DOI: 10.48011/asba.v2i1.1420

Model predictive control for connected and autonomous vehicles at road intersections

Georgios Ninos * **José Diogo Forte de Oliveira Luna** **

* *Dept. of Automation and Systems Engineering, Federal University of Santa Catarina, Florianópolis, Brazil (georgios.ninos@posgrad.ufsc.br)*
** *Federal Institute of Rondônia - Porto Velho Calama Campus, Porto Velho, Brazil (jose.luna@ifro.edu.br)*

Abstract: This work presents a model predictive control (MPC) to coordinate connected and autonomous vehicles (CAVs) with a Vehicle-to-Vehicle (V2V) communication when they enter intersections. With the purpose of minimizing energy as well as passing the intersection smoothly, it uses an individual linear quadratic optimal controller for each CAV, with a predefined path, that will respect mixed-integer linear constraints to guarantee collision avoidance in relation to the nearby vehicles. This method solves different scenarios with a different number of CAVs crossing the intersection, coming from more than one road, including a platoon formation. The results show that MPC is an efficient technique to integrate multiple CAVs to collaborate with the mutual objective of join merging zones without accidents.

Keywords: Autonomous Vehicles; Intersection control; Model Predictive Control.

1. INTRODUCTION

Nowadays, driving has become a costly task in terms of time and fuel spent amid traffic jams that occur mostly on the streets of large cities and busy highways. It can cause the occurrence of human errors in the control of vehicles related to acts of imprudence, stress or weariness. Thus, an option capable of mitigating these issues is the adoption of an Intelligent Transportation System (ITS), together with Connected and Autonomous Vehicles (CAVs). An ITS provides a set of strategies with the ability to improve safety and mobility in urban flow, through integration between transport management and operation L. Greer et al (2018). The CAV control is responsible for its guidance, while environment dynamic constraints and obstacles avoidance are satisfied Qian (2016). In other words, it requires special efforts to plan an optimal trajectory for the car movement, owing to the changing circumstances, such as traffic signs and other individuals on the street lanes.

Concerning CAVs, one of the critical issues for their high-performance is the flow control at intersections of urban routes or highways Mahmassani (2016). This mode of transport has the potential to contribute to the better efficiency in traffic and reduction in the number of road accidents, and as a consequence, there is an urgent need for a reorganization in the traditional ways of traffic control and operation, because they cannot afford an Autonomous Intersection Management (AIM).

Just as important as how to set CAVs control is how to establish communication between them in a way that averts failure in the messages exchanging. To that end, it is possible to choose one or more transmission systems among V2V (vehicle-to-vehicle), V2I (vehicle-to-infrastructure), I2I (infrastructure-to-infrastructure) or V2X (vehicle-to-everything) transmissions. Usually, in AIMs, there are two types of coordination: the centralized one in which there is at least one general task decided by a single central controller, and the decentralized approach in which each vehicle determines its own control policy Rios-torres and Malikopoulos (2015).

In this context, in the last two decades, there has been a substantial integration of technologies advances in wireless communication, digital processing, and detection in traffic management systems, with the goal of increasing their sustainability, safety, and reliability, evidenced in the work of Mahmassani Mahmassani (2016). In addition, Rios-Torres and Malikopoulos Rios-torres and Malikopoulos (2015) and González *et al* González et al. (2016) explore how different applications of coordination and movement control in ITS and CAVs result in shorter spaces between vehicles and faster responses, while at the same time as improving road capacity by identifying the appropriated speed limits. Moreover, Katrakazas *et al* Katrakazas et al. (2015) and Wang and Hussein Wang and Husseim (2012) have directed studies to assist researchers in developing motion planning techniques for autonomous vehicles in real time.

Taking into account the aforementioned characteristics, model predictive control (MPC) provides one of the most promising ways to deal with the dynamic and uncertain nature of trajectory planning and tracking for CAVs Reddy (2016); Liu et al. (2010). In this approach, the CAV control observes future predictions, on a fixed horizon, based on the currently available information. Recent works have applied MPC in autonomous driving field. Reddy (2016) and Liu et al. (2010) present a classic hierarchical control architecture of an individual vehicle that decomposes the controller into a motion planner and a tracking controller. Nolte et al. (2017) proposes a safe trajectory planning even in cases of system failures through MPC for linear parameter-varying systems. Qian et al. (2015) and Makarem and Gillet (2013) suggest decentralized MPC approaches where vehicles solve local optimization problems in parallel, ensuring them to cross the intersection smoothly.

In this paper, a decentralized MPC approach, built upon the work of Makarem and Gillet (2013), solves the intersection control problem involving CAVs, applying collision avoidance constraints in each vehicle using the predicted trajectories of the nearby vehicles in a V2V communication. The proposed strategy addresses the gap concerning the exclusion of the vehicle after the vehicle left the intersection and the extension of the same-lane crash avoidance constraints to the whole prediction horizon, while considering a cost function that weights the speed tracking and the control effort alongside with the platoon formation.

2. INTERSECTION CONTROL PROBLEM

In AIM context, vehicles that enter road junctions or merging highways have to subject themselves to some rules to obtain the better crossing sequence. Commonly, there is a delimited area before the intersection or merging zone, called control zone, where the vehicles exchange data and define a priority list to approach the intersection or merging zone. The intersection and merging control problems are very similar and can be easily treated with the same approaches Rios-torres and Malikopoulos (2015). Fig.1 shows these two different scenarios.

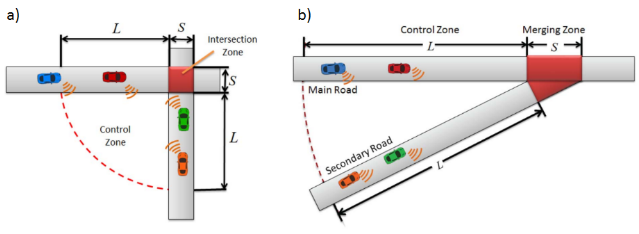

Figure 1. a) Cross intersection b) Merging roadway. Source: Rios-torres and Malikopoulos (2015)

The distance between the entry of the two zones is L, and the region at the center of the intersection has a length S. The main objective of good coordination is to establish safe distances between the vehicles placed at the merging zone to avoid collisions.

It is possible to consider that vehicles close to an intersection follow second-order dynamics to describe their movements, as shown in (1):

$$m_i \ddot{x}_i(t) = u_i(t), \tag{1}$$

where $i = 1, 2, ..., n$, $n \in N$ indexes each vehicle in its specific lane (one side of the road with the orientation pointing to the intersection zone), m_i is the mass of the i-th vehicle, x_i is the position of the i-th vehicle, and u_i is the input of the i-th vehicle. For simplicity, this work will assume that each CAV knows its path and they can only accelerate or decelerate without shifting off the existing pathway nor taking turns. Furthermore, the model does not account for friction or air drag.

3. MPC STRATEGY

MPC is a control strategy that uses the system model to obtain an optimal control sequence, minimizing an objective function Camacho and Bordons (2007). At each sampling instant, there are predictions of the behavior of the system in the prediction horizon N. Although the controller calculates an optimal control sequence that minimizes a cost function for each step of the horizon, it will only use the input values for the current sampling interval.

As a result of this, a discrete-time representation of this model becomes necessary, so it may consider a sampling period T, a sampling instant k and the application of Euler approximation to the (1), to obtain the following discrete-time state-space model for the movement of the CAV:

$$x(k+1) = \mathbf{A}x(k) + \mathbf{B}u(k), \tag{2}$$

where $\mathbf{A}$ is the state matrix, and $\mathbf{B}$ is the matrix related to the input signal, respectively, given by:

$$\mathbf{A} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \tag{3}$$

$$\mathbf{B} = \begin{bmatrix} 0.5T^2 m_i^{-1} \\ \\ T m_i^{-1} \end{bmatrix}. \tag{4}$$

The state $x(k) = [x_{1i}(k)\ x_{2i}(k)]^T$ is composed of the position x_1 and speed x_2 of each vehicle. The input control $u(k)$ represents the net force of the vehicles which is directly related the energy consumption used to move them.

The main idea is to use individual MPC for each vehicle with some assumptions:

- The nearby vehicles share information about their position, speed and intended direction at every time step;
- The CAVs cannot turn to the left or right. They can only move forward in a straight line;
- Each CAV considers that the predictions will not change for all vehicles, except its own;
- Every vehicle optimizes its speed along its path to cross the intersection.

In order to ensure smooth trajectories and limited inputs, this work proposes to minimize the following cost function J in a standard quadratic form:

$$J = \sum_{j=1}^{N} \sum_{i=1}^{n} q_i (v_{ref} - x_{2i}(k+j))^2 + r_i u_i^2(k+j-1), \tag{5}$$

where x_{2i} is the speed of the i-th vehicle along its path during the j-th time step of the horizon, v_{ref} is the desired speed in the optimization horizon N, and u_i is the control input of the vehicles. The two constants q_i and r_i are weights that impact on the importance of the trajectory smoothness and the actuation energy. The first term penalizes the deviation of the speed from the desired speed and the second one penalizes the control effort. By means of constraints, it will be possible to ensure the collision avoidance between the vehicles. Furthermore, this keeps a quadratic MPC problem with mixed-integer linear constraints.

Before establishing constraints, it is necessary to introduce a priority assignment to determine which CAVs will join the intersection first, based in which one has the shorter predicted time arrival τ, estimated by:

$$\tau_i = \frac{d_i(0)}{v_i(0)}, \tag{6}$$

where $d_i(0)$ and $v_i(0)$ denote the distance between the vehicle i and the intersection, and the actual speed of the vehicle i, respectively. If there is a vehicle l in a different road converging to the same intersection, the predicted arrival time τ_l could be calculated in the same way as in (6). To demonstrate this scenario, Fig.2 depicts the priority assignment for two vehicles.

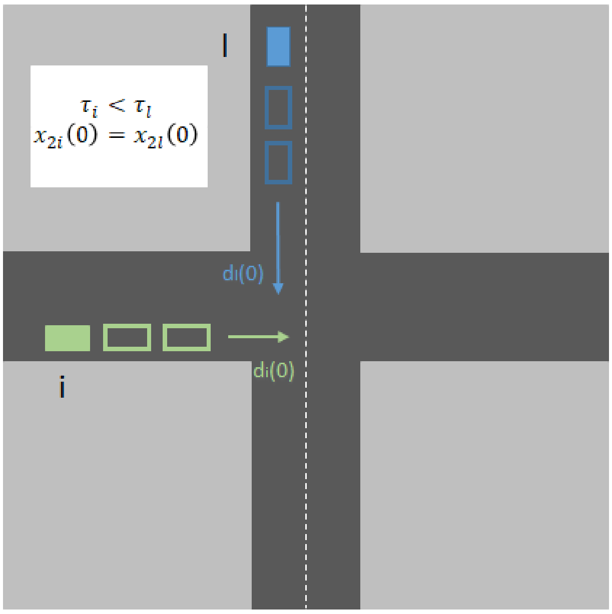

Figure 2. The initial positions are used to determine the priority of the vehicles for passing the intersection. In this case, both vehicles have the same speed.

As seen from Fig.2, τ_i is shorter than τ_l, so, in this case, the vehicle i will be the first to get in the intersection. Thus, the constraint for collision avoidance for crossing vehicles is given by a single mixed-integer linear inequality:

$$\begin{cases} x_{1i}(k+N) - x_{1l}(k+N) \geq \Delta d(1-\delta_i)(1-\delta_l) & \tau_i \geq \tau_l \\ x_{1l}(k+N) - x_{1i}(k+N) \geq \Delta d(1-\delta_i)(1-\delta_l) & \tau_i < \tau_l \end{cases} \tag{7}$$

The distance between the predicted positions for both CAVs in the time step N^{th} must be higher than the minimum distance difference Δd, which guarantees the crossing of vehicles without collision. Thus, in accordance with the priority list, they should decide whether to accelerate or decelerate to enter the intersection zone. As there is no chance the CAVs will collide after they pass through the intersection, the binary variables δ_i and δ_l will assume the value 1, so, the required minimum distance will be 0 when at least one of the compared vehicles crosses the road junction.

When considering more than one vehicle in the same lane, there is the need to add another constraint to ensure that they will not crash. So, the solution is to limit a minimum distance between the one step ahead predicted position for a CAV and the current location for the CAV in front of it, ensuring a better safety to the system. This situation is shown in Fig.3.

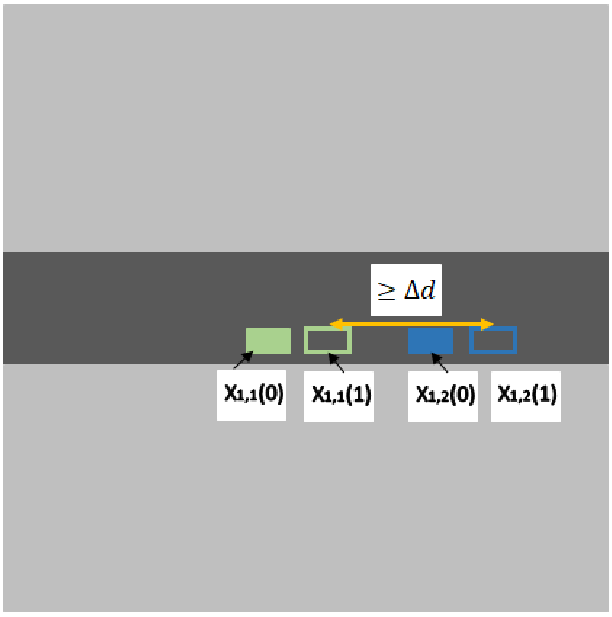

Figure 3. Constraints related to collision avoidance for the vehicles in the same lane.

Constraint (8) represents the collision avoidance for vehicles in the same lane.

$$x_{1i}(k+j) - x_{1i+1}(k+j) \geq \Delta d \ \forall i = 1, 2, ..., n \tag{8}$$

It is important to take into account dynamic limits, like acceleration and braking, and road limits, like traffic signs indicating the maximum speed allowed. So, the linear inequalities constraints that fulfill these requirements are:

$$m_i a_{i,min} \leq u_i(k+j) \leq m_i a_{i,max} \ \forall i = 1, 2, ..., n \tag{9}$$

$$0 \leq x_{2,i}(k+j) \leq v_{i,max} \ \forall i = 1, 2, ..., n \tag{10}$$

When more than two lanes are involved, a new set of constraints has to be leveraged, as shown in (11) and (12) for intersection crossin, and collision avoidance in the same lane, shown in (13).

$$\begin{cases} x_{1i,o}(k+N) - x_{1l,p}(k+N) \geq \Delta d(1-\delta i)(1-\delta l) \\ \quad \forall o = 1, 3; \forall i, \forall l = 1, 2, ..., n; \forall p = 2, 4 \end{cases} \tag{11}$$

$$\begin{cases} x_{1i,o}(k+N) - x_{1l,p}(k+N) \geq \Delta d(1-\delta i)(1-\delta l) \\ \quad \forall o = 2, 4; \forall i, \forall l = 1, 2, ..., n; \forall p = 1, 3 \end{cases} \tag{12}$$

$$\begin{cases} x_{1i}(k+j) - x_{1l}(k+j) \geq \Delta d \\ \forall i = 1, 2, ..., n; \forall l > i \end{cases} \tag{13}$$

In (11) and (12), o represents the lanes in the horizontal highway, and p denotes those in a vertical way. Furthermore, i and l indicate the vehicles placed in each lane of o and p, respectively. In (13), i and l are vehicles in the same lane, but l is always higher than i because there will be a comparison between the distances of different vehicles.

One of the ways to mitigate congestion, reduce energy use and emissions, and improve safety, is through the packing of CAVs in platoons Rios-torres and Malikopoulos (2015). Forming a platoon before entering the intersection and keeping it while passing intersections can decrease the computation time for optimization, just focusing on the leader Makarem and Gillet (2013). Consequently, the cost function (5) must be modified, in order to add a new term:

$$J = J_v + J_u + J_x, \tag{14}$$

with:

$$J_v = \sum_{j=1}^{N} \sum_{i=1}^{n} q_i (v_{ref} - x_{2i}(k+j))^2, \tag{15}$$

which aims to make the i-th CAV follow the speed v_{ref}.

$$J_u = \sum_{j=1}^{N} \sum_{i=1}^{n} r_i u_i^2(k+j-1), \tag{16}$$

which ains to minimize the energy spent to run the CAVs. And:

$$J_x = \sum_{j=1}^{N} \sum_{i=1}^{n-1} s_i (x_{1i}(k+j) - x_{1(i+1)}(k+j))^2, \tag{17}$$

which encourages platooning, where s_i is the weight coefficient that penalizes the distance difference for the vehicles in the same lane. When s_i is large enough, this new cost function makes the vehicles staying together with an average speed.

4. SIMULATION AND RESULTS

The scenario used for the simulations is similar to the cross intersection in Fig.2. The control zone has $L = 150$ m and $S = 8$ m. The lanes have a numerical sequence from 1 to 4, starting on the left side of the intersection zone and running counterclockwise. The simulations ran in the software Matlab, using the toolbox for modeling and optimization Yalmip Löfberg (2004), in association with the optimization solver Gurobi. Table 1 shows the vehicle parameters, as well as the simulation parameters.

The first situation tested how the MPC behaves when four vehicles join the intersection zone, coming from the perpendicular lanes 1 and 4 and giving more importance to the desired speed ($q_i = 2000$ and $r_i = 1$). The result is shown in Fig.4. The initial positions for the cars on each lane were $x_1(0) = [-80\ \ -95]$.

The vehicles closer to the intersection in each lane (black lines) accelerate to cross the junction first, with a priority to the vehicle in lane 1. The remaining vehicles (blue lines) have to decelerate to keep a minimum distance and wait

Table 1. Simulation and vehicles parameters

[HTML]000000 Parameter	[HTML]000000 Value
T	500 ms
N	10
v_{ref}	6 m/s
v_{max}	15 m/s
v_{min}	0 m/s
m	600 kg
a_{max}	3.47 m/s^2
a_{min}	-10 m/s^2
Δd	10 m

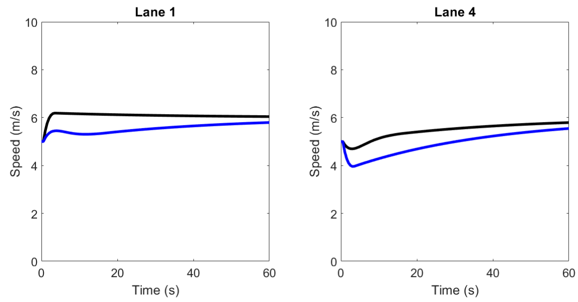

Figure 4. Four different vehicles that come from perpendicular lanes and join the intersection.

for their turns. It is noticeable that all of them try to reach the desired speed (6 m/s).

With the same parameters as before, another simulation experimented the cross intersection with six vehicles divided into two perpendicular lanes. Fig.5 presents the outcomes of this new circumstance. The initial positions for the cars on each lane were $x_1(0) = [-80\ \ -90\ \ -100]$.

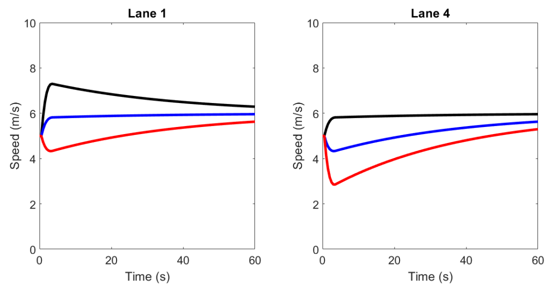

Figure 5. Six different vehicles that come from perpendicular lanes and join the intersection.

The results are similar to the previous situation as the vehicles near the intersection (black lines) attempt to cross the intersection by increasing their speeds, and those far from it (blue and red lines) adjust their acceleration to respect the priority list.

A scenario with twelve vehicles entering the intersection from four different paths, following the (11), (12) and (13), presents an operation with similar speed profiles for the vehicles in the same road, but in opposite lanes, as shown in Fig.6. The initial positions for the cars on each lane were $x_1(0) = [-80\ \ -100\ \ -120]$.

In lane 1 and 3, the second CAVs (blue lines) accelerate to getting closer to the vehicles in front of them (black lines), later, they decelerate to try to match their speed, while the last ones (red lines) decelerate until the leaders pass the intersection. In lane 2 and 4, the chosen solution was

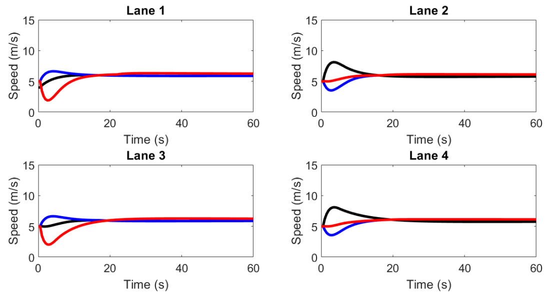

Figure 6. Twelve different vehicles that come from four different lanes.

to prioritize only the entering of the first vehicles (black lines), so they accelerate to be the earliest CAVs to join the intersection zone. Fig.7 depicts the moment before the first vehicles cross the intersection and illustrate the speed profiles in Fig.6.

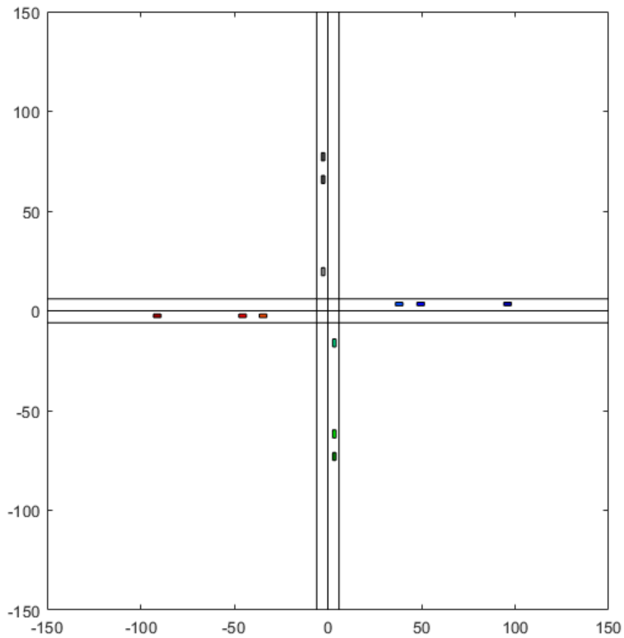

Figure 7. Illustration of the moment before the first vehicles cross the intersection. Lane 1: cars moving from left to right; Lane 2: cars moving from bottom to top; Lane 3: cars moving from right to left; Lane 4: cars moving from top to bottom.

The performance of the platoon formation, defined by the cost function in (14), took into consideration two different sets of weights. First, it assumed $s_i = 900$, forcing the vehicles to stay together, then, $s_i = 1$, giving less importance to the platoon configuration. Fig.8 draws these two environments. The initial positions for the cars on each lane were $x_1(0) = [-80\ -100\ -120]$.

For $s_i = 1$, the vehicles behave as in Fig.7, forming small separate groups. On the other hand, for $s_i = 900$, in each lane the vehicles form a solid platoon to pass the intersection simultaneously. Because of the abrupt variations in acceleration input, necessary for platoon formation, the weight r_i had to assume a higher value ($r_i = 6$), in order to give more importance to energy saving.

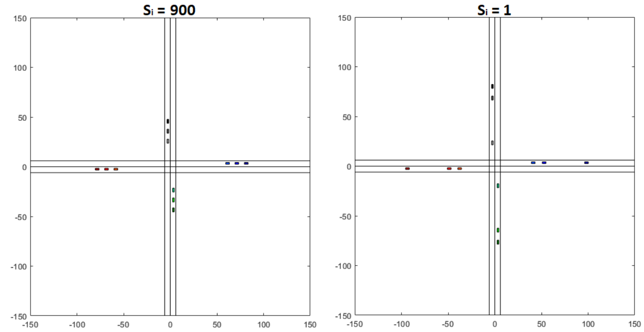

Figure 8. Two sets of weights for platoon formation. Lane 1: cars moving from left to right; Lane 2: cars moving from bottom to top; Lane 3: cars moving from right to left; Lane 4: cars moving from top to bottom.

Fig.9 demonstrates the speed aspects when the platoon formation happens.

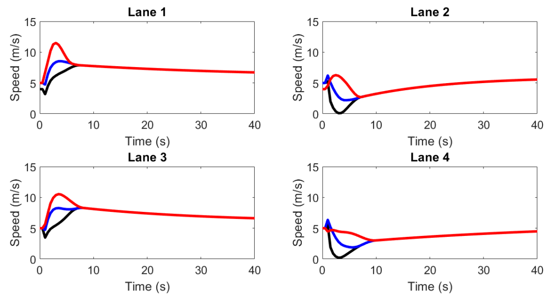

Figure 9. Twelve different vehicles forming four platoons.

When the MPC requires a platoon formation, the vehicles in the last position (red lines) to pass the intersection raise their speed quickly to get closer to the others. Additionally, the two CAVs next to the intersection (black and blue lines) decelerate until there is speed synchrony among all the vehicles in the lane.

This work also measured the inputs of the MPC system in relation to the applied acceleration for each CAV, with and without platoon formation. Fig.10 and 11 describe these results.

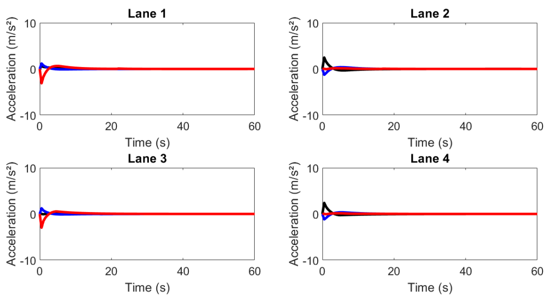

Figure 10. Input acceleration without platoon formation.

As seen in Fig.10 and 11, both configurations lead the inputs to be close to zero, but the platoon formation demands more acceleration in the first moments of the simulation, so, it will require more energy consumption, contrary to what Rios-Torres and Malikopoulos Rios-torres and Malikopoulos (2015) say. However, there is

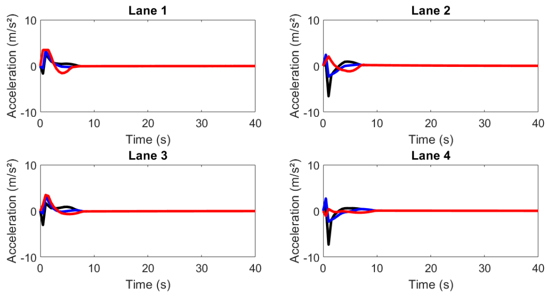

Figure 11. Input acceleration with platoon formation.

the need to consider the absence of the air drag in this study, because it is a factor that impacts a lot in the energy reduction when the vehicles take advantage of the wake created by the heads of the platoon. In addition, it shows the first organization of the CAVs as a large group, and they can reuse the same formation in other intersections. As a consequence of this, the variations will be less frequent.

A comparison with the controller proposed in Makarem and Gillet (2013) was conducted in order to assess the performance of the proposed strategy. The tuning of the controller was proceeded as displayed on Table 2. The initial positions for the cars on each lane were $x_1(0) = [-80\ -100\ -120]$.

Table 2. Tunning and simulation parameters for Makarem and Gillet controller.

Parameter	Value
v_{max}	15 m/s
a_{max}	10 m/s^2
a_{min}	-10 m/s^2
v_{ref}	6 m/s
N	10
q	1
p	1
r	2
Δd	20 m
ΔD	10 m
m	600 kg

The scenario considered twelve vehicles, with three on each lane. The resulting energy consumption is listed on Table 3, compared with the proposed strategy, which was able to reduce the spent energy in 6.70%. This means deploying a smoother control, which implies in more comfort for the users and increased autonomy.

Table 3. Comparison.

Controller	Energy
Makarem and Gillet	367.282 kJ
Proposed	342.681 kJ

5. CONCLUSION

This work proposed a MPC for the coordination of CAVs at intersections. The control minimizes a quadratic cost function, following mixed-integer linear constraints for collision avoidance, in order to guarantee minimum energy consumption and smooth trajectories for the crossing vehicles. It also establishes a priority list to enter the road junction, based on the predicted arrival time at the intersections. Vehicle models obey to second order dynamics as well as their constraints. The simulations exploited different circumstances like the number of vehicles, the approaching for multiple lanes and the platoon formation. The results showed that MPC is an efficient way to deal with uncertain environments to trajectory planning and tracking for CAVs.

For future researches, pending questions on the effect of communication delay and the robustness of the MPC, under model uncertainty and parameter variation, should be addressed. Furthermore, the study must include operation on a combination of intersections, using a significant number of CAVs to simulate real traffic conditions, also given them the ability to make turns. Finally, the air drag and friction will be considered in the platoon formation to measure its influence on energy saving.

REFERENCES

Camacho, E.F. and Bordons, C. (2007). *Model Predictive Control*. Springer-Verlag, London, UK, 2 edition. Advanced Textbooks in Control and Signal Processing.

González, D., Pérez, J., Milanés, V., and Nashashibi, F. (2016). A Review of Motion Planning Techniques for Automated Vehicles. *IEEE Transactions on Intelligent Transportation Systems*, 17(4), 1135–1145. doi:10.1109/TITS.2015.2498841.

Katrakazas, C. et al. (2015). Real-time motion planning methods for autonomous on-road driving: State-of-the-art and future research directions. *Elsevier*, 416–442.

L. Greer et al (2018). Intelligent transportation systems benefits, costs, and lessons learned 2018 update report. Technical report, U.S. Department of Transportation, Washington.

Liu, C. et al. (2010). Experimental tests of autonomous ground vehicles with preview. *International Journal of Automation and Computing*, 342–348.

Löfberg, J. (2004). Yalmip : A toolbox for modeling and optimization in matlab. In *In Proceedings of the CACSD Conference*. Taipei, Taiwan.

Mahmassani, H. (2016). Autonomous vehicles and connected vehicle systems: Flow and operations considerations. *Transportation Science*, 1–23.

Makarem, L. and Gillet, D. (2013). Model predictive coordination of autonomous vehicles crossing intersections. In *16th International IEEE Conference on Intelligent Transportation Systems (ITSC 2013)*, 1799–1804. IEEE.

Nolte, M. et al. (2017). Model predictive control based trajectory generation for autonomous vehicles – an architectural approach. *IEEE*.

Qian, X., Gregoire, J., de La Fortelle, A., and Moutarde, F. (2015). Decentralized model predictive control for smooth coordination of automated vehicles at intersection. In *2015 European Control Conference (ECC)*, 3452–3458. doi:10.1109/ECC.2015.7331068.

Qian, X. (2016). *Model predictive control for autonomous and cooperative driving*. Master's thesis, PSL Research University, Paris, FR.

Reddy, G.P. (2016). *Hierarchical Model Predictive Control for Trajectory Generation and Tracking in Highly Automated Vehicles*. Master's thesis, Faculty of Mechanical, Maritime and Materials Engineering (3mE) · Delft University of Technology, Delft, NL.

Rios-torres, J. and Malikopoulos, A.A. (2015). A Survey on the Coordination of Connected and Automated Vehicles at Intersections and Merging at Highway On-Ramps. *IEEE Transactions on Intelligent Transportation Systems*, 1–12.

Wang, Y. and Husseim, I.I. (2012). *Searching and Classification Using Multiple Autonomous Vehicles*. Springer, London.