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Identification and Switched Control of an Aeropendulum System ⋆

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Abstract: An aeropendulum system built at the Control Research Laboratory of UNESP-Ilha Solteira is presented, identified and controlled. The system has nonlinear dynamics and unknown parameters. An identification method using MatLab® software is used to obtain local linear models and enable the design of automatic controllers using Linear Matrix Inequalities (LMIs). Therefore, the dynamic of the system can be approximated by local linear models at different points of operation combined by membership functions which are unknown in this case. From the approximated mathematical model of the system and the Lyapunov's stability theory, switched controllers considering decay rate and bounds on norm of the state feedback matrices K_i are designed. The switched control does not require knowledge of the membership functions to compose the control signal and it is an important characteristic that solve the control problem of aeropendulum modelled by Takagi-Sugeno fuzzy models. Practical applications illustrate the efficiency of the methodology used, the results of the implementations are shown and compared.

Keywords: Aeropendulum; Identification; Switched control; Takagi-Sugeno fuzzy systems; Uncertain systems; Nonlinear systems; Norm constraint.

1. INTRODUCTION

Pendulums are classic control problems (Job and Jose, 2015) and are naturally nonlinear systems that can be approximated, in some cases, by linear systems around the respective equilibrium point (Enikov and Campa, 2012). There are some variations of pendulums, such as the aeropendulum, which consists of a propeller attached to the motor shaft to produce a thrust force in order to move a rod to a desired position. A description of the aeropendulum system can be found in Veiga (2016). In Job and Jose (2015), besides the description of an aeropendulum, the proportional-integral-derivative (PID) control and linear–quadratic regulator (LQR) control are presented with comparison in terms of the performance obtained for each controller. An observer-based fuzzy regulator for aeropendulum with output feedback is presented in Farooq et al. (2015). In Enikov and Campa (2012), the aeropendulum is presented as a good alternative for a low-cost hands-on experiment with modeling and feedback linearization. The aeropendulum is the focus of discussion in this work with identification and operation of the system with switched control (Souza et al., 2014), that is, the uncertain nonlinear aeropendulum system is identified and described by Takagi-Sugeno fuzzy models.

The physical parameters of the aeropendulum system used in this work are different from the parameters considered in the mentioned works. Therefore, it is necessary to identify them to carry out the controller design. The aeropendulum identification is based on the modeling presented in Job and Jose (2015) considering that the parameters are unknown. The local linear models are obtained for each operation point (Santim et al., 2012). The method of system identification used in this work is similar to the method used to identify a biomechanical system for lower limbs (Teodoro et al., 2019) describing the system in a convex combination of polytopic uncertainties. The lower limb system can be considered similar to pendulum systems.

The description of nonlinear systems by Takagi-Sugeno fuzzy models (Takagi and Sugeno, 1985) is used in modern control theory and allows the representation of nonlinear systems as a convex combination of local linear models (Takagi and Sugeno, 1985) weighted by a membership function (Taniguchi et al., 2001; Santim et al., 2012). A classic control technique applied to the systems described by Takagi-Sugeno fuzzy models is the Parallel Distributed Compensation (PDC) (Wang et al., 1995). However, the membership function needs to be known or estimated for PDC control. For the aeropendulum system used in this work, the membership function is unknown making it impossible to use fuzzy controllers via PDC. A solution to this problem is the use of controllers that do not require the knowledge of the membership functions. Some of the controllers that do not need this knowledge are the robust single gain (Boyd et al., 1994) and the switched controllers (Souza et al., 2014). An explanation and comparison between both the controllers considering actuator saturation and guaranteed cost function is presented in Silva et al.

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(2020). In this paper only the switched controller is used. The switched controller uses the switching law and it is based on minimizing of the derivative of the Lyapunov function and selects the best feedback gain for each instant from a set of pre-calculated gains K_i (Souza et al., 2014). The practical implementation of the switched controller in an uncertain nonlinear aeropendulum system identified by Takagi-Sugeno fuzzy models is presented and the insertion of the bounds on the norm of the state feedback matrices K_i is also presented and allows to obtain better results. The design of the controllers includes decay rate (Boyd et al., 1994), bounds on norm of the state feedback matrices and Linear Matrix Inequalities (Boyd et al., 1994). The numerical resolution of the LMIs is done with the LMILab solver (Gahinet et al., 1994) with MatLab® and Yalmip interface (Löfberg, 2004).

The following notation will be used in the work $x(t) = x$, $y(t) = y$, $u(t) = u$, $z(t) = z$, $\theta(t) = \theta$, $\dot{\theta}(t) = \dot{\theta}$, $\mathbb{K}_{n_r} = \{1, 2, 3, ..., n_r\}$, $n_r \in \mathbb{N}$, is the set of natural numbers. $\alpha = \alpha(z)$ is the system membership function and it is dependent on the premise variable vector z.

2. PRELIMINARY CONCEPTS

2.1 Switched Control for Uncertain Systems Described by Takagi-Sugeno Fuzzy Models

Takagi-Sugeno fuzzy models are described by IF-THEN rules. Such models relate each inputs and outputs of a nonlinear system locally.

$$\text{Rule } i : \text{IF } z_1 \text{ is } \mathcal{M}_1^i \text{ and } \ldots \text{ and } z_{n_z} \text{ is } \mathcal{M}_{n_z}^i,$$
$$\text{THEN } \begin{cases} \dot{x} = A_i x + B_i u, \\ y = C_i x, \end{cases} \tag{1}$$

where $i \in \mathbb{K}_{n_r}$, $j \in \mathbb{K}_{n_z}$, $\mathcal{M}_j^i$ is the fuzzy set j of the rule i, $A_i \in \mathbb{R}^{n_x \times n_x}$, $B_i \in \mathbb{R}^{n_x \times n_u}$ and $C_i \in \mathbb{R}^{n_y \times n_x}$ are the matrices of the local linear models, $z_1 \cdots z_{n_z}$ are the premise variables and correspond to state variables and uncertain system parameters, $x \in \mathbb{R}^{n_x}$ is the state vector, $u \in \mathbb{R}^{n_u}$ is the input vector and $y \in \mathbb{R}^{n_y}$ is the output vector.

Takagi-Sugeno fuzzy modeling consists of the combination of local linear models by means of normalized weights called membership functions $\alpha_i(z)$, $i \in \mathbb{K}_{n_r}$,

$$\alpha_i(z) = \frac{\omega_i(z)}{\sum_{i=1}^{r} \omega_i(z)}, \tag{2}$$

where $\omega_i(z) = \prod_{j=1}^{n_z} \mathcal{M}_j^i(z_j)$, $\sum_{i=1}^{n_r} \omega_i(z) > 0$, $\omega_i(z) \geq 0$, $\sum_{i=1}^{n_r} \alpha_i(z) = 1$ and $\alpha_i(z) \geq 0$ for all $i \in \mathbb{K}_{n_r}$ and $j \in \mathbb{K}_{n_z}$. $\mathcal{M}_j^i(z_j)$ is the weight of the fuzzy set $\mathcal{M}_j^i$ associated to the premise variable z_j.

The nonlinear system combined by local linear models is

$$\begin{cases} \dot{x} = \displaystyle\sum_{i=1}^{n_r} \alpha_i(z)(A_i x + B_i u) = A(\alpha)x + B(\alpha)u, \\ y = \displaystyle\sum_{i=1}^{n_r} \alpha_i(z) C_i x = C(\alpha)\, x. \end{cases} \tag{3}$$

The settling time of a system can be related to the decay rate (Boyd et al., 1994) and is defined as a real number $\gamma \geq 0$, such that

$$\lim_{t \to \infty} e^{\gamma t} ||x|| = 0, \tag{4}$$

for all trajectory x. Using a quadratic Lyapunov candidate, a lower limit can be established for the decay rate of the feedback system. The condition $\dot{V}(x) \leq -2\gamma V(x)$, for all trajectories x, is equivalent to the specifications of a decay rate greater than or equal to γ (Boyd et al., 1994).

The switched controller (Souza et al., 2014) selects a feedback gain $K_i, i \in \mathbb{K}_{n_r}$, from a set of pre-calculated gains. The selection is made through the switching law and it is based on minimizing of the derivative Lyapunov function that returns the value of index $\sigma(t)$. The switched controller and the switching law are presented

$$u = u_\sigma = -K_\sigma x,$$
$$\sigma(t) = \arg^* \min_{j \in \mathbb{K}_{n_r}} \left\{ x^T \bar{Q}_j x \right\}. \tag{5}$$

where $\bar{Q}_j \in \mathbb{R}^{n_x \times n_x}$, $j \in \mathbb{K}_{n_r}$ is an auxiliary matrix used to obtain the index $\sigma(t) \in \mathbb{K}_{n_r}$ which results in the lowest value of $x^T \bar{Q}_j x$.

The Takagi-Sugeno fuzzy model (3) with the feedback control law (5) is

$$\dot{x} = \sum_{i=1}^{r} \alpha_i(z) \{A_i - B_i K_\sigma\} x. \tag{6}$$

The Theorem 1 presents sufficient conditions for the stability of the nonlinear system (6) with decay rate greater than or equal to γ and it is a particular solution of the result of Souza et al. (2014).

Theorem 1. (Souza et al., 2014) Consider a nonlinear system (6). Assume defined positive symmetric matrix $X \in \mathbb{R}^{n_x \times n_x}$, symmetric matrices Z_i, $Q_i \in \mathbb{R}^{n_x \times n_x}$, matrices $M_j \in \mathbb{R}^{n_u \times n_x}$ and a scalar $\gamma \geq 0$ such that the conditions

$$-B_i M_j - M_j^T B_i^T - Z_i - Q_j < 0, \tag{7}$$
$$X A_i^T + A_i X + Z_i + Q_i + 2\gamma X < 0, \tag{8}$$

are feasible, for all i, $j \in \mathbb{K}_{n_r}$. Then the switched control law (5) makes the origin of the system (6) asymptotically stable with a decay rate greater than or equal to γ. $P = X^{-1}$, $\bar{Q}_j = X^{-1} Q_j X^{-1}$ and the controller gains are $K_j = M_j X^{-1}$.

Proof. Considering the following quadratic Lyapunov function

$$V(x) = x^T P x. \tag{9}$$

A lower limit can be established for the decay rate γ (Boyd et al., 1994) of the system (6), then

$$\begin{aligned} &\dot{V}(x) + 2\gamma V(x) \\ &\quad = x^T (A_i^T P + P A_i + 2\gamma P - K_\sigma^T B_i^T P - P B_i K_\sigma) x. \end{aligned} \tag{10}$$

Consider the existence of symmetric matrices $\bar{Z}_i$, $\bar{Q}_j \in \mathbb{R}^{n_x \times n_x}$ such that (Souza et al., 2014)

$$-(P B_i K_j + K_j^T B_i^T P) \leq \bar{Z}_i + \bar{Q}_j, \tag{11}$$

for all i and $j \in \mathbb{K}_{n_r}$.

Then, multiplying (11) by α_i, pre and post multiplying by x^T and x, adding from $i = 1$ to n_r and replacing j by σ, one obtains

$$
\begin{aligned}
-x^T(PB_iK_\sigma + K_\sigma^T B_i^T P)x & \\
= -\textstyle\sum_{i=1}^{r} \alpha_i x^T(PB_iK_\sigma + K_\sigma^T B_i^T P)x & \quad (12)\\
\leq \textstyle\sum_{i=1}^{r} \alpha_i x^T \bar{Z}_i x + x^T \bar{Q}_\sigma x. &
\end{aligned}
$$

From the control law (5), and knowing that the minimum of a set of real numbers is less than or equal to the convex combination of the elements of this set, note that

$$x^T\bar{Q}_\sigma x = \min_{j\in\mathbb{K}_{n_r}} \{x^T\bar{Q}_j x\} \leq \sum_{i=1}^{n_r} \alpha_i x^T \bar{Q}_i x. \quad (13)$$

From (12) and (13), we have

$$-x^T(PB_iK_\sigma + K_\sigma^T B_i^T P)x \leq \sum_{i=1}^{n_r} \alpha_i x^T(\bar{Z}_i + \bar{Q}_i)x. \quad (14)$$

From (10) and (14), for all $x \neq 0$, one has

$$\dot{V}(x) + 2\gamma V(x) < 0, \quad (15)$$

if

$$A_i^T P + PA_i + \bar{Z}_i + \bar{Q}_i + 2\gamma P < 0. \quad (16)$$

Define $X = P^{-1}$, $Z_i = X\bar{Z}_iX$, $Q_i = X\bar{Q}_iX$ and $M_j = K_jX$. Pre and post multiplying (14) and (16) by X the LMIs (7) and (8) are obtained. The proof is concluded.

In the implementations, one can observe that the gain values can be high and make the practical implementations infeasible. In view of this problem, it is necessary to limit the norm of the state feedback matrices K_i . Therefore, an upper limit to controllers norm is imposed (Assunção et al., 2007) and consequently lower gains are obtained with no occurrence of actuator saturation during implementations. The Theorem 2, proposed in the sequence, imposes a bound on the norm of switched controllers and it is based in Assunção et al. (2007) and Buzetti (2017).

Theorem 2. The specification of bounds on the state feedback matrices K_j of the switched controllers, for all $j \in \mathbb{K}_r$, can be described finding the minimum of η, $\eta > 0$, such that $K_jK_j^T < \eta I$. The optimal value of η can be obtained by the solution of the following optimization problem

$$
\begin{aligned}
&\underset{X,M_j}{\text{minimize}} \quad \eta \\
&\text{subject to} \\
&\quad X > I, && (17)\\
&\quad \begin{bmatrix} \eta I & M_j \\ M_j^T & I \end{bmatrix} > 0, && (18)\\
&\quad \text{(set of LMIs)}
\end{aligned}
$$

where the set of LMIs is equal to (7) and (8).

Proof. Using Schur's complement in (18), we have

$$M_jM_j^T < \eta I. \quad (19)$$

Pre and post multiplying (17) by $\sqrt{X}$ one has

$$\sqrt{X}I\sqrt{X} < \sqrt{X}X\sqrt{X} \Rightarrow X < XX. \quad (20)$$

Pre and post multiplying (17) by K_j and K_j^T, respectively, we have

$$K_jIK_j^T < K_jXK_j^T. \quad (21)$$

From (19)-(21) and $M_j = K_jX$, follows that

$$K_jIK_j^T < K_jXK_j^T < K_jXXK_j^T = M_jM_j^T < \eta I. \quad (22)$$

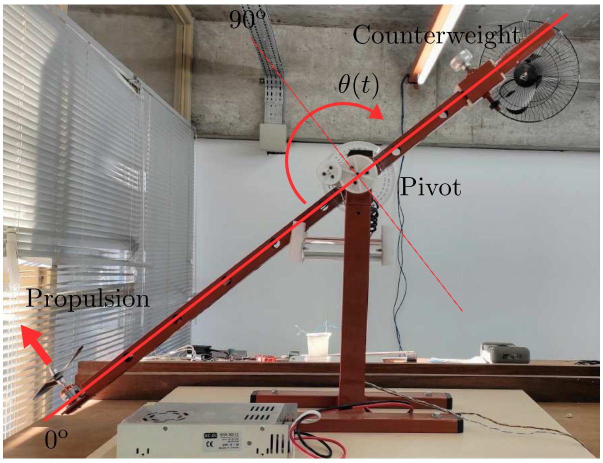
Then $K_jK_j^T < \eta I$. The proof is concluded.

2.2 Description of Aeropendulum System

The aeropendulum system, shown in Figure 1, is composed of a rod with rotational movement of approximately 90° of rotation. The rod is fixed on a pivot and the inert position is considered as reference. At one end, there is a propeller attached to a brushless motor shaft to produce a thrust force in order to control the angular position of the pendulum. At the other end, there is a counterweight with variable position and mass that allows to increase or reduce the produced torque. A linear potentiometer is used as a sensor to provide the angular position and it is fixed at the pivot. This system was built at the Control Research Laboratory of São Paulo State University (UNESP).

The rod of the aeropendulum is made of aluminum and it is 1 meter long. The pivot is positioned 0.6 meters away from the propellant and 0.4 meters from the variable mass. This asymmetry creates a center of mass that does not coincide with the position of the pivot. The potentiometer rotates by means of a gear system whose mechanical ratio provides better sensitivity and increases the range rotation from 90° to 300°. More specification about making of aeropendulum is shown in Veiga (2016).

Figure 1. Aeropendulum system.



The brushless motor is common in unmanned aerial vehicles because it does not use commutator brushes ensuring high durability and power with low weight and volume. The motor is able to rotate only in one direction and it is driven by an Electronic Speed Controller (ESC) driver powered by a 12V DC source attached inside the rod. The ESC converts the 12V DC into an alternating signal to power the motor. The input signal control $u(t)$ corresponds to ESC signal with a 5V pulse-width modulated signal between 1000 μs and 1800 μs with frequency of 500Hz. Finally, a Q2-USB data acquisition device is used for real-time control and communication of the system with Simulink interface from MatLab® and Quarc®.

3. AEROPENDULUM SYSTEM IDENTIFICATION AND CONTROL

3.1 Aeropendulum System Identification

This section presents the method for obtaining local linear models used to design the controllers of the aeropendulum system. The dynamics of the system is nonlinear and, in Job and Jose (2015), a state space model for the aeropendulum is presented. However the system parameters are unknown, and thus have to be identified. The state space model for the aeropendulum is

$$\begin{bmatrix}\dot{\theta}\\ \ddot{\theta}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\dfrac{m\,l\,g\,d\,sin(\theta)}{J\,\theta} & \dfrac{c}{J}\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\end{bmatrix} + \begin{bmatrix}0\\ \dfrac{K_m}{J}\end{bmatrix}u,$$
$$y = [1\ \ 0]\begin{bmatrix}\theta\\ \dot{\theta}\end{bmatrix}, \tag{23}$$

where u is the system input, y is the system output and, finally, θ and $\dot{\theta}$ are the system state variables angular displacement and angular velocity, respectively. Note that in (23) one has a nonlinear dynamic in the state space model structure $\dot{x} = f(x,u)$ with the term $\sin(\theta)$. The system parameters are described in Table 1.

Table 1. Aeropendulum system parameters.

	Description
m	Pendulum mass (kg)
l	Pendulum length (m)
d	Distance from pivot to center of mass (m)
c	Viscous damping Coefficient (Nms/rad)
J	Moment of inertia (Kgm^2)
g	Gravity acceleration (m/s^2)
K_m	Gain of propeller (Nm/μs)

The system is linearised around the operating point and described by the corresponding transfer function

$$\frac{\Theta(s)}{U(s)} = \frac{K_m/J}{s^2 + (c/J)s + (m\,l\,g\,d/J)}. \tag{24}$$

The parameters of the transfer function are unknown and simplified by the coefficients $a_{21} = m\,l\,g\,d/J$, $a_{22} = c/J$ and $b_{21} = K_m/J$. These coefficients are estimated using the MatLab® software for ramp-type entries. Besides the system not being modelled, it has a dead zone, that is, an initial input u_d is required to take the system out of the rest. For each test, different values u_d are observed to take the aeropendulum out of the dead zone. The first step is determining the time and the corresponding value u_d that system comes out of the rest. The input $u(t)$ used in identification is obtained from $u(t) = u_t(t) - u_d$, where $u_t(t)$ is the total input including dead zone. In Figure 2 the entries used in identification can be seen. Figure 3 presents the corresponding output for each test.

The procedure for obtaining the input and output data of the open-loop system are also repeated for other values of $u_t(t)$ respecting the physical limitations of the system. Hereupon the number of identified points is increased. Input values $u_t(t)$ are varied between 1145 μs to 1162 μs. The transfer function coefficients (24) are identified for all curves similar to Figures 2 and 3. Among all the coefficients obtained in the identification, the respective maximum and minimum were selected. The maximum and the minimum found are presented following

$$a_{211} = 2.9138, \quad a_{221} = 1.9776, \quad b_{211} = 0.0481,$$
$$a_{212} = 1.6243, \quad a_{222} = 0.8289, \quad b_{212} = 0.1076. \tag{25}$$

All possible combinations between maximum and minimum coefficients (25) are combined. The transfer functions for each combination are obtained and transformed into

Figure 2. Inputs $u(t)$ used to identify the aeropendulum for $u_t(t) = 1160\,\mu s$ in open-loop system.

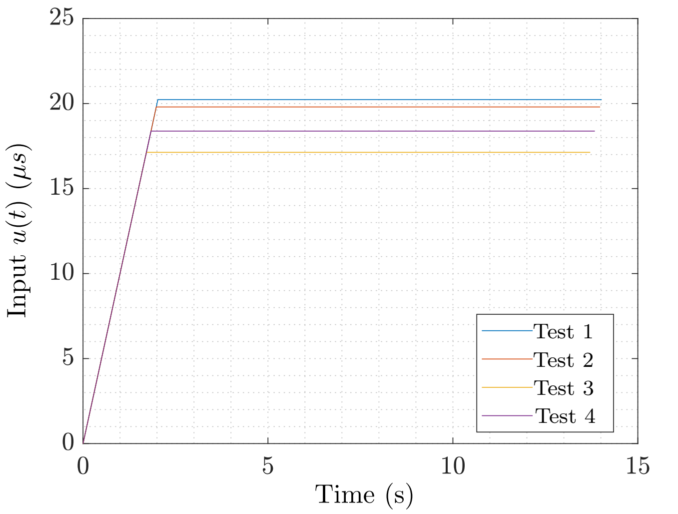
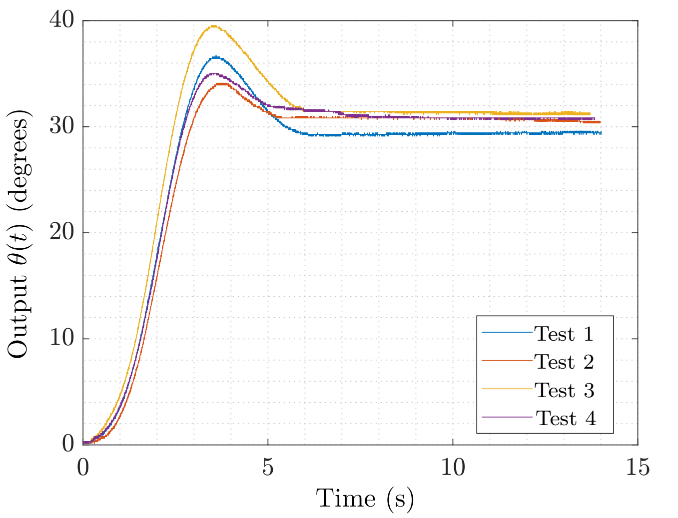


Figure 3. Outputs $\theta(t)$ of open-loop system used to identify the aeropendulum.



a state space system through realization in controllable canonical form. In this way, a Takagi-Sugeno fuzzy model is obtained with $n_r = 2^3$ rules considering the number of possible combinations between maximum and minimum values. The vertices A_i and B_i, $i \in \mathbb{K}_{n_r}$, for each local model of the Takagi-Sugeno fuzzy system (1) are

$$\begin{aligned}
A_1 = A_2 &\begin{bmatrix}0 & 1\\ -2.9138 & -1.9776\end{bmatrix},\\
A_3 = A_4 &\begin{bmatrix}0 & 1\\ -2.9138 & -0.1076\end{bmatrix},\\
A_5 = A_6 &\begin{bmatrix}0 & 1\\ -1.6243 & -1.9776\end{bmatrix},\\
A_7 = A_8 &\begin{bmatrix}0 & 1\\ -1.6243 & -0.1076\end{bmatrix},\\
B_1 = B_3 = B_5 = B_7 &= \begin{bmatrix}0\\ 0.0481\end{bmatrix},\\
B_2 = B_4 = B_6 = B_8 &= \begin{bmatrix}0\\ 0.1076\end{bmatrix}.
\end{aligned} \tag{26}$$

3.2 Switched Control Implementation in Aeropendulum System

The switched control, shown in Subsection 2.1, is applied to the nonlinear system aeropendulum described by Takagi-Sugeno fuzzy model.

Remark 1. A reference tracking control is desired for the aeropendulum control. The equilibrium states for the system (23) is $x_0 = \begin{bmatrix}\theta_d & 0\end{bmatrix}$, where θ_d is θ desired. For the equilibrium states to be the origin of the system, a constant is subtracted in θ in order to track a reference angle

$$\begin{bmatrix}\bar{\theta} \\ \dot{\bar{\theta}}\end{bmatrix} = \begin{bmatrix}\theta - \theta_d \\ \dot{\theta}\end{bmatrix}. \tag{27}$$

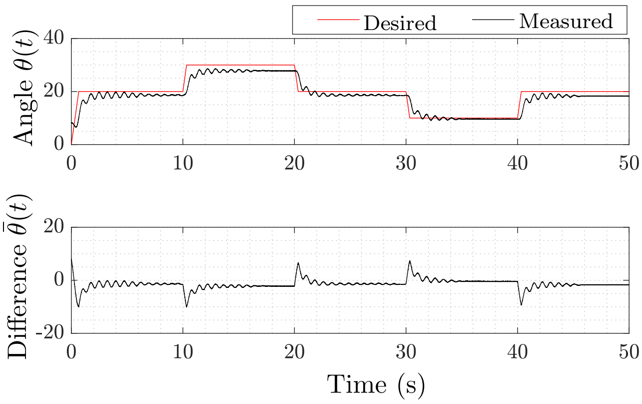
The first implementation is related to a controller design without norm constraint, that is, the Theorem 1 is used with decay rate $\gamma = 0.5$. In the second implementation, the norm constraint is included to controller design. The Theorem 2 is used and a parameter related to the bound on norm of the state feedback matrices K_i is minimized with decay rate $\gamma = 2$. For both implementations, the local models (26) is used. After that, it is possible to analyse and compare the results for each implementation.

The solution using Theorem 1 with $\gamma = 0.5$ is

$$\begin{aligned} K_1 &= [667.5933 \ 374.5367]\,, \\ K_2 &= [453.0564 \ 255,0055]\,, \\ K_3 &= [672.9152 \ 371.3710]\,, \\ K_4 &= [519.7875 \ 288.6839]\,, \\ K_5 &= [708.6723 \ 405,6153]\,, \\ K_6 &= [394.0755 \ 226.7174]\,, \\ K_7 &= [646.8805 \ 360.1018]\,, \\ K_8 &= [479.6544 \ 267.9240]\,. \end{aligned} \tag{28}$$

In Figures 4 and 5, the implementation of the switched control with gains (28) applied to the aeropendulum system is presented. The Figure 4 shows angular position $\theta(t)$ and angular difference $\bar{\theta}(t)$ between measured and desired values. Figure 5 shows the control signal $u_t(t)$ and the switching index $\sigma(t)$.

Figure 4. Angle $\theta(t)$ and angular difference $\bar{\theta}(t)$ for switched control desired with Theorem 1.

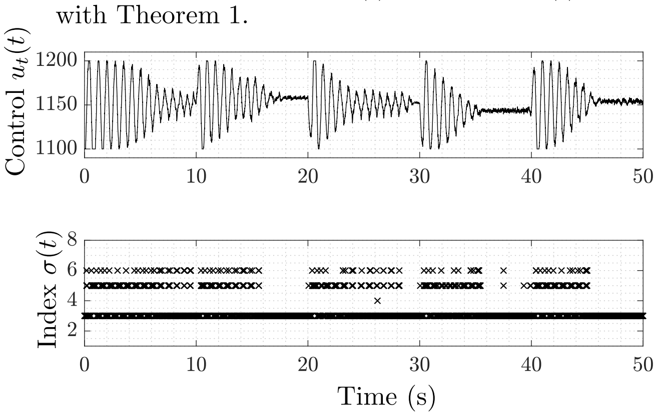


Analysing Figures 4 and 5, one can observe a small steady-state error, saturation of the control signal and a high value of settling time. In order to improve results and eliminate the actuator saturation, a norm constraint is now considered in the controller design. For that, the Theorem 2 is used in the next controller design.

The solution using Theorem 2 with a suitable decay rate $\gamma = 2$ is

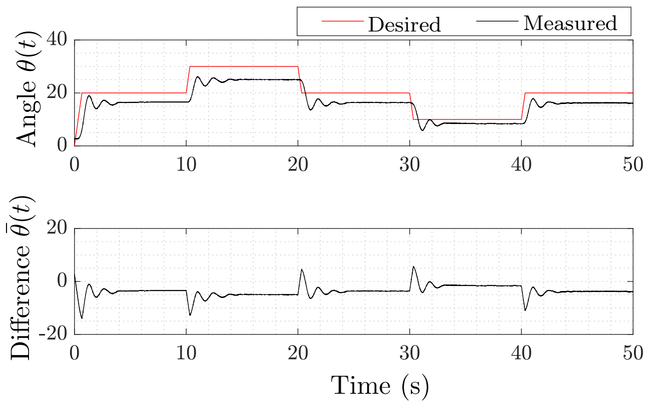
$$\begin{aligned} K_1 &= [190.9680 \ 72.8494]\,, \\ K_2 &= [142.6710 \ 62.3343]\,, \\ K_3 &= [241.7246 \ 82.8745]\,, \\ K_4 &= [186.6029 \ 69.7999]\,, \\ K_5 &= [154.8685 \ 66.5432]\,, \\ K_6 &= [116.5921 \ 59.0799]\,, \\ K_7 &= [208.8359 \ 75.7092]\,, \\ K_8 &= [161.1946 \ 64.8745]\,. \end{aligned} \tag{29}$$

Figure 5. Switched control $u_t(t)$ and index $\sigma(t)$ desired with Theorem 1.



In Figures 6 and 7, the implementation of the switched control with norm constraint is applied to aeropendulum system. The gains (29) are used in the implementation.

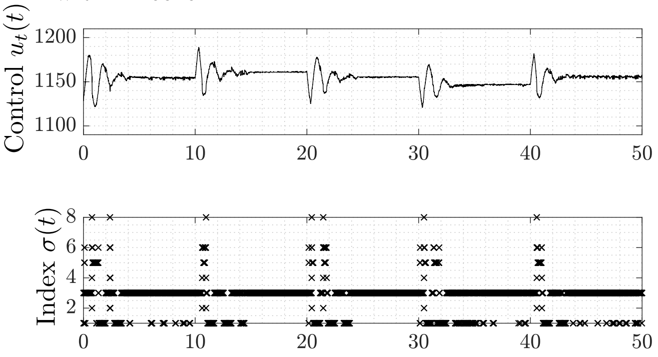
Figure 6. Angle $\theta(t)$ and angular difference $\bar{\theta}(t)$ for switched control desired with Theorem 2.



From the Figures 6 and 7, one can observe that the steady-state error is slightly bigger than the Figure 4, however the control signal is smoother and the actuator saturation is eliminated. The inclusion of the norm constraint decreases the settling time because it makes possible to use higher decay rates. The controller effort without norm constraint is greater than controller effort with norm constraint as it can be seen from Figures 5 and 7.

Remark 2. Just a referencial changing is used in order to obtain the desired values in implementations. The steady-state error observed in implementaions can be explained by the fact of the identification method does not consider the system as a reference tracking problem. The further away the desired position is from the origin the greater is the steady-state error.

Figure 7. Switched control $u_t(t)$ and index $\sigma(t)$ desired with Theorem 2.



4. CONCLUSIONS

The aeropendulum system built at the Engineering College of UNESP Ilha Solteira presents a good operation and the identification obtained allows to control the aeropendulum system with switched control becoming it stable. The optimization of the bound on norm of the state feedback matrices K_i allows to obtain better results with grater decay rate. The transient response is better and the elimination of actuator saturation is observed due to the decrease of the bound on norm of the state feedback matrices. As future works to further improve the results, other restrictions in the design of switched controller can be considered as actuator saturation and performance indices, the system can be modelled as a reference tracking problem and integrators can be also added to the controller design in order to mitigate the observed steady-state error.

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