SYNTHESIS OF ROBUST CONTROL SYSTEMS WITH DYNAMIC ACTUATORS AND SENSORS USING A STATIC OUTPUT FEEDBACK METHOD

Bruno Sereni^{*}, Roberto K. H. Galvão[†], Edvaldo Assunção^{*}, Marcelo C. M. Teixeira^{*}

*São Paulo State University (UNESP) - Department of Electrical Engineering 1370 Prof. José Carlos Rossi Avenue Ilha Solteira, São Paulo, Brazil

[†]Instituto Tecnológico de Aeronáutica (ITA) - Divisão de Engenharia Eletrônica 50 Marechal Eduardo Gomes Square São José dos Campos, São Paulo, Brazil

Emails: bruno.sereni@unesp.br, kawakami@ita.br, edvaldo.assuncao@unesp.br, marcelo.teixeira@unesp.br

Abstract— In this paper, we propose a strategy for the robust stabilization of uncertain linear time-invariant (LTI) systems considering sensors and actuators whose dynamics cannot be neglected. The control problem is addressed by defining an augmented system encompassing the plant, sensor and actuator dynamics. The central idea of the proposed method lies in the fact that the actual plant states, measured by sensors, are not available for feedback, and thus, the problem can be regarded as a static output feedback (SOF) control design. Then, SOF gain matrices are computed through a two-stage method, based on linear matrix inequalities (LMIs). Intending to illustrate the efficacy and explore the main features of the proposed technique, some computational examples are presented in an application of the method for the design of a robust controller for the classic benchmark problem of the two-mass-spring problem. The examples cover the case of asymptotic stabilization of known and uncertain system model, in addition to decay rate inclusion and incomplete system state information.

Keywords— Robust control, Static output feedback, Linear time-invariant systems, Linear matrix inequalities

1 Introduction

With the development of more sophisticated technology and equipment, it tends to become increasingly relevant to consider the dynamics of sensors and actuators in modern control problems. For instance, sensors, actuators, control law, and control surfaces could affect the stability and performance of modern lightweight aircraft (Yang et al., 2018; Al-Jiboory et al., 2017; Stanford, 2016), due to aeroservoelasticity, a design characteristic inherent of such airplanes (Botez et al., 2008). In fact, the negative impact of neglecting of sensor and actuator dynamics has been long known (Young and Kokotovic, 1982; Leitmann et al., 1986). However, considering such additional dynamics in the control design often leads to a problem of increased difficulty, which may not be solvable by applying standard control techniques.

This design issue can be approached by considering an augmented system in which the resulting state vector comprises plant, sensor and actuator dynamics. However, the implementation of full state feedback control laws would be hindered because: (1) the actual plant states measured by the sensors would not be available for feedback and (2) additional sensors (possibly with non-negligible dynamics) would be required to measure the actuator states. Within this context, we show that this control design problem can be cast into the form of static output feedback (SOF), which can be readily implemented without the need for additional sensors or state estimators.

The SOF control technique has been object of great interest, with several papers written in the past decades, as seen in Sadabadi and Peaucelle (2016), Syrmos et al. (1997), and references therein. Among the available methods, strategies based on linear matrix inequalities (LMIs) have been proving to be an interesting approach for SOF (Spagolla et al., 2019; Frezzatto et al., 2018). In fact, the LMIs can be efficiently solved via convex optimization tools, and have been widely applied in different control problems (Gahinet and Apkarian, 1994; Teixeira et al., 2003; Oliveira and Peres, 2006). However, the synthesis of output feedback controllers is still an open problem, with different approaches proposed in the literature. In particular, the two-stage method, introduced in Peaucelle and Arzelier (2001) and Mehdi et al. (2004), has been receiving great attention in the past decades, due to its simple vet effective approach (Agulhari et al., 2012; Sereni et al., 2020).

The present paper proposes the use of an LMIbased two-stage SOF method to deal with linear time-invariant (LTI) systems whose actuator and sensor dynamics should not be neglected. System performance is also taken into account, by considering a decay rate specification in the control design. It is assumed that the dynamics models of the plant, sensors and actuators may be subject to parametric uncertainties. The effectiveness of the proposed strategy is evaluated in numerical examples involving the classic benchmark model of a system with one rigid-body mode and one flexible mode (Wie and Bernstein, 1992).

2 Problem Description

Consider the linear time-invariant (LTI) system described as

$$\dot{x}(t) = A(\alpha)x(t) + B(\alpha)z(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and $z(t) \in \mathbb{R}^m$ are the state and control vectors, respectively.

Moreover, $A(\alpha) \in \mathbb{R}^{n \times n}$ and $B(\alpha) \in \mathbb{R}^{n \times m}$ are uncertain matrices which can be represented in a polytopic domain $\mathscr{D} =$ $\left\{(A, B)(\alpha):(A, B)(\alpha) = \sum_{1}^{N} \alpha_r(A_r, B_r), \alpha \in \wedge_N\right\},$ where (A_r, B_r) denotes the *r*-th polytope vertex, and *N* is the number of vertices of the polytope. \mathscr{D} is parameterized in terms of a vector $\alpha = (\alpha_1, ..., \alpha_N),$ whose parameters α_r are unknown constants belonging to the unitary simplex set $\wedge_N = \left\{\alpha \in \mathbb{R}^n: \sum_{r=1}^{N} \alpha_r = 1; \alpha_r \geq 0; r = 1, ..., N\right\}.$

In this paper, we assume the existence of q sensors, with dynamics described by

$$\dot{v}_i(t) = a_{v,i}v_i(t) - a_{v,i}\left(\sum_{j=1}^n c_{i,j}x_j(t)\right),$$
 (2)

where $v_i(t)$ are the sensor outputs, $a_{v,i} < 0$ are time-invariant (but possibly uncertain) parameters for i = 1, 2, ..., q, and $c_{i,j}$ are known constants for j = 1, 2, ..., n.

Additionally, it is considered that the control vector z(t) is applied through m actuators whose dynamics are described by

$$\dot{z}_k(t) = a_{z,k} z_k(t) - a_{z,k} \left(\sum_{l=1}^p d_{k,l} u_l(t) \right),$$
 (3)

where $z_k(t)$ are the actuator outputs, $u_l(t)$ are the actuator commands, $a_{z,k} < 0$ are timeinvariant (but possibly uncertain) parameters for k = 1, 2, ..., m, and $d_{k,l}$ are known constants for l = 1, 2, ..., p.

The problem consists in designing a control law u(t) = Lv(t) where $v(t) \in \mathbb{R}^q$ is the vector of sensor outputs, $u(t) \in \mathbb{R}^p$ is the vector of actuator commands, and $L \in \mathbb{R}^{p \times q}$ is a convenient gain matrix to be determined in order to ensure the closed-loop asymptotic stability of the overall system. Figure 1 presents a block diagram that illustrates the control system configuration.

3 Proposed Strategy

For dealing with this control problem, we propose the definition of the augmented state vector $w(t) \in \mathbb{R}^{n+q+m}$ defined as

$$w(t) = \begin{bmatrix} x'(t) & v'(t) & z'(t) \end{bmatrix}'$$

such that one may describe the combined plant, actuators, and sensors dynamics according to the following augmented state-space model

$$\dot{w}(t) = \bar{A}(\alpha)w(t) + \bar{B}(\alpha)u(t), \qquad (4)$$

where

$$\bar{A}(\alpha) = \begin{bmatrix} A(\alpha) & 0_{n \times q} & B(\alpha) \\ -A_v(\alpha)C(\alpha) & A_v(\alpha) & 0_{q \times m} \\ 0_{m \times n} & 0_{m \times q} & A_z(\alpha) \end{bmatrix},$$
$$\bar{B}(\alpha) = \begin{bmatrix} 0_{n \times p} \\ 0_{q \times p} \\ -A_z(\alpha)D \end{bmatrix},$$

with

$$A_{v}(\alpha) = \operatorname{diag} \left\{ a_{v,1}, a_{v,2}, \dots, a_{v,q} \right\},$$

$$A_{z}(\alpha) = \operatorname{diag} \left\{ a_{z,1}, a_{z,2}, \dots, a_{z,m} \right\},$$

$$C = \begin{bmatrix} c_{1,1} & c_{1,2} & \dots & c_{1,n} \\ c_{2,1} & c_{2,2} & \dots & c_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{q,1} & c_{q,2} & \dots & c_{q,n} \end{bmatrix},$$
(5)

and

$$D = \begin{bmatrix} d_{1,1} & d_{1,2} & \dots & d_{1,p} \\ d_{2,1} & d_{2,2} & \dots & d_{2,p} \\ \vdots & \vdots & \ddots & \vdots \\ d_{m,1} & d_{m,2} & \dots & d_{m,p} \end{bmatrix}$$

Then, considering an output vector y(t) = v(t), for the augmented system (4), we may write $y(t) = \overline{C}w(t)$, where $\overline{C} = [0_{q \times n} \quad I_{q \times q} \quad 0_{q \times m}]$. Now, the aforementioned problem may be addressed as a static output feedback control design with u(t) = Ly(t).



Figure 1: Closed-loop block diagram.

3.1 SOF Control Design

In order to derive the desired SOF gain, L, that will provide asymptotic stability to the closedloop augmented system (4), we apply the twostage LMI-based SOF design strategy presented in Sereni et al. (2018), which was chosen due to its capability of coping with system model uncertainty and also ensuring enhanced performance in terms of a minimum decay rate criterion.

The decay rate associates the convergence rate of the system state to the origin with the evolution of a time-dependent exponential function $e^{\sigma t}$. In what follows, σ will correspond to either β (in the first design stage) or γ (in the second design stage). For a formal and complete description of the decay rate criterion, the reader is referred to Boyd et al. (1994).

3.2 The Two-Stage Method

The two-stage method, originally introduced in Peaucelle and Arzelier (2001) and Mehdi et al. (2004), consists of first obtaining a robust state feedback (SF) gain K, and then, using such gain as an input information for the second stage, in which the SOF gain is derived. A minimum decay rate specification is imposed in both stages of design, and they are defined as positive constants β and γ , respectively.

• First Stage: State Feedback Design

In the first stage, we consider the control law as u(t) = Kw(t), then system (4) in closed-loop is represented by $\dot{w}(t) = [\bar{A}(\alpha) + \bar{B}(\alpha)K]w(t)$.

As well known in the LMI literature (Boyd et al., 1994), a sufficient condition for this uncertain state-feedback system to be quadratically stable, considering the inclusion of a decay rate greater than or equal to $\beta > 0$, is the existence of a symmetric matrix W > 0 and a matrix Z, such that

$$\bar{A}_r W + W \bar{A}'_r + \bar{B}_r Z + Z' \bar{B}'_r + 2\beta W < 0 \quad (6)$$

holds for r = 1, 2, ..., N. A stabilizing gain is then obtained as $K = ZW^{-1}$.

• Second Stage: Output Feedback Design

In the second stage, the previously obtained robust state feedback gain K is used as an input parameter for the design of the robust output feedback gain $L \in \mathbb{R}^{p \times q}$. Such gain may be obtained by solving a set of sufficient LMI conditions stated in Theorem 1.

Theorem 1 (Sereni et al., 2018) Assuming that there exists a state feedback gain K such that $\bar{A}(\alpha) + \bar{B}(\alpha)K$ is asymptotically stable, then there exists a stabilizing static output feedback gain L such that $\overline{A}(\alpha) + \overline{B}(\alpha)L\overline{C}(\alpha)$ is asymptotically stable, considering a decay rate greater than or equal to $\gamma > 0$, if there exist symmetric matrices $P_r > 0$, and matrices F_r , G_r , H and J such that

holds for r = 1, 2, ..., N*, and*

$$\begin{bmatrix} \Upsilon_{ij}^{1i} \\ \Upsilon_{ij}^{2i} \\ \bar{B}'_r F'_s + J\bar{C}_r + J\bar{C}_s + \bar{B}'_s F'_r - 2HK \\ * & * \\ \begin{bmatrix} -G_r - G'_r - G_s - G'_s & * \\ \bar{B}'_r G'_s + \bar{B}'_s G'_r & -2H - 2H' \end{bmatrix} < 0 \quad (8)$$

holds for r = 1, 2, ..., N - 1, and s = r + 1, r + 2, ..., N, where $\Upsilon_{ij}^{11} = \bar{A}'_r F'_s + F_r \bar{A}_s + K' \bar{B}'_r F'_s + F_r \bar{B}_s K + \bar{A}'_s F'_r + F_s \bar{A}_r + K' \bar{B}'_s F'_r + F_s \bar{B}_r K + 2\gamma P_r + 2\gamma P_s$, and $P_r + P_s - F'_r - F'_s + G_r \bar{A}_s + G_r \bar{B}_s K + G_s \bar{A}_r + G_s \bar{B}_r K$. In the synthesis condition, the robust static output feedback gain is given by $L = H^{-1}J$.

Proof: See (Sereni et al., 2018). \Box

It is worth mentioning that in the particular case where the control design does not require a minimum decay rate specification, it is possible to choose $\beta = \gamma = 0$ in (6) and in Theorem 1. With this, only the asymptotic stabilization will be guaranteed with the designed SOF controller.

Moreover, the minimum decay rate does not have to be set to the same value in both design stages, since each stage is performed independently (Sereni et al., 2018).

The aforementioned fact grants the two-stage method the characteristic of being only a sufficient condition for SOF design, since the choose of a stabilizing first stage SF gain is not unique, nor limited to the use of (6). In fact, there are examples available in the literature, such as hitand-run methods (Arzelier et al., 2010), that exploit such feature by employing some heuristics on the search for the stabilizing SF gain.

Indeed, particularly in our proposed method, the closed-loop system guaranteed minimum decay rate is solely related to γ in Theorem 1. Due to that, the value specified for β in the first stage represents an heuristic approach, since no guarantees on the feasibility or associated performance in the second stage are provided, although the SOF design success for a particular γ is sensible to the value defined for β in the earlier stage, as empirically verified during the performed experiments.

4 Examples

In order to illustrate the efficacy of the proposed strategy, we present four computational examples involving a two-mass-spring system, which can be regarded as a simple model for the dynamics of a system with one rigid-body mode and one flexible mode (Wie and Bernstein, 1992). In the following examples, the LMIs are programmed with MATLAB software, and solved via YALMIP interface (Lofberg, 2004), using the SDPT3 solver (Toh et al., 1999).

4.1 Example 1: Stabilization

Consider a system of two bodies with masses m_1 and m_2 , connected by a spring with stiffness constant k, as in Wie and Bernstein (1992). The dynamics of the system, considering that a control force z(t) is applied on body 1, can be represented in the state-space form (1), as

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-k}{m_1} & \frac{-k}{m_1} & 0 & 0 \\ \frac{k}{m_2} & \frac{-k}{m_2} & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_1} \\ 0 \end{bmatrix} z(t),$$

$$(9)$$

where $x_1(t)$ and $x_2(t)$ are the positions of body 1 and body 2, respectively; and $x_3(t)$ and $x_4(t)$ are the corresponding velocities. Moreover, the state is measured with sensors whose dynamics are described as in (2) and the control signal is applied by an actuator with dynamics described as in (3).

The parameters m_1 , m_2 , and k, as well as the matrices with the actuators and sensors parameters in (5), are assumed to be $m_1 =$ $m_2 = 1.0$ kg, k = 1.0 N/m, $A_v =$ diag{-5, -5, -5, -5}, $A_z = -5$, C = I_{4x4} and D = 1.

Before employing the proposed method, the results of a simpler approach based on a decentralized control design (Zecevic and Siljak, 2010) are presented. The idea of this strategy is to impose structure constrains on the problem variables to apply a state-feedback technique using only the available measurements. For this purpose, the following structure is imposed on the matrix variables W and Z in (6):

$$\bar{W} = \begin{bmatrix} W_x & 0_{4\times 4} & 0_{4\times 1} \\ 0_{4\times 4} & W_F & 0_{4\times 1} \\ 0_{1\times 4} & 0_{1\times 4} & W_z \end{bmatrix} \text{ and } \bar{Z} = \begin{bmatrix} 0_{1\times 4} & Z & 0 \end{bmatrix}$$
(10)

In that way, a gain $\bar{K} = [K_x \ K_F \ K_z]$, with $K_x = K_z = 0$, could be derived by making $\bar{K} = \bar{Z}\bar{W}^{-1}$. However, if these structural constraints are imposed in LMIs (6), with $\beta = 0$, the problem has no feasible solution.

Now, using the strategy proposed in this paper, we address the issue as a SOF problem. For that, an augmented system as in (4) is considered. Then, with $\beta = 0$ in (6), for the first stage of design, the following state feedback gain K is derived

$$K = \begin{bmatrix} -2.3879 & -0.4405 & -4.3230 & -1.6019 \\ 0.7126 & 0.3687 & 2.2717 & 0.4723 & -0.2407 \end{bmatrix}.$$
 (11)

By using this gain matrix in the second stage, via Theorem 1, considering no decay rate specification $(i.e \ \gamma = 0)$, one may derive the SOF gain L as follows

$$L = \begin{bmatrix} -0.9959 & -0.1793 & -2.3679 & -1.1066 \end{bmatrix}.$$
(12)

The resulting control law stabilizes the augmented system with the sensors and actuator, since the eigenvalues of $\bar{A} + \bar{B}L\bar{C}$ are $-0.4361, -5, -5, -5, -7.6867, -0.2089 \pm 1.1490j$ and $-0.7297 \pm 2.4277j$.

4.2 Example 2: Decay Rate

In addition to stability, we show that improved performance can also be enforced for the closedloop system. For that, we consider a minimum decay rate in both first and second stages of project, β and γ , respectively.

For illustration, consider the same system and parameters used in Example 1. Then, a SOF gain matrix L can be computed considering decay rates $\beta = 0.2$ in (6), and $\gamma = 0.2$ in Theorem 1, LMIs (7) and (8). As result, the following gain matrix is obtained:

$$L = \begin{bmatrix} -2.3217 & 0.6543 & -3.3938 & -1.9717 \end{bmatrix}.$$
 (13)

Figure 2 presents the transient response of the closed-loop system, obtained via simulation with arbitrarily defined initial condition x(0) = $\begin{bmatrix} -0.1 & 0 & -0.2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. For comparison purposes, the responses provided by both controllers, with (13) and without decay rate specification (12), are presented. One may observe that with the inclusion of a minimum decay rate a faster transient response is obtained. Although, as Figure 3 shows, this improvement comes at cost of a higher amplitude in the control command, u(t).

4.3 Example 3: Polytopic Uncertainties

In this next example, we consider uncertainties in the model of the two-mass-spring system. We assume that the mass of body 1, the position sensors parameters $a_{v,1}$ and $a_{v,2}$ in $A_v(\alpha)$, and the actuator parameter $a_{z,1}$ in $A_z(\alpha)$ are uncertain, laying within the intervals 0.9 kg $\leq m_1 \leq$ 1.1 kg, $-6 \leq a_{v,1}, a_{v,2}, a_{z,1} \leq -4$. The parameters $a_{v,3}, a_{v,4}, m_2$ and k are known and have the same values as in Example 2.

Therefore, having four uncertain parameters, the uncertain augmented system may be represented as a convex combination of sixteen vertices,



Figure 2: Transient response of the two-massspring system with SOF control designed with minimum decay rate $\gamma = 0.2$ (—) and without decay rate restriction (- -).



Figure 3: Control commands with minimum decay rate $\gamma = 0.2$ (—) and without decay rate restriction (- -).

which are generated by combining the maximum and minimum values of m_1 , $a_{v,1}$, $a_{v,2}$, and $a_{z,1}$. By applying the proposed strategy, and solving (6) with $\beta = 0.2$ and the LMIs in (7) and (8), considering $\gamma = 0.2$, one can derive the robust SOF controller as

$$L = \begin{bmatrix} -0.1856 & -0.1642 & -1.7503 & -0.1197 \end{bmatrix}.$$
(14)

Figure 4 presents the eigenvalues of all 16 vertices of the resulting closed-loop system polytope, as well as of the open-loop configuration. As one can see, the designed robust SOF matrix gain (14) managed to allocate all closed-loop eigenvalues to the left of the dashed vertical line at s = -0.2(see the inset in Figure 4). Therefore, the minimum decay rate constraint specified in the controller design has been properly enforced. In fact, note that some of the open-loop eigenvalues are originally placed way out of the prescribed design bounds.

4.4 Example 4: Partial State Vector Measurement

In this final example, we address the case where only a subset of the system state variables are available for measurement, i.e. q < n. Consider



Figure 4: Eigenvalue placement of the open-loop (\circ) and closed-loop (\times) system with the robust SOF controller (14).

that the two-mass-spring system is fitted with only q = 2 sensors, which are responsible for measuring the position and velocity of the mass m_1 (which corresponds to the state variables x_1 and x_3 , respectively). Also, assume that the system parameters values and uncertainties intervals are the same as in Example 3.

Then, defining the augmented system accordingly, a robust SOF gain

$$L = \begin{bmatrix} -0.5014 & -1.8308 \end{bmatrix}$$
(15)

is able to be obtained with $\gamma = 0.2$ in (7) and (8) by using $\beta = 0.15$ in (6)¹, following the proposed method. Observe that, in this case, we have a 7th order augmented system, but the SOF gain matrix is composed of only two entries, since we considered only two state variable measurements available for feedback.

In addition, with a simulation analysis with the same initial condition considered in Example 2, the transient responses of all 16 vertices of the closed-loop two-mass-spring system polytope with robust SOF gain matrices (15) and (14) are obtained and presented in Figure 5. As one may observe, both closed-loop systems show very similar dynamics responses. In fact, for the particular example addressed, no noticeable loss of performance occurred by employing only two sensors for state variable measurement. At this point, it is crucial to emphasize that the transients represented by the red lines are achieved with a SOF controller designed considering the absence of two of state variable measurements. The presented results underline the main feature of the proposed method, which consists in synthesizing a gain matrix that is capable of providing robust stability in the presence of model uncertainty and sensor/actuator dynamics, even with incomplete information about the system state.

¹After testing Theorem 1 for different values of β in the first stage, $\beta = 0.15$ was the highest value found to lead for feasibility success in the second stage with $\gamma = 0.2$.



Figure 5: Transient response of all 16 vertices of the two-mass-spring system polytope with robust SOF controller using four sensors (Example 3, - -) and only two sensors (Example 4, --).

5 Conclusion

This paper proposed a robust control strategy that takes into account sensor and actuator dynamics through the use of a SOF synthesis method. We showed with a two-mass-spring example that it may not be possible to find a solution when using a state-feedback decentralized control strategy, whereas our method was able to provide a stabilizing controller. Moreover, our technique can also be applied in control designs that consider improved dynamics performance and/or polytopic uncertainties in the plant model. Furthermore, incomplete state vector measurement can also be coped with our method. It is also worth mentioning that the proposed strategy can be extended to handle nonlinear system dynamics since the adopted SOF method can be easily modified to address gain-scheduled control problems. In future works, the strategy can be adapted to consider more complex actuators and sensors dynamics, such as high order filters. The study about how to exploit the particular structure of the augmented system in the context of SOF design, specially involving the two-stage method could also be investigated, along with the application of the method on aircraft models. Extensions to optimization problems regarding performance criteria such as disturbance rejection can also be analyzed.

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