating envelope is limited by two lines that are associated with the maximum and minimum angular speed ω_{max} and ω_{min} .

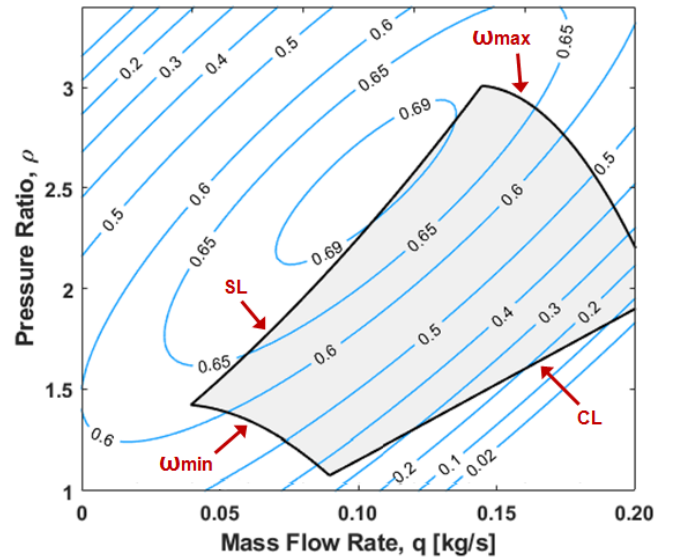


Fig. 5. Compressor efficiency map.

In this case study, the lines that limit the operating envelope are approximated by Equations (11) to (14).

$$SL = 30.75q^2 + 9.45q + 1.00 \quad (11)$$

$$CL = 7.50q + 0.40 \quad (12)$$

$$\omega_{max} = -242.50q^2 + 69.00q - 1.90 \quad (13)$$

$$\omega_{min} = -112.50q^2 + 7.50q + 1.300 \quad (14)$$

The PHM system updates the RUL prediction of each compressor at the end of each operation cycle. We assume that the increment in the degradation level of each compressor in each operation cycle is a random variable that follows a Gamma distribution with scale parameter θ and shape parameter β . Gamma distributions have been adopted in many works to model degradation processes (Van et al., 2012), (Schirru et al., 2010), (Rodrigues, 2018). Table 2 shows the scale and the shape parameters used to model the degradation increment of each compressor. Table 3 shows the current degradation level of each compressor, which were arbitrarily chosen for this example. A failure threshold $FT = 100$ is considered, i.e. a compressor fails whenever its degradation level reaches the failure threshold. A safety level of $S = 5\%$ is adopted, i.e. the RUL prediction corresponds to the operation cycle in which the failure probability is equal to or higher than S .

Table 2. Compressor Degradation Parameters

Parameter	Description	Value
θ	Scale Parameter	1.0
β	Shape Parameter	4.0

Table 3. Current Degradation Levels

Compressor	Degradation
1	54.32
2	25.08
3	80.83
4	27.84
5	52.61
6	50.01
7	65.79

According to Table 1, the compressors must operate at a pressure ratio of 2.5 and provide a total mass flow rate of 1.1 kg/s. Finally, it is assumed that the duration of a maintenance intervention in each compressor is two operation cycles.

A stress effect approach is used to model the influence of load distribution in RUL predictions. A stress model that includes the effect of the mass flow rate q assigned to each compressor in the scale parameter θ is adopted (Duan and Wang, 2019). The variable scale parameter is computed according to Equation (15).

$$\theta(q_i) = \theta_{ref} \cdot \exp \left[\gamma \cdot \left(1 - \frac{q_i}{q_{nom}} \right) \right] \quad (15)$$

where θ_{ref} is the scale parameter of the gamma distribution for nominal load (see Table 2), and γ is a model coefficient. In this paper, a $\gamma = -1.3$ is used.

5.3 Cost Minimization without Considering RUL Predictions

As mentioned earlier, in this load distribution optimization problem, each compressor represents a class of the MMKP, each candidate operation point (q, ρ) represents one item, and each future operation cycle t represents one dimension of the capacity vector C . Therefore, the goal is to find the mass flow rate (item) that will be assigned to each compressor (class) to minimize total power consumption, subject to the maintenance capability constraint.

For comparison purposes, we first solve the load optimization problem without considering the maintenance capability constraint. This result shows the minimum power consumption that can be obtained to meet the gas compressor station output requirements. For each compressor, 13 candidate values for q were used. It can be seen from Figure 5 that, for a pressure ratio of 2.5, the range of values that q can assume is limited by the surge line (SL) and the maximum speed line (ω_{max}). Based on Equations (11) and (13), the range of q is $[0.115, 0.188]$.

Table 4 shows the mass flow rate assigned to each compressor and corresponding power consumption. It can be seen that the desired mass flow rate is obtained with a total power consumption of 275.13 kW.

Table 4. Load Distribution without Considering RUL Predictions

Compressor	q_i [kg/s]	$P_C^{(i)}$ [kW]
1	0.152	36.93
2	0.158	39.70
3	0.158	39.70
4	0.158	39.70
5	0.158	39.70
6	0.158	39.70
7	0.158	39.70

Figure 6 shows the expected number of compressors needing a repair intervention in each operation cycle for the solution obtained without considering RUL predictions in the optimization model. It can be seen that in some operation cycles the maintenance personnel capacity would be exceeded. Also, it can be noticed that compressors 1, 5, and 6 would be simultaneously out of service during operational cycle 18. In this situation, the remaining compressors could not provide the desired mass flow rate.

5.4 Cost Minimization Considering RUL Predictions

Now, the proposed MMKP approach is used to solve the same load distribution problem. However, in this new experiment, the expected number of compressors needing a repair intervention is limited to one. Table 5 shows the mass flow rate assigned to each compressor and corresponding power consumption. It can be seen that the desired mass flow rate is obtained with a total power consumption of 276.53 kW.

Figure 7 shows the expected number of compressors needing a repair intervention in each operation cycle for the

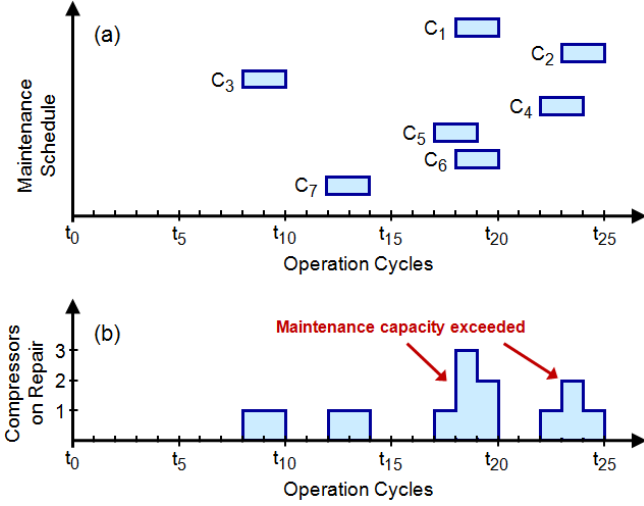


Fig. 6. RUL predictions for each compressor for the load distribution obtained without considering RUL predictions in the optimization model. (a) Expected maintenance interval for each compressor. (b) Number of compressors needing a repair intervention in each operation cycle exceeds maintenance personnel capacity.

Table 5. Load Distribution Considering RUL Predictions

Compressor	q_i [kg/s]	$P_C^{(i)}$ [kW]
1	0.164	42.84
2	0.152	36.93
3	0.158	39.70
4	0.164	42.84
5	0.152	36.93
6	0.146	34.45
7	0.164	42.84

solution obtained with the proposed model that takes RUL predictions into account. When compared with the previous solution, the solution obtained with the proposed model presented a small increase of 0.508% in power consumption. However, the maintenance personnel capacity is not violated.

6. CONCLUSIONS

In this paper, we presented a model to solve the load distribution optimization problem that takes into account the RUL predictions obtained from a PHM system to limit the number of components that will be simultaneously on repair. The proposed model uses an approach based on the MMKP (Multiple-choice Multidimensional Knapsack Problem) to find a solution with minimum cost that does not violate the maintenance personnel repair capacity.

A case study considering a gas compressor station with seven compressors was used to illustrate the application of the proposed model. The results show that the use of RUL predictions to define the load distribution allows the proposed model to find solutions that do not violate the maintenance personnel capacity constraint. Although a small increase in power consumption is observed, the proposed model distributes the repair demand and avoids the situation in which multiple components are failed and

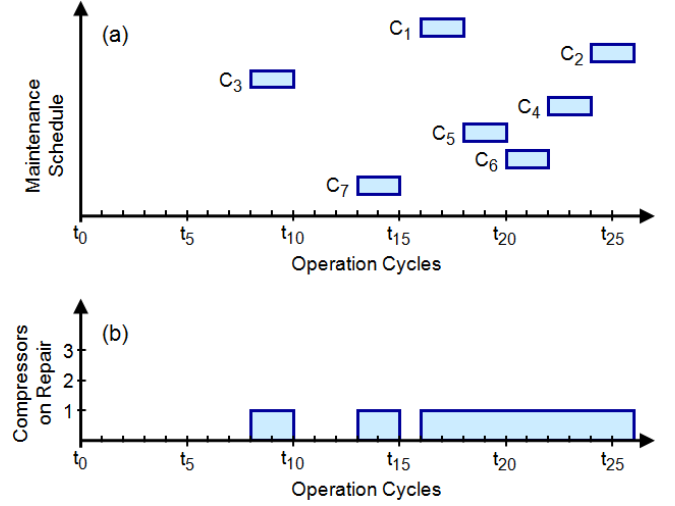


Fig. 7. RUL predictions for each compressor for the load distribution obtained with the proposed model. (a) Expected maintenance interval for each compressor. (b) Number of compressors needing a repair intervention in each operation cycle does not violate the maintenance personnel capacity constraint.

the remaining components do not provide the desired mass flow rate.

Future works may extend the scope of this paper by investigating the impact of uncertainties in maintenance duration and RUL predictions. Another opportunity for future research is to investigate the use of approximated algorithms to solve the MMKP.

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REFERENCES

- Akbar, M.M., Rahman, M.S., Kaykobad, M., Manning, E., and Shoja, G. (2006). Solving the multidimensional multiple-choice knapsack problem by constructing convex hulls. *Computers & Operations Research*, 33(5), 1259–1273. doi:http://dx.doi.org/10.1016/j.cor.2004.09.016.
- Babbar, A., Ortiz, E.M., Syrmos, V.L., and Arita, M.M. (2009). Advanced diagnostics and prognostics for engine health monitoring. In *Proceedings of the 2009 IEEE Aerospace Conference*, 3518–3527. IEEE, Big Sky, Montana.
- Boškoski, P. and Juričić, D. (2013). Modeling localized bearing faults using inverse gaussian mixtures. In *Proceedings of the 2013 Annual conference of the prognostics and health management society*, 107–113. PHM society, New Orleans, Louisiana.
- Cassity, J., Aven, C., and Parker, D. (2012). Applying weibull distribution and discriminant function techniques to predict damaged cup anemometers in the 2011 PHM competition. *International Journal of Prognostics and Health Management*, 3, 1–7.
- Celaya, J.R., Saxena, A., and Goebel, K. (2012). Uncertainty representation and interpretation in model-based

- prognostics algorithms based on Kalman filter estimation. In *Proceedings of the 2012 Annual conference of the prognostics and health management society*, 427–436. PHM society, Minneapolis, Minnesota.
- Cortinovis, A., Mercangöz, M., Zovadelli, M., Pareschi, D., Marco, A.D., and Bittanti, S. (2016). Online performance tracking and load sharing optimization for parallel operation of gas compressors. *Computers & Chemical Engineering*, 88, 145–156. doi:10.1016/j.compchemeng.2016.01.012.
- Dalle Ave, G., Hernandez, J., Harjunkoski, I., Onofri, L., and Engell, S. (2019). Demand side management scheduling formulation for a steel plant considering electrode degradation. *IFAC-PapersOnLine*, 52(1), 691–696.
- Duan, F. and Wang, G. (2019). Optimal design for constant-stress accelerated degradation test based on gamma process. *Communications in Statistics - Theory and Methods*, 48(9), 2229–2253. doi:10.1080/03610926.2018.1459718.
- El Mejdoubi, A., Chaoui, H., Sabor, J., and Gualous, H. (2018). Remaining useful life prognosis of supercapacitors under temperature and voltage aging conditions. *IEEE Transactions on Industrial Electronics*, 65(5), 4357–4367. doi:10.1109/TIE.2017.2767550.
- Eleftheroglou, N., Mansouri, S.S., Loutas, T., Karvelis, P., Georgoulas, G., Nikolakopoulos, G., and Zarouchas, D. (2019). Intelligent data-driven prognostic methodologies for the real-time remaining useful life until the end-of-discharge estimation of the lithium-polymer batteries of unmanned aerial vehicles with uncertainty quantification. *Applied Energy*, 254, 113677. doi:https://doi.org/10.1016/j.apenergy.2019.113677.
- Gebraeel, N. (2010). Prognostics-based identification of the top k units in a fleet. *IEEE Transactions on Automation and Science Engineering*, 7, 37–48.
- Goebel, K., Daigle, M., Abhinav, S., Sankararaman, S., Roychoudhury, I., and Celaya, J. (2007). *Prognostics: The Science of Making Predictions*. CreateSpace, New York, 1 edition.
- Gomes, J.P.P., Leão, B.P., Vianna, W.O.L., Galvão, R.K.H., and Yoneyama, T. (2012). Failure prognostics of a hydraulic pump using Kalman filter. In *Proceedings of the 2012 Annual conference of the prognostics and health management society*, 464–468. PHM society, Minneapolis, Minnesota.
- Kong, F., Wang, Y., Deng, Q., and Yi, W. (2010). Minimizing multi-resource energy for real-time systems with discrete operation modes. In *2010 22nd Euromicro Conference on Real-Time Systems*, 113–122.
- Kumar, S. and Cortinovis, A. (2017). Load sharing optimization for parallel and serial compressor stations. In *2017 IEEE Conference on Control Technology and Applications (CCTA)*, 499–504.
- Lin, X., Basten, R.J.I., Kranenburg, A.A., and van Houtum, G.J. (2017). Condition based spare parts supply. *Reliability Engineering & System Safety*.
- Louen, C., Ding, S.X., and Kandler, C. (2013). A new framework for remaining useful life estimation using support vector machine classifier. In *Proceedings of the 2013 Conference on Control and Fault-Tolerant Systems*, 228–233. Curran Associates, Nice, France.
- Mohammad, R., Kalam, A., and Amari, S.V. (2013). Reliability of load-sharing systems subject to proportional hazards model. In *Proceedings of the 2013 Annual Reliability and Maintainability Symposium (RAMS)*, 1–5. Orlando.
- Moser, M., Jokanović, D.P., and Shiratori, N. (1997). An algorithm for the multidimensional multiple-choice knapsack problem. *IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences*, 80, 582–589.
- Paparella, F., Domínguez, L., Cortinovis, A., Mercangöz, M., Pareschi, D., and Bittanti, S. (2013). Load sharing optimization of parallel compressors. In *2013 European Control Conference (ECC)*, 4059–4064. Zurich. doi:10.23919/ECC.2013.6669697.
- Penna, J.A.M., Nascimento Jr., C.L., and Rodrigues, L.R. (2012). Health monitoring and remaining useful life estimation of lithium-ion aerospace batteries. In *Proceedings of the 2012 IEEE Aerospace Conference*. IEEE, Big Sky, Montana.
- Rodrigues, L.R. (2018). Remaining useful life prediction for multiple-component systems based on a system-level performance indicator. *IEEE/ASME Transactions on Mechatronics*, 23(1), 141–150. doi:10.1109/TMECH.2017.2713722.
- Rodrigues, L.R., Gomes, J.P.P., and Alcântara, J.F.L. (2018). Embedding remaining useful life predictions into a modified receding horizon task assignment algorithm to solve task allocation problems. *Journal of Intelligent & Robotic Systems*, 90(1), 133–145. doi:10.1007/s10846-017-0649-8.
- Sbihi, A. (2007). A best first search exact algorithm for the multiple-choice multidimensional knapsack problem. *Journal of Combinatorial Optimization*, 13, 337–351.
- Schirru, A., Pampuri, S., and Nicolao, G.D. (2010). Particle filtering of hidden gamma processes for robust predictive maintenance in semiconductor manufacturing. In *2010 IEEE International Conference on Automation Science and Engineering*, 51–56. doi:10.1109/COASE.2010.5584518.
- Van, P.D., Voisin, A., Levrat, E., and Iung, B. (2012). Condition-based maintenance with both perfect and imperfect maintenance actions. In *Proceedings of the 2012 Annual conference of the prognostics and health management society*, 256–264. PHM Society, Minneapolis, Minnesota.
- Zagorowska, M., Thornhill, N., Haugen, T., and Skourup, C. (2018). Load-sharing strategy taking account of compressor degradation. In *2018 IEEE Conference on Control Technology and Applications (CCTA)*, 489–495. Copenhagen. doi:10.1109/CCTA.2018.8511550.