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Nonlinear Grey-box Identification of a Landing Gear based on Drop Test Data

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Abstract: In many aircraft applications, specially on an antiskid control design, it is important to understand and consider the gear walk phenomenon, which is characterized by the deflection on the landing gear structure due the high braking force acting at the tire contact with the ground. This phenomenon is observed on drop tests, and its prediction on landing gear design depends on an adequate evaluation of the equivalent stiffness and damping of the structure, which is difficult, since they depend on the mechanism configuration. In this paper, it is presented a grey-box identification methodology for estimating these parameters of the landing gear, based on simulated data of a drop test. As the drop tests are mandatory obligatory for certificating modern aircraft according to e.g. Federal Aviation Regulations (FARs) by the Federal Aviation Administration (FAA), we hope to introduce a method based on measurements that are available at the design phase. The method will be useful to decrease men/hour costs and increase reliability by enabling better and more accurate anti-skid design.

Keywords: Grey box modelling, Recursive identification, Identification for control, Aerospace, Vehicle dynamic systems.

1. INTRODUCTION

Landing gears are critical parts of the hydromechanical subsystem of an aircraft. During braking, their purpose is to dissipate kinetic energy efficiently in order to ensure proper aircraft performance. The anti-skid system orchestrates braking given the inputs by the pilot. Its primary purpose is to avoid wheels to block and thus maximize brake efficiency. However, landing gears have rich nonlinear dynamics which difficults the design of anti-skid control laws (Rahmani and Behdinan, 2020). Torsional and translational vibrations are observed when the mechanical dynamics of such complex multibody systems is measured. Shimmy happens when the torsional structural dynamics is poorly damped (Rahmani and Behdinan, 2019), while the gear walk phenomenon is observed when translational deflection amplitude in the landing gear is relatively high due to the braking force acting at the tire contact with the ground (Gualdi et. al., 2008). In aircraft braking simulations, we must efficiently evaluate or predict the gear walk phenomenon. The deflection observed during gear walk is limited by the stiffness and damping of the ensemble composed of suspension and structure of the landing gear. In this context, it is of utmost importance to be able to model and simulate efficiently the aforementioned phenomena so that design meets specified requirements.

At a robust design for antiskid control, the gear walk phenomenon must be considered, since the estimation of braking force depends on the angular velocity of the wheel and the velocity of the wheel hub. Pritchard (2001) remarks the relevance of the study of the dynamics of landing gear, mainly due the effects of vibration and shimmy induced by braking, highlighting its criticality on aircraft safety. Krüger et. al. (1997) give a complete review of landing gear requirements and operational conditions. They describe the drop test and remark its importance to analyze the stiffness and inertial of each element, to evaluate the behavior of shock absorber and of the wheel and tire. Sinou et. al. (2006) study an experimental approach of friction-induced vibration on aircraft brake, remarking the number of researches on this subject. Luo and Zhao (2018) propose a spatial landing gear mechanism to achieve higher stiffness and higher strength, demonstrating that these properties are closely dependents of the geometry and the constraints between the bodies.

We may observe many studies that are dedicated to understanding the dynamics of landing gears and improve its performance, since it is a critical system on aircraft security (Krüger et. Al., 1997). Moreover, the physical phenomena involving the tires have important effects over the system dynamics and they must be considered on developed models. Van Slagmaat (1992) develops a nonlinear model for landing gear simulation, using the magic formula for tire dynamics. Yadav and Singh (1995) present an optimal anti-skid braking control based on a one-step ahead prediction of the braking force required, aiming to reduce the landing run. Gualdi et. al. (2008) feature a multi-body landing gear model, demonstrating the effect of gear walk, to be employed as a design tool for anti-skid landing gear braking control. D'Avico et. al. (2017) presents a control-oriented model of landing gear, using the Burckhardt tire model for determining the braking forces. This model is validated experimentally, as presented for D'Avico et. al. (2018). The suppression of gear walk phenomenon is studied by Yin et. al. (2019), who propose an anti-skid control that minimize the maximum gear walk angle, satisfying other constraints. Jiao et. al. (2019) propose an anti-skid brake control with identification of the runway characteristics and the tire conditions, so that it is possible to estimate the maximum friction force. They obtain, as result, a considerable improvement of the braking efficiency when compared to algorithms based only on wheel deceleration. Chen et. al. (2018) present an improved braking control algorithm, which considerers both the wheel deceleration and longitudinal slip, to enhance the robustness and the efficiency of braking process. Tourajizadeh and Zare (2016) propose a robust and optimal nonlinear control of shimmy vibration, remarking the importance of minimizing this vibration on aircraft performance and security. Somakumar and Chandrasekhar (1999) propose an intelligent anti-skid brake controller based on neural network, with learning, nonlinear mapping and pattern-recognition abilities. It defines the brake torque after analyzing the runway condition, so that the braking is optimum.

The design of a landing gear is evaluated on many tests, among which there is the drop test. It consists in lifting the landing gear in a specific test bed and dropping it from a height that will cause a desired impact velocity. Among the measurements commonly made, the horizontal force is very important as it affects directly the gear walk phenomenon (Wang et. al., 2017). Xue et. al. (2012) present a method of optimizing the damper coefficient of an amphibious landing gear by means of simulation and drop test, which illustrates the large application of this test on aircraft design. Shixing et. al. (2011) study a drop test of a landing gear with a MRF (magnetorheological fluid) damper, assessing the influence of this component on the dynamics and performance of the landing gear. This kind of damper is also focus of Li et. al. (2015), who present a magneto-rheological damper structure for landing gear, commenting its main advantages, such as adjustable damping force, simple structure and independence of external energy. Wei et. al. (2014) develop a more complex model for landing gear fall dynamics. The model presented has two degrees of freedom and adds viscous friction and grip effects to the Coulomb friction model.

The drop test may also be used for evaluating the gear walk and predicting the structural stiffness and damping of the landing gear, by identification techniques. Fallah et. al. (2008) demonstrate the importance and influence of structural parameters in the design and control of vibrations on aircraft landing gears. This phenomenon is not considered on ground vehicles, because the braking forces are not of the same scale. The drop test measurement system is also focus of some recent researches, as the presented by Pytka et. al. (2019). They feature a dynamometer wheel for landing gear tests. The measurement system is designed for obtaining the vertical load and longitudinal forces acting in the wheel, as well as the moments around all the axes.

Batill and Bacarro (1988) feature a nonlinear identification of a single degree of freedom of landing gear system, applying Newtonian Iteration. It considers the suspension and structure both linear and nonlinear, when is used the hydropneumatic damper on the airplane. The identification process on mechanisms is most explored on robotic systems. These methods are well explored by Wu et. al. (2010), who give an overview of dynamic parameter identification of serial and parallel robots, summarizing the main methods used and advantages and disadvantages of each one. Oliviers and Campion (1997) propose a methodology for parameters identification in a nonlinear model of a robot with flexible arms. So, both inertial and elastic parameters are estimated, and the kinematics must consider the displacements due to flexibility of the bodies. Díaz-Rodríguez et. al. (2010) present a methodology for dynamic parameters identification of a 3 degree of freedom (DOF) parallel robot. They explain that not all the parameters may be properly identified, and they apply the weighted least squares method for determining the relevant ones. This method is also used by Bahloul et. al. (2018) on an identified model for a 6-DOF industrial robot, based on the inverse dynamic equations. Gao et. al. (2018) present a parameter identification method based on Denavit-Hartenberg model, validated on a 6-DOF industrial robot. They suggest a modified least squares algorithm, designed for minimize the residual movement uncertainties and the application of singular value decomposition for determining the parameters most relevant.

This paper aims to present a nonlinear identification methodology for a four degree of freedom landing gear model that includes the gear walk phenomenon. The proposed identification is based on simulated drop test data, as we have seen its importance and application on aircraft design. Moreover, all the variables used on identification may be measured or estimated on this test, allowing its application on experimental data. In the aforementioned state of the art review, we note that the identification process is applied on low-complexity models or specific components of the landing gear. So, the main contribution of this paper is evaluating of a grey-box identification methodology on an enhanced complexity model, aiming to be able to precisely identify parameters on a real drop test data. We propose a analytical model of the mechanism and identify its parameters using a combined Kalman filter and derivatives approximation technique.

The remainder of this paper is organized as follows. In Section 2 we present the analytical modeling of the landing gear and its equations of motion using the Lagrange formulation. In the following section, the grey-box identification algorithms devised for this work are presented, highlighting their application on multibody systems parameters estimation. Subsequently, we explain the landing gear simulation and how we obtain the corresponding measured variables applied on the identification process. Finally, the results of the dynamic simulation and of the parameters estimation are presented, explaining the relationship with the physical phenomena and analyzing the precision of the identification. At the conclusion, we comment about future research possibilities and the use of different methods of modelling.

2. LANDING GEAR SYSTEM

The landing gear is modelled, in this work, as a planar mechanism, with concentrated inertias. The stiffness and damping of its structure and suspension system are considered as a rotational spring and damper, which represents the stiffness and damping of the structure. In the system, there are three bodies: a concentrated mass representing the airplane with coordinates (x,z), a bar with length L representing the structure of the landing gear, with coordinates (x_b, z_b, θ), and the wheel with coordinates (x_w, z_w, ϕ), radius R, and an applied braking torque T, as shown in Figure 1.

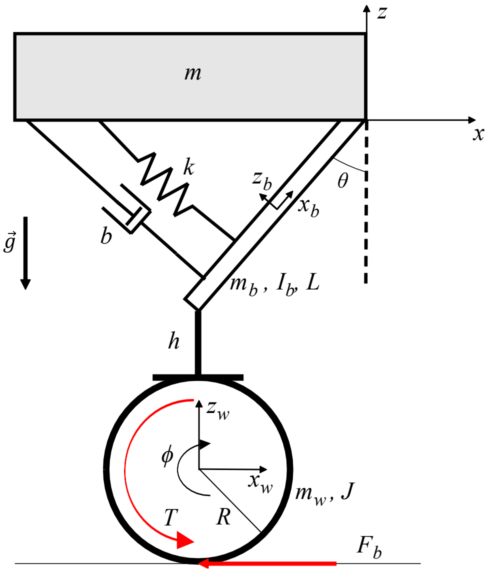

Figure 1: Landing gear model

We use the Lagrange formulation in order to obtain the dynamic equations of motion for the landing gear. To this end, the kinematic relations between the generalized coordinators are presented as

$$\begin{cases} \dot{x}_b = \dot{x} - \frac{L}{2}\dot{\theta}\cos\theta \\ \dot{z}_b = \dot{z} + \frac{L}{2}\dot{\theta}\sin\theta \\ \dot{x}_w = \dot{x} - L\dot{\theta}\cos\theta \\ \dot{z}_w = \dot{z} + L\dot{\theta}\sin\theta \end{cases} \quad (1)$$

We may rewrite some parameters, for simplifying equations, as follows:

$$M_1 = (m + m_b + m_w) \quad (2)$$

$$M_2 = \left(\frac{m_b}{4} + m_w\right)L^2 + I_b \quad (3)$$

$$M_3 = (m_b + 2m_w)L \quad (4)$$

So, the kinetic energy E_k and potential energy E_p of the system may be described as:

$$E_k = \frac{1}{2}M_1(x^2 + z^2) + \frac{1}{2}M_2\dot{\theta}^2 + \\ + \frac{1}{2}M_3\left(-\dot{x}\dot{\theta}\cos\theta + \dot{z}\dot{\theta}\sin\theta\right) + \frac{1}{2}J\dot{\phi}^2 \quad (5)$$

$$E_p = M_1 g z - M_3 g\cos\theta - m_w g h + \frac{k(\theta - \theta_0)^2}{2} \quad (6)$$

The Lagrange Equation is defined at Eq. 7, where q_i indicates each generalized coordinate and Q_i the external forces or moments related to q_i.

$$\frac{d}{dt}\frac{\partial}{\partial \dot{q}_i}\left(E_k - E_p\right) - \frac{\partial}{\partial q_i}\left(E_k - E_p\right) = Q_i \quad (7)$$

Appling the Lagrange Equation for each generalized coordinate, namely x, z, θ and ϕ, it is possible to obtain the system dynamic equations.

$$M_1\ddot{x} - M_3\ddot{\theta}\cos\theta + M_3\dot{\theta}^2\sin\theta = 0 \quad (8)$$

$$M_1\ddot{z} + M_3\ddot{\theta}\sin\theta + M_3\dot{\theta}^2\cos\theta + M_1 g = F_z \quad (9)$$

$$-M_3\ddot{x}\cos\theta + M_3\ddot{z}\sin\theta + M_2\ddot{\theta} + M_3 g\sin\theta \\ + k(\theta - \theta_0) + b\dot{\theta} = F_z L\sin\theta + F_b L\cos\theta \quad (10)$$

$$J\ddot{\phi} + c\dot{\phi} = F_b R - T \quad (11)$$

The braking force is not considered on the first equation, because the prototype on test is confined. So, this force is equilibrated by the kinematic constraint imposed by the apparatus. The braking force is considered proportional to the vertical load, which is due the elastic condition of the tire. So, it may be written as:

$$F_b = \mu F_z \quad (12)$$

The parameter μ depends on the slip (λ), which relates the wheel hub velocity and the wheel angular velocity as

$$\lambda = \frac{\dot{x}_w - \dot{\phi}R}{\dot{x}_w} \quad (13)$$

There are many models relating the coefficient μ and the slip λ, among which, one of the most widespread is the Burckhardt model, described by Harifi et. al. (2008).

$$\mu(\lambda) = \vartheta_1(1 - e^{-\lambda\vartheta_2}) - \lambda\vartheta_3 \quad (14)$$

The relationship between μ and λ for dry asphalt is presented in Figure 2. For this kind of ground, the parameters ϑ_1, ϑ_2 and ϑ_3 are, respectively, 1.2801, 23.99 and 0.52 (Harifi et. al., 2008).

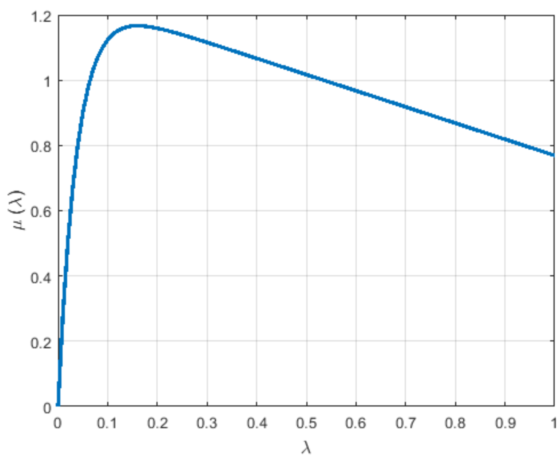

Figure 2: Relationship between the friction coefficient and slip

Having defined the analytical model, we are ready to state the parameter estimation algorithm.

3. GREY-BOX ESTIMATION ALGORITHM

The grey-box identification has as an objective the determination of unknown parameters, through measured or estimated states. When the *ne* dynamic equations of a multibody system are known, it is possible to identify *np* linear independent parameters that allow to rewrite the equations as a matrixial multiplication, for each sample *j*:

$$\boldsymbol{\varphi}(j)\Theta = f(j) \tag{15}$$

where $\Theta \in R^{np}$ is a vector with the smallest set of linear independent parameters of the model, $\boldsymbol{\varphi}(j) \in R^{ne \times np}$ is a matrix with only known terms, as measured or estimated positions, velocities and accelerations. The vector $f \in R^{ne}$ is composed by independent terms, which includes external forces and moments and others that are not related to unknown parameters.

Since the measured or estimated states may be affected by noise, it is important that Φ and F are as large as possible, considering all points available. So, concatenating the matrix equations, the system may be described as

$$\begin{bmatrix} \boldsymbol{\varphi}(1) \\ \vdots \\ \boldsymbol{\varphi}(\mathrm{j}) \\ \vdots \\ \boldsymbol{\varphi}(\mathrm{N}) \end{bmatrix} \Theta = \begin{bmatrix} f(1) \\ \vdots \\ f(\mathrm{j}) \\ \vdots \\ f(\mathrm{N}) \end{bmatrix} \tag{16}$$

Generally, the equation 16 may be written as

$$\boldsymbol{\Phi}\Theta = F \tag{17}$$

which may be then treated effectively. The identification problem resembles to obtain the vector of parameters Θ with measurements made in **Φ** and F. For the grey-box identification, many methods may be used for estimating the vector of parameters Θ. The simplest is the Batch Least Squares (LS) algorithm, or the Penrose-Moore pseudo-inverse. So, the estimated vector is:

$$\widehat{\Theta} = (\Phi^T\Phi)^{-1}\Phi^T F \tag{18}$$

The recursive approach of Least Squares may already be used in this grey-box identification, with suitable precision. With some modifications this method converges to the Kalman Filter (KF) for parameter estimation, which may be employed for online or offline identification (Söderström and Stoica, 1988). This algorithm is recursive and described on these equations, for each sample *j*

$$\varepsilon(j) = F(j) - \hat{F}(j) = F(j) - \Phi(j)\widehat{\Theta}(j-1) \tag{19}$$

$$K(j) = \frac{P(j-1)\Phi(j)^T}{[1 + \Phi(j)P(j-1)\Phi(j)^T]} \tag{20}$$

$$P(j) = P(j-1) - K(j)\Phi(j)P(j-1) + R_1 \tag{21}$$

$$\widehat{\Theta}(j) = \widehat{\Theta}(j-1) + K(j)\varepsilon(j) \tag{22}$$

where *P* is the covariance matrix, ε is the prediction error and *K* is the gain. In each iteration, the vector of parameters is corrected with a factor proportional to the error between the actual and the estimated vector *F*. In order to execute the identification procedure given by the equations (19)-(22), the matrix *P(0)* and the vector $\hat{\theta}(0)$ must be initiated at iteration *j* = 0. In most cases, *P(0)* is properly defined as a diagonal matrix with large entries and the initial estimation of vector $\hat{\theta}$ may be defined as zero, so that the estimator does not converges to a local minimum (Billings, 2013).

It is important to note that the trace of *P* decreases along the iterations and tends to zero if R_1 is not considered. So, when the system is time-varying or in online applications, it is important to define a matrix R_1 with large entries, so that the trace of *P* remains in a large value. In offline applications or time-invariant systems, R_1 may be defined as zero.

4. DROP TEST SIMULATIONS

For obtaining the data used on identification, the complete dynamics of landing gear mechanism is simulated, such explained previously in the section 2. The simulation aims to reproduce the drop test. This test consists of simulate the moment the aircraft landing gear touches the ground, only with vertical movement. The initial conditions are set to be equal the real test. So, the wheel receives an initial angular velocity, so that there is braking force and the ensemble is released from a certain height.

The dynamical equations are, then, numerically solved so that the generalized coordinates and their first derivatives are found. The accelerations associated to each coordinate must be estimated as:

$$\ddot{q}_k = \frac{\dot{q}_{k+1} - \dot{q}_{k-1}}{2\Delta t} \tag{23}$$

The tires are considered elastic elements, with internal damping. So, in the simulation, the vertical load is calculated as proportional to its vertical deformation and deformation rate. However, since the normal force on the ground is measured on drop tests, its value is computed on the simulation. Beyond that, the set of measured variables on drop test includes also the vertical acceleration of the sprung mass and the vertical displacement of the wheel hub, which may lead to the angle between the suspension and the sprung mass, and its derivatives.

The results of the simulation of the drop test, with the actual parameters are presented in the following section.

5. RESULTS

In the present section, we describe the results of applying the grey-box estimation procedure for the landing gear case with simulated drop-test data. The present grey-box identification is based on simulated data from a landing gear drop test. The main goal is to determine the unknown parameters from the model, such as the inertial and, mainly, the equivalent damping and stiffness of the ensemble of suspension and structure of the landing gear.

The results of the dynamical equations are the shown in figures 3 and 4. As the landing gear structure works similarly to a vehicular suspension, it is expected that the z-coordinate of the sprung mass presents a damped oscillatory movement.

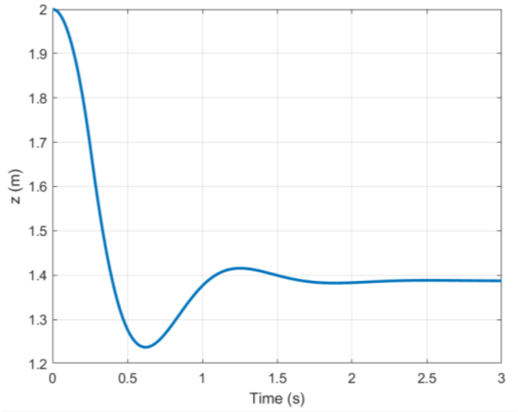

Figure 3: Vertical displacement of the sprung mass

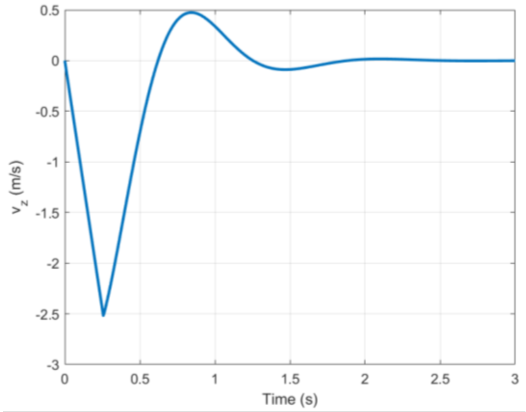

Figure 4: Vertical velocity of sprung mass

The angle θ, defined between the structure and the vertical axis, has a nonzero value initially and increases toward the maximum value when the wheel touches the ground and the tire force begins to actuate, producing a momentum that acts positively at this angle, as seen in figure 5. The angle θ is limited by the stiffness of the structure and the suspension spring.

Since the dynamical equations are known, it is possible to write them as a matrix multiplication. So, the Lagrange equations obtained for the landing gear dynamics could be written as:

$$\begin{bmatrix} \ddot{x} & 0 & -\ddot{\theta}\cos\theta + \dot{\theta}^2\sin\theta & 0 & 0 & 0 & 0 \\ \ddot{z}+g & 0 & \ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta & 0 & 0 & 0 & 0 \\ 0 & \ddot{\theta} & -\ddot{x}\cos\theta + \ddot{z}\sin\theta + g\sin\theta & \theta-\theta_0 & \dot{\theta} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \ddot{\phi} & \dot{\phi} \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ k \\ b \\ J \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ F_z \\ F_z L\sin\theta + F_z\mu L\cos\theta \\ F_z\mu R - T \end{bmatrix} \quad (24)$$

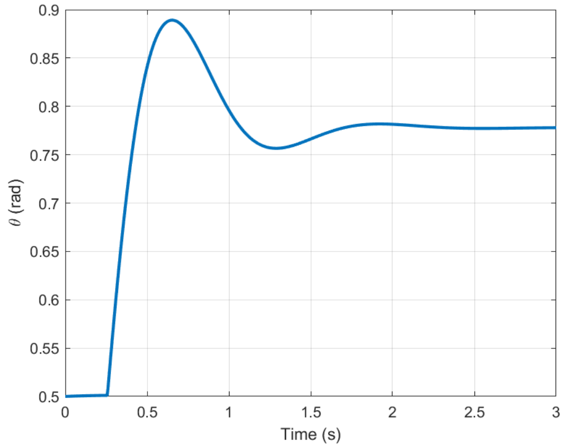

Figure 5: Angle between the vertical axis and the concentrated mass of the structure of landing gear

Once it is supposed the vector in the right side is known, by means of measured or estimated variables, we may affirm that there are two decoupled systems, which are the sprung mass with suspension and the landing gear wheel. We may assume this because the parameters of each system do not affect the dynamics of the other, as observed in the matrix in the left side of the equation 24. The parameters associated to the wheel are usually known or they may be easily measured, so that we may neglect the last equation and the parameters *J* and *c* on the identification process.

It is important to note that it is not possible to predict the mass, length and moment of inertia of each body, but only the combination of them, described by the parameters M_1, M_2 and M_3, in equations 4, 5 and 6. The set of estimated parameters should be as small as possible and any parameter should not be linear combination of others.

We may also do a deduction related to the horizontal displacement of the sprung mass. This variable remains so close to zero in all the simulation and the real drop test, because it is realized on a confined space. So, even if this variable is estimated, its value should be so small that could be lost in the associated noise, harming the identification process. Consequently, the horizontal displacement of the wheel hub is also small, and the tire has a slip near to 1 in all the test. Given the above, the equation 24 may be adapted to:

$$\begin{bmatrix} \ddot{z}+g & 0 & \ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta & 0 & 0 \\ 0 & \ddot{\theta} & \ddot{z}\sin\theta + g\sin\theta & \theta-\theta_0 & \dot{\theta} \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ k \\ b \end{bmatrix} = \begin{bmatrix} F_z \\ F_z L\sin\theta + F_z\mu L\cos\theta \end{bmatrix} \quad (25)$$

Using all the data generated on the dynamics, the process of estimation becomes precise as the results show, reducing the error of estimation on both approaches. In the Kalman Filter estimation, the initial covariance matrix is defined as a diagonal matrix with 1e3 as entries. The estimated vector of parameters is initialized as zero. The table 1 presents the comparative analysis of the results, considering that there is no measurement noise.

Table 1: Estimation results without measurement noise

Parameter (Unit)	Value	LS	LS Error (%)	KF	KF Error (%)
M_1 (ton)	10.2	10.2000	0.0002	10.2001	0.0005
M_2 (kg.m²)	126	124.6235	-1.0925	124.6235	-1.0924
M_3 (kg.m)	150	151.8450	1.2300	151.8456	1.2304
k (kN/rad)	450	446.4083	-0.7981	446.4083	-0.7982
b (kN.s/rad)	90	89.8058	-0.2157	89.8058	-0.2157

The efficiency of the proposed method is evidenced on the small values of the errors of estimation. Since the main goal is to determine the angular stiffness of the ensemble suspension and structure, it is important to evaluate its error, which is less than 1% in terms of absolute value. The errors associated to inertial parameters are due to the approximations adopted on this methodology. However, as they are small, it may be affirmed that these approximations do not harm the estimation. It should be observed also that both approaches present similar results. So, the definition of which one must be used may follow other criteria that not the precision.

Considering that the measurement may be affected by noises, we present two different results. In the first one, presented on table 2, it is considered that the measured data has normal distribution with standard deviation of 0.1%. In the second one, presented on table 3, the standard deviation adopted is 1%.

All errors associated to estimated parameters, including stiffness and damping of the landing gear, have the same magnitude of the case without noise. These errors are small and adequate, which denotes that the proposed methodology remains efficient, even if there are measure noise associated to the sensors.

Table 2: Estimation results with measurement noise of 0.1%

Parameter (Unit)	Value	LS	LS Error (%)	KF	KF Error (%)
M_1 (ton)	10.2	10.2000	0.0002	10.2000	0.0005
M_2 (kg.m²)	126	124.6341	-1.0841	124.6341	-1.0841
M_3 (kg.m)	150	151.9179	1.2786	151.9184	1.2790
k (kN/rad)	450	446.3996	-0.8001	446.3996	-0.8001
b (kN.s/rad)	90	89.8052	-0.2164	89.8052	-0.2164

Table 3: Estimation results with measurement noise of 1.0%

Parameter (Unit)	Value	LS	LS Error (%)	KF	KF Error (%)
M_1 (ton)	10.2	10.2001	0.0008	10.2001	0.0011
M_2 (kg.m²)	126	124.6797	-1.0478	124.6797	-1.0478
M_3 (kg.m)	150	156.4681	4.3121	156.4687	4.3125
k (kN/rad)	450	445.6982	-0.9560	445.6982	-0.9560
b (kN.s/rad)	90	89.7471	-0.2810	89.7471	-0.2810

It may be observed that in the second scenario the precision of the estimation of inertial parameters is worse than in the other cases, specially for the parameter M_3, but the error is not high (less than 5%). However, the stiffness and damping of the suspension of the landing gear are estimated with errors of same magnitude of the ones observed on the noise-free scenario, which leads to conclude that the methodology proposed is robust, even if the sensors have not a measurement noise. These parameters are the main ones in the analysis of the gear walk phenomenon.

The figure 6 shows a comparative between the measured and simulated data for the angle of the structure, using parameters estimated presented on table 3.

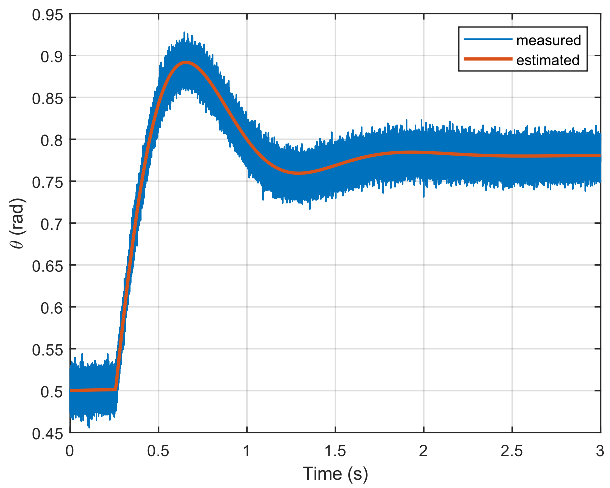

Figure 6: Comparison between measured and estimated angle of the landing gear structure

6. CONCLUSIONS

This paper presented a methodology that may be used effectively on the identification of structural parameters of landing gears. Even on scenarios with large scale of measurement noise, the stiffness and damping of landing gear structure are predicted with small deviations, which do not affect the gear walk simulation. Since the simulated data was obtained from a complete model, which considerer even the tire dynamics, we presume that the methodology may be applied on a real drop test data. This application is important for practical robust control design of aircraft braking systems.

There are two main future possibilities of contributions related to the present paper. The first one is the application of the identification process with the formulation provided by different methods of dynamic modelling, specially bond graphs. This technique allows the integration of models of the different systems, by the coupling of power inputs and outputs (Karnopp et. al., 2012). So, it is possible to construct and identify parameters of an entire aircraft, as well as associate predictive control algorithms for its automation.

The second one is the improvement of the identification process. In complex systems, with many parameters, it is important to demonstrate and understand which ones are identifiable, and then, apply identification process focused on these ones. The identifiability analysis enables one to infer whether the estimation of the parameters based on measurements are unique, thus granting greater confidence in the parameters estimates with respect to fitting error amplitudes. In other words, the identifiability property holds if a set of model parameters will map to different set of measurements (Grewal and Glover, 1976). In the case of landing gear modeling, it is important to have such theoretical confirmation as this model is used to certificate aircrafts.

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