DOI: 10.48011/asba.v2i1.1209

Control of a MIMO Magnetic Levitation System using Exponential Control Barrier Function ⋆

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Abstract: This work presents the control of a magnetic levitation system. The system is constituted by a Y shape metal plate that must be levitated by electromagnetic attractive forces. The system is nonlinear, open loop unstable and Multiple-Input/Multiple-Output (MIMO), whose inputs are represented by attractive forces generated from three electromagnets and outputs are represented by three plate positions. The proposed control structure uses Quadratic Programming (QP) to combine performance/stability objectives, represented by an arbitrary nominal control law, and safety constraints, represented by Control Barrier Functions (CBFs). The arbitrary nominal control law applied is determined by feedback linearization. Multiple safety constraints with relative-degree greater than one were applied. One way to deal with this is to use Exponential Control Barrier Functions (ECBFs). The results of this control structure applied to the magnetic levitation system are obtained through numerical simulations and indicate that performance/stability objectives are reached and safety constraints are respected.

Keywords: Control Barrier Function, Quadratic Programming, Magnetic Levitation System, Multivariable Control, Feedback Linearization.

1. INTRODUCTION

Magnetic levitation systems can be used in several engineering applications such as magnetic bearings, high precision positioning platform, aerospace shuttles, maglev trains, steel and semiconductor manufacturing plants and educational purposes (Yu and Li (2014)). The most popular and widely used scheme for the magnetic levitation system consists of a metal body, such as a ball, plate or disk suspended by a voltage-controlled magnetic field obtained from an electromagnet (Hajjaji and Ouladsine (2001)). The objective is to keep the metal body at a prescribed reference level. The electromagnet current may be increased until the magnetic force produced compensate the gravitational force acting on the metal body. This system is nonlinear, open loop unstable and Single-Input/Single-Output (SISO) (Barie and Chiasson (1996)).

The magnetic levitation system scheme analyzed in this work is based on the experimental apparatus described in Fujii et al. (1994) and Tsujino et al. (1999). The system is constituted by a Y shape metal plate that must be levitated by electromagnetic attractive forces. The system is also nonlinear and open loop unstable, however, it is a Multiple-Input/Multiple-Output (MIMO) system. The inputs are represented by attractive forces generated from three electromagnets and the outputs are represented by three plate positions. A controller must be designed so that the plate positions track reference inputs with adequate performance.

The works related to control of magnetic levitation systems typically are proposed to satisfy performance/stability objectives, i.e., tracking a desired reference input. Several control techniques are proposed and applied in the literature, such as sliding mode (Al-Muthairi and Zribi (2004)), fuzzy logic (Benomair and Tokhi (2015)), model predictive control (Karampoorian and Mohseni (2010)), backstepping (Liu and Zhou (2013)), neural network (M. Aliasghary and Teshnehlab (2008)) and H_∞ control (Tsujino et al. (1999)). However, this work uses a control structure that simultaneously satisfy performance/stability objectives and safety constraints.

The first study related to safety of dynamical systems was done by Nagumo in the 1940s. This study provided necessary and sufficient conditions for set invariance (Nagumo (1942)). In the 2000s, barrier certificates were introduced to prove safety of nonlinear and hybrid systems (Prajna and Jadbabaie (2004), Prajna (2006), Prajna and Rantzer (2005)). The term "barrier" is related to barrier functions, which, in optimization problems, are added to cost functions to avoid undesirable regions. In Tee et al. (2009), the concept of barrier Lyapunov function was presented, which guarantees set invariance or safety using a "Lyapunov-like" approach. The notion of a barrier certificate was extended to a "control" version to yield the first definition of a Control Barrier Function (CBF), as presented in Wieland and Allgower (2007). In Romdlony and Jayawardhana (2016),

⋆ Authors thank Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) for Grant 2017/22130-4.

barrier functions were combined with Control Lyapunov Function (CLF), creating control Lyapunov barrier function, that guarantees simultaneously safety and stability.

The most recent formulation related to CBF is presented in details in Ames et al. (2019) and Ames et al. (2017). This methodology imposes new conditions on CBF, making the problem less restrictive. It combines CLF or an arbitrary nominal control law, related to performance/stability objectives and CBF, related to safety conditions represented by a safe set. These objectives can be integrated with Quadratic Programming (QP). Several applications using this formulation are proposed in the literature, such as adaptive cruise control (Xu et al. (2017), Mehra et al. (2015)), bipedal walking robot (Hsu et al. (2015)), robotic manipulator (Rauscher et al. (2016)), two-wheeled inverted pendulum (Gurriet et al. (2018)), quadrotors (Wu and Sreenath (2016)) and multi-robot systems (Wang et al. (2017)).

The control structure proposed in this work for the MIMO magnetic levitation system follows the formulation described above. The performance/stability objectives are that the plate positions track the reference inputs. To satisfy this, it is applied feedback linearization to generate an arbitrary nominal control law. The safety conditions are considered to guarantee that the plate positions never exceed predetermined values. To satisfy this, multiple safety conditions with relative-degree greater than one are applied. One way to deal with this is to use Exponential Control Barrier Functions (ECBFs) as described in Nguyen and Sreenath (2016) and Ames et al. (2019). The results of this control structure applied to the magnetic levitation system are obtained through numerical simulations.

In section 2, the modeling of MIMO magnetic levitation system is described. The nominal control law, the concepts of CBF and ECBF, and the final control structure are presented in section 3. Simulation results and conclusions are presented in sections 4 and 5, respectively.

2. SYSTEM MODELING

The schematic diagram of the magnetic levitation system is presented in Fig. 1. The system is constituted by a Y shape plate made of aluminum with small pieces of iron mounted at the edges and that must be levitated by electromagnetic forces. The inputs are represented by attractive forces generated from three electromagnets. The controller provides voltage command signals V_1, V_2 and V_3 that are converted to proportional current signals i_1, i_2 and i_3 by power amplifiers in order to generate the corresponding attractive forces F_1, F_2 and F_3. The outputs are represented by three plate positions r_1, r_2 and r_3, measured by gap sensors mounted below the edges of the plate.

The coordinate axis of the plate X_V, X_p and X_r are presented in Fig. 2. x_v is the vertical gap length between the electromagnet and the plate at the origin O, right above the center of gravity G, while θ_p and θ_r are the pitching and rotating angles, respectively. The parameters of the magnetic levitation system are presented in Fig. 3. M is the mass of the plate, J_p and J_r are the moments of inertia around the origin O in pitching X_p and rolling X_r

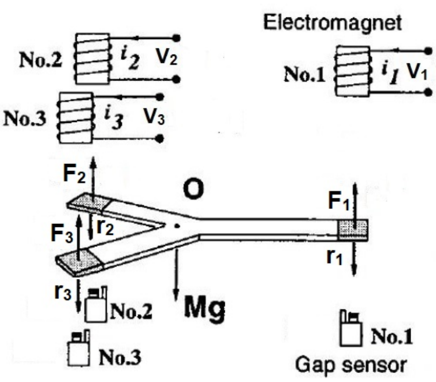


Fig. 1. Schematic diagram of the magnetic levitation system (Tsujino et al. (1999))

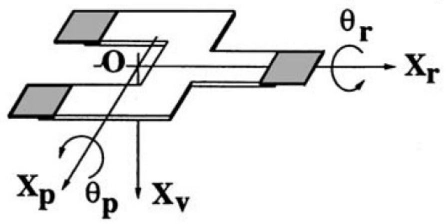


Fig. 2. Coordinate axis of the plate (Tsujino et al. (1999))

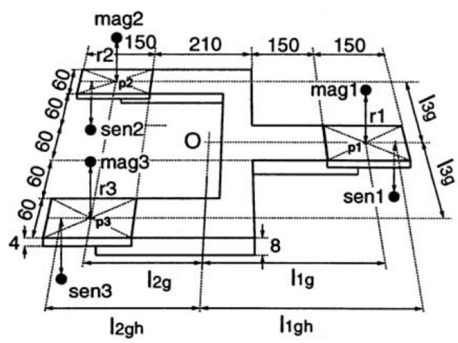


Fig. 3. Parameters of the magnetic levitation system (Tsujino et al. (1999))

directions, respectively, g is the gravitational acceleration and k_1, k_2 and k_3 are the constants related to each electromagnet (Tsujino et al. (1999)). Other parameters can be seen directly in the Fig. 3.

Under several idealized assumptions, the equations of vertical, pitching and rotating motions can be described respectively as

$$M\ddot{x}_v = Mg - (F_1 + F_2 + F_3), \tag{1}$$

$$J_p\ddot{\theta}_p = F_1 l_{1g} - (F_2 + F_3)l_{2g} - Mgd\sin\theta_p, \tag{2}$$

$$J_r\ddot{\theta}_r = (F_2 - F_3)l_{3g} - Mgd\sin\theta_r, \tag{3}$$

where d is the distance between the origin O and the center of gravity G.

The plate positions r_1, r_2 and r_3 have the same directions as x_v and are given by

$$r_1 = x_v - l_{1g}\tan\theta_p, \tag{4}$$

$$r_2 = x_v + l_{2g} \tan\theta_p - l_{3g} \tan\theta_r, \quad (5)$$
$$r_3 = x_v + l_{2g} \tan\theta_p + l_{3g} \tan\theta_r, \quad (6)$$

and the electromagnets attractive forces can be written as a nonlinear function of the input voltages V_j and plate positions r_j, such that:

$$F_j := k_j \left(\frac{V_j}{r_j}\right)^2 \quad j = 1, 2, 3. \quad (7)$$

The system can be represented by:

$$\dot{x} = f(x) + g(x)u, \quad (8)$$
$$y = o(x), \quad (9)$$

where $x = \begin{bmatrix} x_v & \theta_p & \theta_r & \dot{x}_v & \dot{\theta}_p & \dot{\theta}_r \end{bmatrix}^T \in D \subset \mathbb{R}^n$ is the state vector, $u = [F_1 \;\; F_2 \;\; F_3]^T \in U \subset \mathbb{R}^m$ is the input vector, $y = [r_1 \;\; r_2 \;\; r_3]^T \in D \subset \mathbb{R}^{no}$ is the output vector,

$$f(x) = \begin{bmatrix} \dot{x}_v & \dot{\theta}_p & \dot{\theta}_r & g & \dfrac{-Mgd\sin\theta_p}{J_p} & \dfrac{-Mgd\sin\theta_r}{J_r} \end{bmatrix}^T, \quad (10)$$

$$g(x) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\dfrac{1}{M} & -\dfrac{1}{M} & -\dfrac{1}{M} \\ \dfrac{l_{1g}}{J_p} & -\dfrac{l_{2g}}{J_p} & -\dfrac{l_{2g}}{J_p} \\ 0 & \dfrac{l_{3g}}{J_r} & -\dfrac{l_{3g}}{J_r} \end{bmatrix}, \quad (11)$$

and

$$o(x) = \begin{bmatrix} x_v - l_{1g} \tan\theta_p \\ x_v + l_{2g} \tan\theta_p - l_{3g} \tan\theta_r \\ x_v + l_{2g} \tan\theta_p + l_{3g} \tan\theta_r \end{bmatrix}. \quad (12)$$

3. CONTROL STRUCTURE

This section presents the concepts of CBF, ECBF and the final control structure that integrates the nominal control law and the ECBFs through QP.

3.1 Nominal Control Law - Feedback Linearization

As previously mentioned, a nominal control law must be designed so that the plate positions r_1, r_2 and r_3 track reference inputs r_{1d}, r_{2d} and r_{3d} to satisfy performance/stability objectives. It is applied a feedback linearization.

The central idea of feedback linearization is to algebraically transform a nonlinear system dynamics into a (fully or partly) linear one, so that a linear control techniques can be applied. It amounts to canceling the nonlinearities in a nonlinear system so that the closed-loop dynamics becomes a linear form, and a desired linear dynamics can be imposed (Slotine and Li (1991)).

The system modeling is represented in (8) and (9). A nominal control input $u_{no} \in \mathbb{R}^3$ must be designed to make the output $y \in \mathbb{R}^3$ tracks a desired trajectory $y_d \in \mathbb{R}^3$, while keeping the whole state $x \in \mathbb{R}^6$ bounded, where $y_d = [r_{1d} \;\; r_{2d} \;\; r_{3d}]^T$, and its time derivatives up to a sufficiently high order are assumed to be known and bounded. In the model described, the output is not directly related to the control input, so it is applied the input-output linearization approach (Slotine and Li (1991)).

To generate a direct relationship between the output y and the input u_{no}, the output must be differentiated twice, such that:

$$\ddot{y} = f_y(x) + g_y(x)u_{no}, \quad (13)$$

where $f_y(x) \in \mathbb{R}^3$ and $g_y(x) \in \mathbb{R}^{3\times3}$ are nonlinear functions of the state.

The control input u_{no} is determined to cancel the nonlinearities. To do this, we have:

$$u_{no} = g_y(x)^{-1} [v - f_y(x)], \quad (14)$$

where $v \in \mathbb{R}^3$ is a new input to be determined using a linear control technique. Letting $e = y - y_d$ be the tracking error, we choose:

$$v = \ddot{y}_d - k_{c1}e - k_{c2}\dot{e}, \quad (15)$$

with k_{c1} and k_{c2} being positive constants.

3.2 Control Barrier Function

As previously mentioned, safety conditions must be imposed in the system such that the plate positions r_1, r_2 and r_3 never exceeds predetermined values.

Safety requires that "bad" things do not happen, such as invariance of a set C. Any trajectory starting inside an invariant set will never reach the complement of the set (Ames et al. (2019)). Safety can be mathematically related to CBFs.

A barrier function $h(x)$ vanishes on the set C boundary, i.e., $h(x) \to 0$ as $x \to \partial C$. If $h(x)$ satisfy Lyapunov-like conditions, then forward invariance of C is guaranteed (Ames et al. (2019)). The natural extension of a barrier function to a system with control inputs is a CBF (Wieland and Allgower (2007)). In CBFs, we impose inequality constraints on the derivative to obtain entire classes of controllers that render a given set forward invariant.

In particular, we consider a set C defined as the superlevel safe set of a continuously differentiable function $h(x) : D \subset \mathbb{R}^n \to \mathbb{R}$ yielding (Ames et al. (2019)):

$$\begin{aligned} C &= \{x \in D \subset \mathbb{R}^n : h(x) \geq 0\}, \\ \partial C &= \{x \in D \subset \mathbb{R}^n : h(x) = 0\}, \\ \mathrm{Int}(C) &= \{x \in D \subset \mathbb{R}^n : h(x) > 0\}. \end{aligned} \quad (16)$$

The definition of safety is given by:

Definition 1. Let $u = k(x)$ be a feedback controller such that (8) is locally Lipschitz. For any initial condition

$x_0 \in D$ there exists a maximum interval of existence $I(x_0)$ such that $x(t)$ is the unique solution to (8) on $I(x_0)$. The set C is forward invariant if for every $x_0 \in C$, $x(t) \in C$ for $x(0) = x_0$ and all $t \in I(x_0)$. The system (8) is safe with respect to the set C if the set C is forward invariant (Ames et al. (2019)).

Considering $L_f h = \nabla h(x) \cdot f(x)$ and $L_g h = \nabla h(x) \cdot g(x)$, the formal definition of CBF is given by:

Definition 2. Consider the control system (8) and the set $C \subset \mathbb{R}^n$ defined by (16) for a continuously differentiable function $h(x) : \mathbb{R}^n \to \mathbb{R}$. The function $h(x)$ is called a CBF defined on the set D with $C \subseteq D \subset \mathbb{R}^n$, if there exists an extended class κ functions α such that (Ames et al. (2017))

$$\sup_{u \in U} \left[L_f h(x) + L_g h(x) u + \alpha(h(x))\right] \geq 0, \forall x \in D. \tag{17}$$

Given a CBF $h(x)$, for all $x \in D$, define the set (Ames et al. (2017))

$$K_{cbf}(x) = \left\{u \in U : L_f h(x) + L_g h(x) u + \alpha(h(x)) \geq 0\right\}. \tag{18}$$

With this definition, we have the following corollary:

Corollary 1. Consider a set $C \subset \mathbb{R}^n$ defined by (16) and let $h(x)$ be an associated CBF for the system (8). Then any locally Lipschitz continuous controller $u : D \to U$ such that $u(x) \in K_{cbf}(x)$ will render the set C forward invariant (Ames et al. (2017)).

Equation (18) shows that the input u only influences the system for $L_g h(x) \neq 0$, so $h(x)$ has to be designed such that $\dot{h}(x)$ depends directly on u, i.e., relative-degree one. However, in several systems, such as robotics, the constraints have relative-degree greater than one, i.e., $L_g h(x) = 0$. In this work, the constraints have relative-degree two. To deal with this, it is necessary to apply ECBFs.

3.3 Exponential Control Barrier Function

CBFs for high-relative degree safety constraints were studied in Hsu et al. (2015), Wu and Sreenath (2015) and Nguyen and Sreenath (2016). The results in Wu and Sreenath (2015) only extended to position-based safety constraints with relative-degree two. In Hsu et al. (2015), a backstepping-based method is applied to arbitrary high relative-degree safety constraints. However, backstepping-based CBF design for higher relative-degree systems (greater than two) is challenging and has not been attempted. In Nguyen and Sreenath (2016), the concept of ECBFs was first introduced as a way to easily enforce high relative-degree safety constraints.

Consider the system (8) with initial condition x_0 with the goal to enforce the forward invariance of the safe set C defined in (16) and supposing that $h(x)$ has arbitrarily high relative-degree $r \geq 1$. Considering $L_f^r h(x) = L_f L_f^{r-1} h(x) = \nabla(L_f^{r-1} h(x)) \cdot f(x)$ and $L_g L_f^{r-1} h(x) = \nabla(L_f^{r-1} h(x)) \cdot g(x)$, the r-th time-derivative of $h(x)$ is given by (Ames et al. (2019)):

$$h^{(r)}(x, u) = L_f^r h(x) + L_g L_f^{r-1} h(x) u, \tag{19}$$

with $L_g L_f^{r-1} h(x) \neq 0$ and $L_g L_f h(x) = L_g L_f^2 h(x) = \cdots = L_g L_f^{r-2} h(x) = 0, \forall x \in D$. Next, we define,

$$\eta_b(x) := \begin{bmatrix} h(x) \\ \dot{h}(x) \\ \ddot{h}(x) \\ \vdots \\ h^{(r-1)}(x) \end{bmatrix} = \begin{bmatrix} h(x) \\ L_f h(x) \\ L_f^2 h(x) \\ \vdots \\ L_f^{(r-1)} h(x) \end{bmatrix}, \tag{20}$$

and assume that, for a given $\mu \in U_\mu \subset \mathbb{R}$, u can be chosen such that $L_f^r h(x) + L_g L_f^{r-1} h(x) u = \mu$. This choice of u is possible since by the relative-degree of $h(x)$ we have $L_g L_f^{r-1} h(x) \neq 0, \forall x$ and moreover μ is a scalar (while $u \in U \subset \mathbb{R}^m$). With this, the above dynamics of $h(x)$ can be written as the linear system (Ames et al. (2019)),

$$\begin{aligned} \dot{\eta}_b(x) &= F \eta_b(x) + G \mu, \\ h(x) &= H \eta_b(x), \end{aligned} \tag{21}$$

where

$$F = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, G = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, H = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}. \tag{22}$$

Clearly, if we choose a state feedback style $\mu = -K_\alpha \eta_b(x)$ then $h(x(t)) = H e^{(F - G K_\alpha)t} \eta_b(x_0)$.

We now have everything setup to define ECBF (Ames et al. (2019)).

Definition 3. Given a set $C \subset D \subset \mathbb{R}^n$ defined as the superlevel set of a r-times continuously differentiable function $h : D \to \mathbb{R}$, then h is an ECBF if there exists a row vector $K_\alpha \in \mathbb{R}^r$ such that for the control system (8),

$$\sup_{u \in U} \left[L_f^r h(x) + L_g L_f^{r-1} h(x) u\right] \geq -K_\alpha \eta_b(x) \tag{23}$$

$\forall x \in \text{Int}(C)$ results in $h(x(t)) \geq C e^{(F - G K_\alpha)t} \eta_b(x_0) \geq 0$ whenever $h(x_0) \geq 0$.

3.4 Integrating Nominal Control Law and ECBF Through QP

The final control structure integrates the nominal control law and the ECBFs through QP.

The aim of the combined control law is to apply the nominal control whenever possible, which is formulated as an optimization problem, minimizing the error (Rauscher et al. (2016))

$$e_u = u_{no} - u \tag{24}$$

between the nominal control u_{no}, shown in (14), and the applied control u. The squared norm of the error

$$\|e_u\|^2 = u^T u - 2u_{no}^T u + u_{no}^T u_{no} \tag{25}$$

is used as objective function. We neglect the last term of (25), since it is constant in a minimization with respect to u. As the problem is MIMO and we impose safety conditions in all plate positions r_1, r_2 and r_3, it is applied multiple ECBFs. So we consider the following QP based controller (Rauscher et al. (2016), Ames et al. (2019)):

$$\begin{aligned}
u^*(x) = \underset{(u,\mu)\in\mathbb{R}^{m+l}}{\arg\min} \ & u^T u - 2u_{no}^T u \\
s.t. \ & L_f^r h_1(x) + L_g L_f^{r-1} h_1(x) u = \mu_1 \\
& \vdots \\
& L_f^r h_l(x) + L_g L_f^{r-1} h_l(x) u = \mu_l \\
& \mu_1 \geq -K_{\alpha_1} \eta_{b_1}(x) \\
& \vdots \\
& \mu_l \geq -K_{\alpha_l} \eta_{b_l}(x)
\end{aligned} \tag{26}$$

where m is the number of control inputs, l is the number of safety constraints or ECBFs and μ_j, $\eta_{b_j}(x)$ and K_{α_j} are defined in (20) and (21), for $j = 1, 2, ..., l$. It is important to highlight that the constraints in the QP enforces the condition given by (23) for ECBF.

4. SIMULATION RESULTS

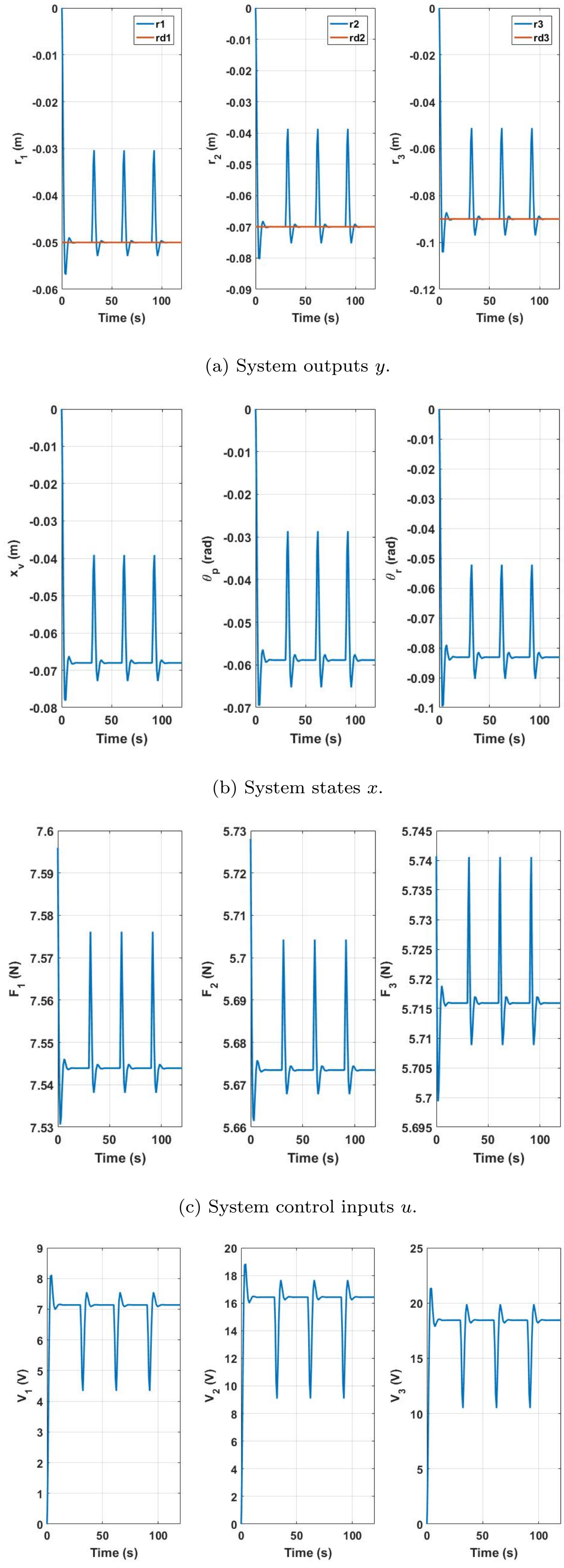
The behaviour of the magnetic levitation system with the proposed control structure was verified through numerical simulations with Matlab/Simulink. We consider two simulation experiments. The parameters of the magnetic levitation system, described in Tsujino et al. (1999), are $l_{1g} = 0.306\ m$, $l_{2g} = 0.203\ m$, $l_{3g} = 0.120\ m$, $M = 1.93\ Kg$, $g = 9.81\ m/s^2$, $J_p = 6.43 \times 10^{-2}\ kgm^2$, $J_r = 1.82 \times 10^{-2}\ kgm^2$, $k_1 = 3.70 \times 10^{-4}\ Nm^2/V$, $k_2 = 1.03 \times 10^{-4}\ Nm^2/V$, $k_3 = 1.36 \times 10^{-4}\ Nm^2/V$ and $d = 3.24 \times 10^{-3}\ m$.

In the simulation 1, the parameters for the nominal control law were $k_{c1} = 1$ and $k_{c2} = 1$. The plate starts at $r_{10} = 0m$, $r_{20} = 0m$ and $r_{30} = 0m$ and must be levitated above this point according to the input references $r_{d1} = -0.050m$, $r_{d2} = -0.070m$ and $r_{d3} = -0.090m$. It is important to highlight that this values are negative because the positive direction of X_V coordinate axis, shown in Fig. 2, is down, so a levitation above the initial condition will be represented by a negative value.

Initially, only the nominal control law was applied, i.e., the safety conditions or ECBFs were not considered, and input disturbances were applied periodically. The simulation results are shown in Fig. 4. In Fig. 4a, the input references are presented in red. It can be observed that the system outputs track the references, according to performance/stability objectives, however the periodic input disturbances generate high amplitude errors. The input voltages, shown in Fig. 4d, can be obtained using (7).

The amplitude of errors in system outputs due to disturbances must be limited. This can be solved applying three safety constraints ($l = 3$) given by

$$h_1(x) = (r_{1max})^2 - (r_1 - r_{d1})^2, \tag{27}$$



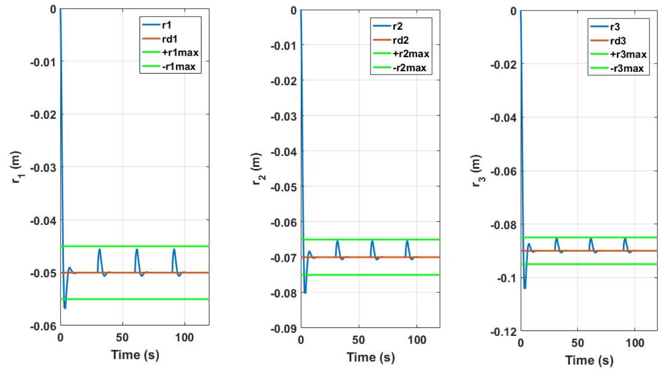
(a) System outputs y.

(b) System states x.

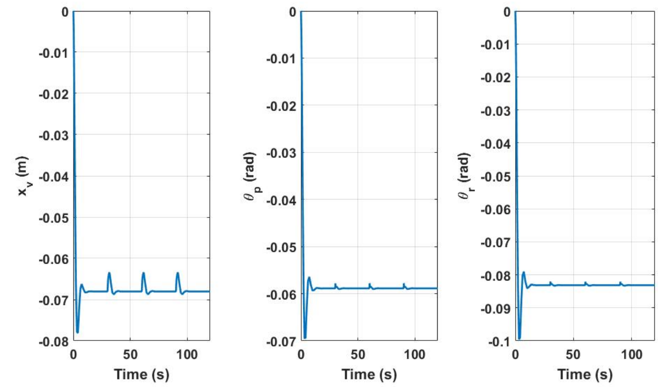
(c) System control inputs u.

(d) System input voltages V.

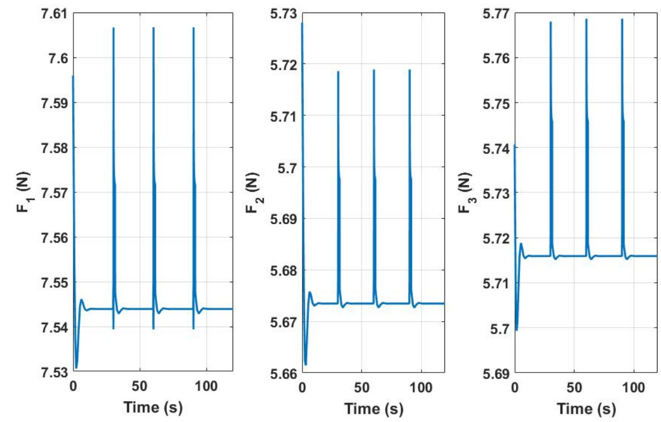
Fig. 4. Simulation 1 - results for the nominal control law without safety conditions.



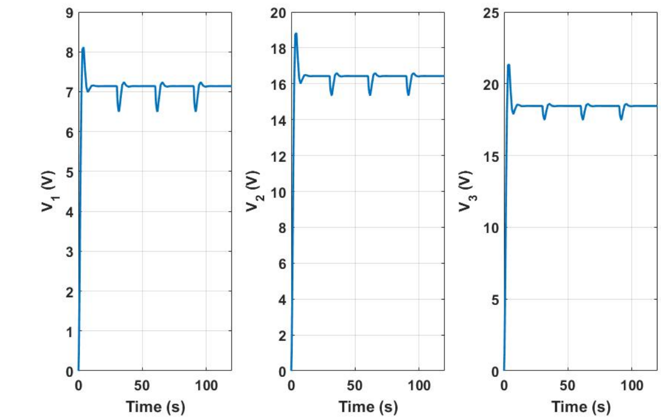
(a) System outputs y.



(b) System states x.

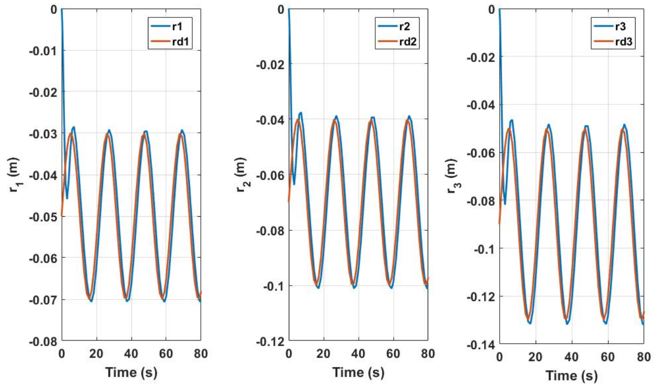


(c) System control inputs u.

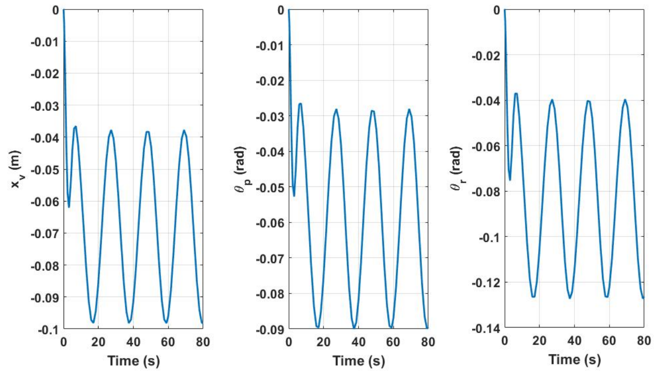


(d) System input voltages V.

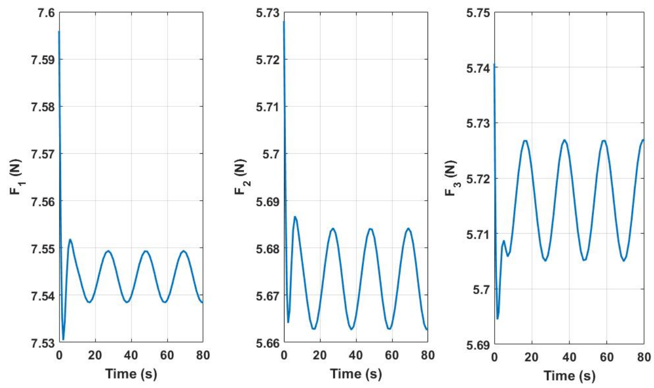
Fig. 5. Simulation 1 - results for the nominal control law with safety conditions.



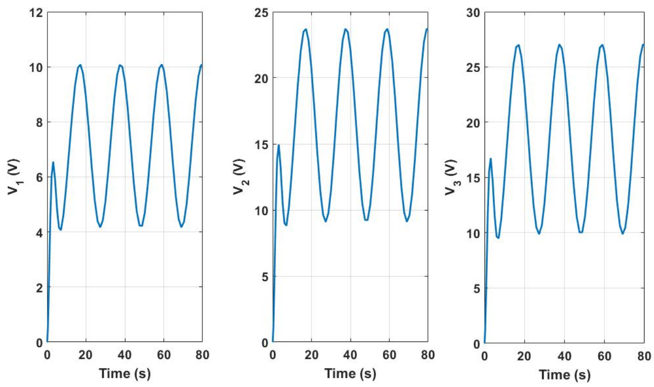
(a) System outputs y.



(b) System states x.

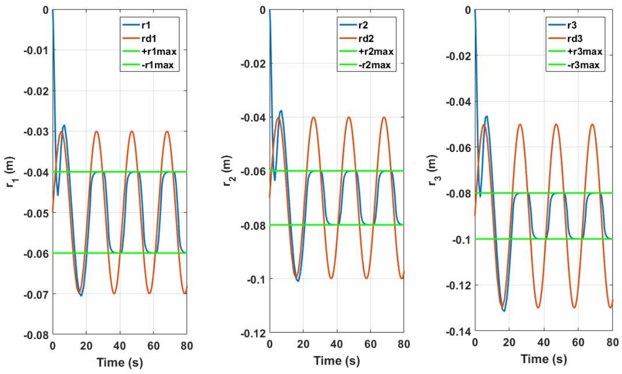


(c) System control inputs u.

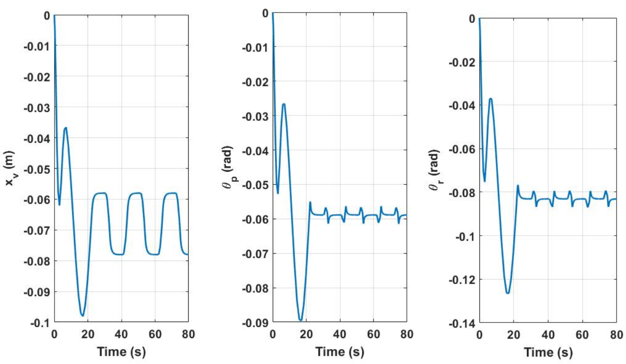


(d) System input voltages V.

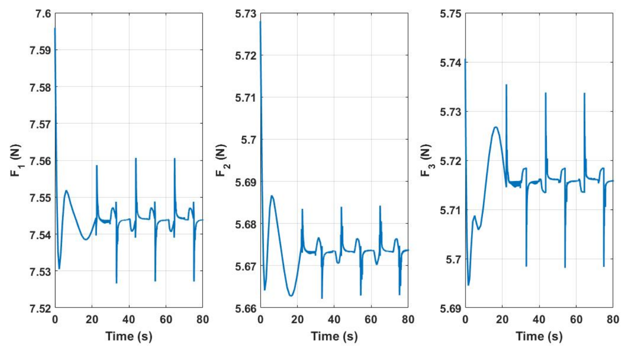
Fig. 6. Simulation 2 - results for the nominal control law without safety conditions.



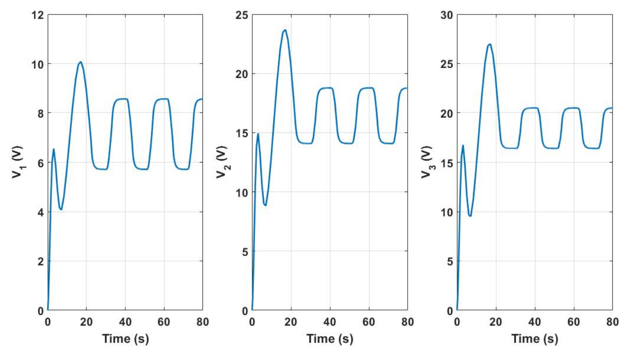
(a) System outputs y.



(b) System states x.



(c) System control inputs u.



(d) System input voltages V.

Fig. 7. Simulation 2 - results for the nominal control law with safety conditions.

$$h_2(x) = (r_{2max})^2 - (r_2 - r_{d2})^2, \tag{28}$$

$$h_3(x) = (r_{3max})^2 - (r_3 - r_{d3})^2, \tag{29}$$

where r_{1max}, r_{2max} and r_{3max} are arbitrary maximum range of the plate positions relative to the input references r_{d1}, r_{d2} and r_{d3}. However, the CBFs $h_1(x)$, $h_2(x)$ and $h_3(x)$ have relative-degree two, i.e., only the second derivative depends on the control input u and $L_g h(x) = 0$. So the safety conditions must be solved using ECBFs as described anteriorly.

Therefore, the QP-based controller defined in (26) was applied to satisfy the safety constraints. The QP was solved recursively using Matlab function quadprog. In the simulation experiment, the parameters for the ECBFs and QP were $r_{1max} = r_{2max} = r_{3max} = 0.005m$ and $K_{\alpha_1} = K_{\alpha_2} = K_{\alpha_3} = [100 \;\; 100]$. The parameters for the nominal control law were the same applied in Fig. 4. The simulation results are shown in Fig. 5. In Fig. 5a, the input references are presented in red and the maximum ranges of the plate positions are presented in green. It can be observed that the system outputs track the references, according to performance/stability objectives and respect the safety constraints due to ECBFs. As the amplitudes of the system outputs are limited, the control inputs are less aggressive and present lower amplitudes when ECBFs are applied. It is important to highlight that the ECBFs are only applied after the system outputs track the reference input for the first time.

In the simulation 2, the plate starts at $r_{10} = 0m$, $r_{20} = 0m$ and $r_{30} = 0m$ and must be levitated according to sinusoidal input references $r_{d1} = 0.02\sin(0.3t) - 0.05[m]$, $r_{d2} = 0.03\sin(0.3t) - 0.07[m]$ and $r_{d3} = 0.04\sin(0.3t) - 0.09[m]$, where t represents time. The parameters for the ECBFs and QP were $r_{1max} = 0.01m$ related to -0.05, $r_{2max} = 0.01m$ related to -0.07, $r_{3max} = 0.01m$ related to -0.09 and K_{α_1} and K_{α_2}, K_{α_3} were the same used in simulation 1. The parameters for the nominal control law were the same applied in Fig. 4. The simulation results for only nominal control law and with safety conditions are shown, respectively, in Figs. 6 and 7. Again, the outputs are tracked accordingly, respecting the performance/stability specifications and the safety constraints due to ECBFs.

5. CONCLUSIONS

This work presents the control of a magnetic levitation system, constituted by a Y shape metal plate that must be levitated by electromagnetic attractive forces. The control structure proposed uses QP to combine performance/stability objectives, represented by an arbitrary nominal control law, and safety constraints, represented by CBFs. The arbitrary nominal control law applied is determined by feedback linearization. Multiple safety constraints with relative-degree greater than one were applied. To deal with this, it was applied ECBFs. The numerical simulations demonstrate that the proposed control structure reach performance/stability objectives and respect safety constraints. In all simulation experiments, the plate positions track the reference inputs with adequate performance and when ECBFs are applied, the maximum ranges of the plate positions are not exceeded.

ACKNOWLEDGEMENTS

Authors thank Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) for Grant 2017/22130-4.

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