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Metaheuristic Search for Optimum Cost-Benefit Resilience Level by Redundancy Adding ⋆

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Abstract: Modern systems have become increasingly more complex, and their analysis becomes significantly more complex. Many practical aspects of complex network tools have mainly been applied to critical infrastructure, in particular, to study power systems' resilience. Blackout prevention, system resilience, and restoration consider the ability of the system's self-healing. The self-healing strategies depend, basically, on the existence of extra lines to re-route energy. Some studies suggested that there is an optimum cost-benefit point when adding power lines redundancies to a system considering the systems' resilience. One method to solve this optimisation problem is the use of a metaheuristic algorithm. These algorithms combine exploration and exploitation on the search for a solution. In this paper, a Chu-Beasley genetic algorithm is used to search for the optimum cost-benefit level of redundancy in a system. The system used is from the Repository of Distribution Systems (REDS), and the function used to evaluate the resilience considers an efficiency coefficient so that the resilience by cost curve would have a maximum point. This experiment is executed as a topological analysis. The expected results were obtained using estimated curves from Monte-Carlo simulations for a wide range of combination of parameters. The results from three different parameters of efficiency coefficient were compared to the expected values obtained. The results show that there is a best cost-benefit level of redundancy when an efficiency level is determinate. Also, the GA used has excellent performance for finding this point.

Keywords: Complex networks, Cost-benefit, Genetic algorithms, Resilience, Self-healing.

1. INTRODUCTION

Modern systems have become increasingly bigger and more complex as a result of the technological development that has been happening (Gallos and Fefferman, 2015). Consequently, their analysis became significantly more complex. An useful modelling for this kind of system is using complex networks (Bessani et al., 2018). The complex network modelling is capable of integrating and unifying the relationship between structure and dynamic (Costa et al., 2011). The usefulness of the complex network modelling is shown by the wide number of applications modelling systems such as cloud computation (Travieso et al., 2015), an epidemic transmission system (Zhu et al., 2017), global tourism flow (Lozano and Gutiérrez, 2018), energy distribution networks (Bessani et al., 2018), optical communication networks (Choji Freitas et al., 2012) and others systems (Costa et al., 2011).

Many practical aspects of complex network tools have mainly been applied to critical infrastructures, in particular, to study power systems' resilience (Bessani et al., 2019). Critical infrastructures are defined as systems with

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a major importance to maintain vital societal functions, physical integrity and security, health, social and economic welfare. The disruption of an critical infrastructure is likely to have cascading effects on others critical infrastructures (Yusta et al., 2011; Bessani et al., 2019). Thus, becoming clear the need for resilience studies for such systems, as resilience is most commonly defined as the capacity of a system to retain its normal operation when failures, errors and new conditions happen (Almoghathawi et al., 2019).

Blackout prevention, system resilience and restoration consider the ability of the system to recover from possible contingencies. This recovery capability depends, basically, on the existence of backup lines to re-route the power flow. The studies Ribeiro et al. (2018) and Ribeiro et al. (2019) suggested that there is an optimum cost-benefit point when adding lines redundancies to a system considering the system's resilience.

Every addition of lines redundancies to a system has a cost. Also, after an amount of investment in the system's redundancy, the investment needed to have a significant increase in the system's resilience by adding redundancies becomes higher. Considering that, if an efficiency level limit for the investment is defined, it is possible to look for the last point with higher or equal efficiency to the limit by penalising the points with higher and lower efficiency. This way, the problem is turned into an optimisation problem.

One method to solve optimisation problems that have mathematical formulation with uncertain, stochastic and dynamic information is the use of metaheuristic algorithms (Bianchi et al., 2009). Some of the metaheuristic are nature-inspired and some are not (Bianchi et al., 2009), but all of them use two phases of the search process called exploration and exploitation (Mandal, 2018). Exploration is a global search that focus on finding promising areas of the search space. Meanwhile, exploitation is a local search that focus on searching over the promising areas of the search space (Mandal, 2018).

Furthermore, the metaheuristics can be divided in two categories based on the number of initial solutions used. In the single-solution-based category the optimisation starts with a single candidate solution that is evolved. In the population-based category the optimisation starts with a set of solutions (which is usually called population) and this population is evolved (Mandal, 2018). The population-based metaheuristics have advantages over the single-solution-based, as they have a greater exploration capability and are less likely to stick in local optima (Mandal, 2018).

One example of population-based metaheuristics are the Swarm Intelligence (SI) algorithms (Mandal, 2018). They are based on the behaviour of swarms, flocks, herds or schools of animals and insects, where one agent navigates using a simulated social and collective intelligence (Mandal, 2018). Another example of a population-based metaheuristics is the Genetic Algorithms (GA) (Mandal, 2018). In the GA a solution to the problem is called individual and a set of them is called population. Every iteration of the algorithm is called a generation, on it an operations is applied to individuals of the current population to create possible members of the next generation (Bianchi et al., 2009).

One GA is the Chu-Beasley GA (CBGA) (Chu and Beasley, 1997). On it a tournament is made with individuals of the current population to select the parents (individuals used on the creation of new individuals) and their combination will only provide one child (new individual) (Chu and Beasley, 1997). Moreover, the next generation is created of a combination of the current generation and the children created. A child is only added to the current population if it is more fit than the least fit individual of the population. When a child is added to the population the least fit individual of the current population is removed of the population (Chu and Beasley, 1997).

In this paper, a Chu-Beasley genetic algorithm (CBGA) is used to search for the optimum cost-benefit level of redundancy in a distribution system. It is a topological analysis without considering lines flow limit and nodes consumption.

The remainder of this paper is organised as follows. Section 2 presents the CBGA used and the methodology of the fitness function used. Moreover, it also shows the description of the simulations performed and explains how the expected values used for comparison where obtained. Section 3 shows the results obtained from the simulations and the expected results obtained. Finally, section 4 presents the conclusions of this paper.

2. METHODOLOGY

For this work the scenario analysed was an only topological study were all kind of losses were not considered and the simple existence of a route from the node to a source was enough to supply the node's demand. For it, the expected results were obtained using estimated curves from Monte Carlo simulations for a wide range of parameters combination.

In the following subsections the evaluation of resilience used as fitness functions is explained. Also, the CBGA and its use are addressed. The system used for the simulations and the method for adding redundancy to the system are shown. Finally, the simulations made, including the experimental search and the way that the expected values were obtained, are explained.

2.1 Evaluation of Resilience

The evaluation of resilience is based on Ribeiro et al. (2018) and Quattrociocchi et al. (2014). On it the resilience of three different topology of system (square grid, small-world and scale-free) were analysed using the mean and squared mean error of multiple runs of a number of failures and self-healing executions. On Quattrociocchi et al. (2014) the failure is the removal of an edge and on Ribeiro et al. (2018) the failure could be the removal of an edge, the removal of a node or an random choice between the previous. At the end of each execution of a failure and self-healing a variable FoS (the number of nodes connected to a source over the total number of nodes) was saved and used to evaluate the resilience of the system. The resilience in Ribeiro et al. (2018) was measured by the number of fails needed to make the system reach the values of FoS equal to 0.9, 0.5 and 0.1.

For this work, the same process of obtaining the variable FoS was used. However, the measure of resilience was considered to be the integral of the curve FoS × *number of fails*. Thus generating only one value as the measure of the resilience. It must be noted that since the resilience measure is obtained by mean of a number of simulations its value is non-deterministic. This means, that the value obtained is the actual value of resilience plus an error value that converges to zero as the number of simulations done increases.

2.2 Resilience × Cost Curve

Since the search is for the best cost-benefit redundancy level, the curves used for analysis were based on the *resilience* × *cost* curve. For this curve, an estimation of the cost was needed. On that account the cost for a redundancy level was considered to be proportional to the total length of the redundancies added to the system. With this cost estimation and using the resilience evaluation before an *resilience* × *cost* curve was obtained.

This *resilience* × *cost* curve had a behaviour similar to an log curve, and for that reason, an optimum cost-benefit point was not clear. However, considering that the *resilience* is a function of the *cost*, $f_{res}(cost)$, and if an efficiency level a for the investment in resilience is specified, then a new function $f^*_{res} = f_{res}(cost) - a \cdot cost$ can be defined. This function's derivative is $f^*_{res}{}' = f_{res}(cost)' - a$, which shows that the function f^*_{res} will have a maximum point, that is the point where the growth rate of the *resilience* match the efficiency level. This point, is the optimum cost-benefit point for the specified efficiency level. This was adopted since most of companies have policies that determinate the minimum investment return rate (efficiency level) that an investment must have so that the investment is made.

2.3 Chu-Beasley Genetic Algorithm (CBGA)

As it was explained in the introduction, the CBGA is a population-based metaheuristic. The differences between CBGA and others GAs are that on the CBGA only one child is produced from a recombination operation and instead of the children forming the new population, the new population is formed by the combination of the children and the current population. This combination is done by only adding a child to the population if the child is better fit than the least fit member of the population. When a child is added to the population, the least fit member of the population is removed from it (Chu and Beasley, 1997). The general idea of the CBGA implemented for this work is shown the flowchart in figure 1.

Specifically for this paper, the CBGA searched over the redundancy levels between 0 and 1 with a precision of 0.01. This was done, as for most problems the difference of 0.001 between redundancy levels does not make much difference in the resilience and in the cost. This way an individual is a redundancy level to be added to the system.

In addition to that, the fitness function used was the f^*_{res} described on the previous subsection. Moreover, the recombination done to generate children was done by calculating the mean of its two parents and rounding it to the nearest value. The parents for the recombination where chosen by tournament between five individuals.

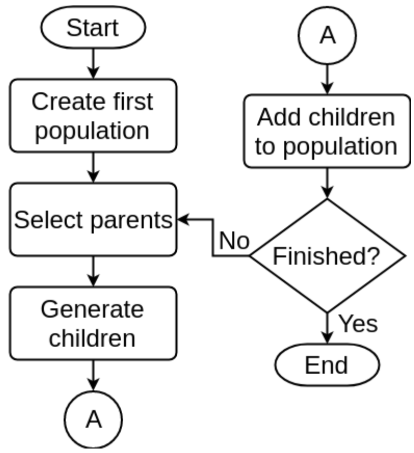

Figure 1. Flowchart showing the operation of the CBGA implemented for this work.

2.4 Repository of Distribution Systems (REDS)

The REDS is a repository of test-cases of distribution systems publicly available for reporting and comparing researches results on problems such as capacitor placement, load balancing, power flow solution, network reconfiguration, etc (Kavasseri and Ababei). The system used in this paper is the bus_83_11, which topology can be seen in the figure 2. This system has eleven sources, eight three consumers nodes and thirteen backup edges.

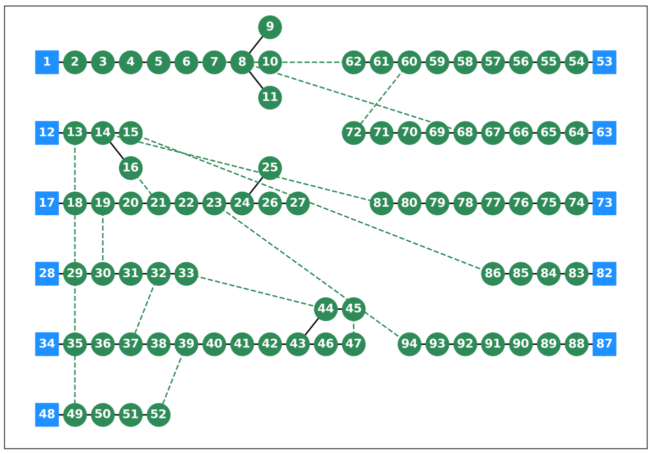

Figure 2. Topology of the system bus_83_11 used for analysis.

2.5 Small-world Method

The small-world method of adding redundancies to the system its an adaptation of a small-world topology graph generator. The small-world topology is where the graph has high efficiency and small diameter and mean geodesic distance Watts and Strogatz (1998).

The adaptation made is that instead of adding random edges on a ring graph to create a small-world graph, the random edges are added to the bus_83_11 graph. The function adapted was the *newman_watts_strogatz_graph* from the NetworkX library of Python (Hagberg et al., 2008). This function has a parameter p which is the probability of adding a new edge for each edge in the system. This variable controls the number of backup edges added to the system, thus this variable was adopted as being r the redundancy level.

This way, the small-world method adds new redundancy edges to the graph in a way that for each edge (u, v) of the base graph a random value between 0 and 1 is drawn. If this value is smaller than the redundancy level r, a new edge (u, w) is added. On this new edge (u, w), w is a randomly chosen node of the graph. By doing that, it adds new edges to the system in the same way as proposed in (Newman and Watts, 1999).

2.6 Expected Results

To have a comparison reference for the results, it was needed to estimate the expected results values. For that purpose, the evaluation of resilience was run for values of r varying from 0.00 to 1.00 with a step of 0.01. Then, these 101 points were used to estimate the *resilience* $\times$ *cost* and $r \times$ *cost* curves. The estimations were done using a polynomial function of degree 120 through the function *polyfit* of the NumPy library of Python (van der Walt et al., 2011). From these estimations, it was possible to find the maximum point of the function f^*_{res} for the efficiency level a equal to 0.3, 0.5 and 0.8, thus obtaining a value o *cost* that would instantiate a level of redundancy.

2.7 Simulation Description

For the simulation and test of the efficiency of the CBGA implemented the algorithm was run a number of times for the efficiency levels of 0.3, 0.5 and 0.8. All executions were made using the parameters of table 1. The results of the runs were compared with the expected results values and between themselves, calculating the mean and the mean squared error (MSE). Moreover, the computational time of the executions was also analysed using mean and MSE. The convergence of the CBGA was analysed by plotting the mean and MSE of the best scores for each generation over the executions.

Table 1. Values of Parameters of the CBGA

Parameter	Value
Evaluation repetition	100
Population size	10
Generation limit	10
Recombination rate	0.5
Mutation rate	0.2

3. RESULTS

The estimated curves used to estimate the results can be seen in the figures 3 and 4. These figures show the estimated curves of *resilience* $\times$ *cost* and $r \times$ *cost* and the 101 experimental points used to estimate them. On them, it becomes clear that the *resilience* $\times$ *cost* curve has a behaviour similar to a log curve.

The figure 5 shows the curves of the function f^*_{res} for a equal to 0.0, 0.3, 0.5 and 0.8. For $a = 0.0$ the function f^*_{res} is equal to the curve *resilience* $\times$ *cost*. In the figure it is possible to see how the original estimated curve does not have a maximum points, however when the efficiency level is determined the resulting curves have clear maximum points. The mathematical definition of the function f^*_{res} guarantees that the maximum point will be the point where the return of resilience given an investment will be equal to the efficiency coefficient a. The expected results obtained from these estimated curves can be seen in the table 2.

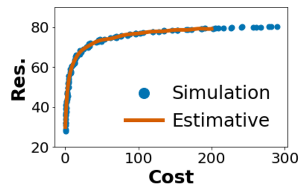

Figure 3. Resilience and cost curve, experimental points in blue and estimated curve in orange.

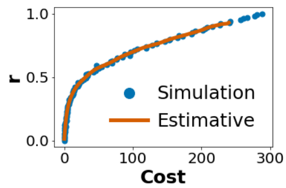

Figure 4. Redundancy level and cost curve, experimental points in blue and estimated curve in orange.

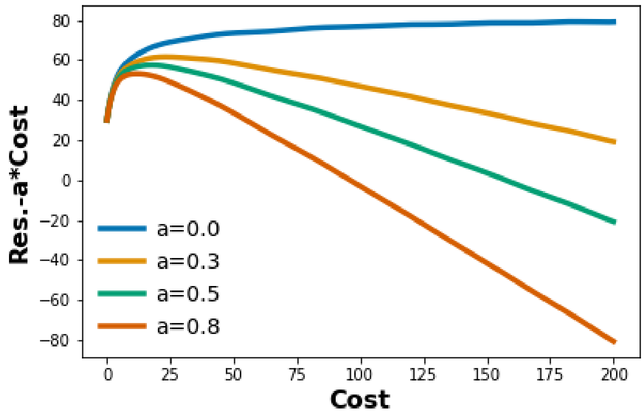

Figure 5. Curves of the equation $f^*_{res} = resilience - a \cdot cost$ by *cost* with different values of a as shown in the legend.

The results of the simulations made to test the precision of the CBGA can be seen on the figure 6. In it the dashed lines indicate the expected value and the dots the values obtained. From it, it is clear that most of the iterations reach values really close to the expected, and the ones that are farthest, are not very far. A better analysis of this can be seen on table 2, where the mean and the mean squared error (MSE) of the results obtained are shown.

In the table 2 for the efficiency level of 0.3 the mean is 0.17 away from the expected value, however, it must be noted that as it can be seen on the figure 5 the curve for $a = 0.3$ has a very smooth inclination, thus being more susceptible to the resilience inference error. Considering it, it becomes clear why for this efficiency level the results were not so good as the other. For the efficiency levels of 0.5 and 0.8, both means were really close to the expected value. For all three efficiency levels the MSE was small, showing the efficiency of the algorithm used.

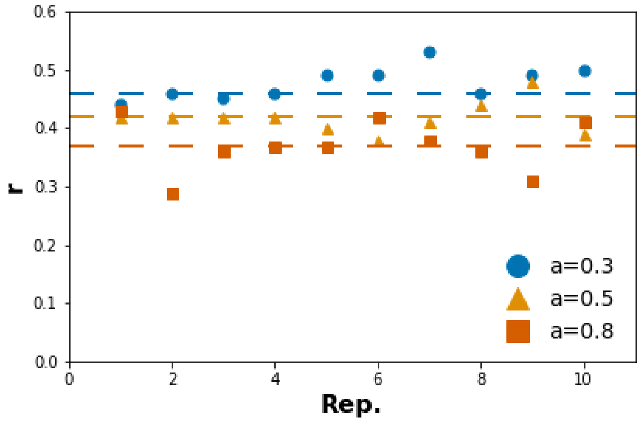

Figure 6. Best cost-benefit redundancy levels obtained for ten connectives runs of the GA. The dashed line are the respective expected values and the values obtained are represented by dots accordingly to the legend of the image.

Table 2. Results Obtained and Expected

Efficiency Level	**Results Obtained Mean**	**Results Obtained MSE**	**Expected Results**
0.3	0.477	0.026	0.46
0.5	0.418	0.026	0.42
0.8	0.370	0.042	0.37

The convergence of the CBGA can be seen in the figure 7. The dots are the means of the best scores for the ten executions of the AG and the shade is the MSE of the best scores. As it can be seen on the figure, the CBGA starts with a small MSE that in just a few iterations get even smaller reaching the ending value. This shows that the algorithm has a good convergence and probably could be run with fewer generations without reducing the quality of the final result.

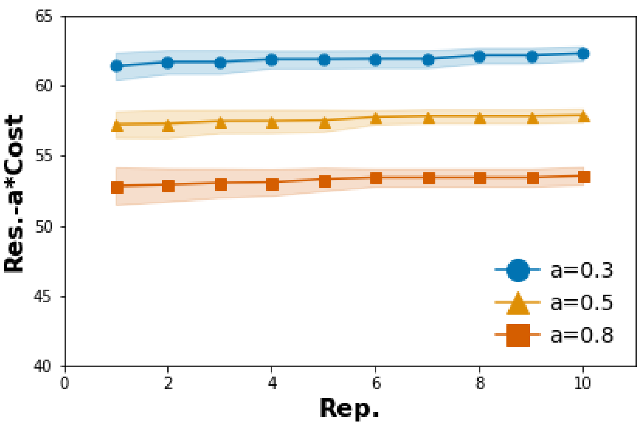

Figure 7. Evolution of the best score over the interaction. It is shown by the mean in points and the MSE in the shade. Each efficiency level is plotted with a different colour and shape as shown in the legend.

The table 3 shows the mean and MSE values of the computational time for the three efficiency levels. From it, it is seen that the mean time is very close for all the levels, being that all of them took almost three hours. Also, the MSE is very small for all simulations.

Table 3. Computational Time of the Simulations

Efficiency Level	**Mean Time**	**MSE**
0.3	02:49:12	00:08:55
0.5	03:00:04	00:12:59
0.8	03:02:16	00:09:26

4. CONCLUSIONS

The results show that there are optimum cost-benefit point when adding resilience to a system with the small-world method for different investment efficiencies. Moreover, it is clear that the Chu-Beasley Genetic Algorithm is a good option to be used to find the optimum cost-benefit point with a given efficiency level, as it has a low varying execution time, presents a good convergence for a solution and good final solution.

Referring specifically to the optimum cost-benefit point, it can be seen from the expected result estimation and from the actual results obtained that for a big band of efficiency levels (from 0.3 to 0.8) the optimum point is between the 0.3 and 0.5 levels of redundancy. This result is consistent with the observations of Ribeiro et al. (2018) and Ribeiro et al. (2019).

Possibles followings works would be the analysis of other methods of adding redundancies in the system (e.g. using a method based on the scale-free topology), consider weighs and limitations of the system and the use of other search methods to make the search for the optimum point.

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