

An Extended Kalman Filter implementation for estimating the mass of passengers vehicles

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Abstract: The vehicle mass is an important information to guarantee drivability, performance feel, fuel economy and safety. In this sense, the application of strategies to reach these goals depends on how accurate the vehicle operating information is, requiring a precise and robust algorithm. This paper proposes an offline mass estimation method of passenger cars, using the Extended Kalman Filter. The method combines the filter approach whereby only the speed is provided as a measurement with a parameter estimation to adjust possible modelling errors and to calculate a proposed engine torque. Tests in city routes were done in controlled dynamics to guarantee the effectiveness of the estimation and the simulation results presented errors below 5%.

Keywords: Mass Estimation, Extended Kalman Filter, Vehicle Operating Information, Nonlinear Systems

1. INTRODUCTION

Fuel economy, drivability, and safety of passengers are prioritized areas for today's automotive manufacturers (Holm, 2011). In this respect, it is well-known that the dynamic behavior of a vehicle is affected by its mass and the road slope (Lundin and Olsson, 2012). Thus, many proposed fuel-saving techniques are dependent on measurements of road conditions and variation of vehicle mass. However, using extra sensors to obtain these quantities increases the financial cost of the solution. In this way, several studies propose the use of software and microprocessor technology to estimate road and vehicle mass variations by using data that is already available in the vehicle.

Zarringhalam *et al.*, (2012) present an approach to estimate mass in passenger cars based on suspension dynamics and compare four estimation techniques, namely: (i) Recursive Least Squares (RLS), (ii) Recursive Kalman filter (RKF), (iii) gradient parameter estimation and (iv) Extended Kalman filter (EKF). They highlight the advantages of using EKF instead of others, like consistent performance and applicability to nonlinear systems with noisy measurements. Bae, Ryu and Gerdes, (2001) compare two methods of estimating road grade of ground vehicles using GPS signal. The results with the engine torque measurement are used in a longitudinal balance to produce a recursive estimation of vehicle mass and other parameters. Winstead and Kolmanovsky, (2005) propose the estimation of road grade and mass in a mid-size vehicle using the EKF powered by a engine torque uncertainty model, in addition to controlling the vehicle speed through a model predictive controller in order to enhance parameter identification. Paulsson, (2016) estimates the mass and road grade applying a method based on RLS associated with vehicle longitudinal dynamics, whose the engine torque input signal

come from vehicle's CAN-bus. The author works with a mid-size vehicle and also show the robustness of the RLS method. Fathy, Kang and Stein, (2008) work with the same method in a instrumented SUV, but a fuzzy supervisor is added to the estimator to determine whether the vehicle's data is under appropriate conditions to the estimator.

Regarding truck longitudinal models, Andreas Eriksson, (2009) works with heavy vehicles and uses an adaptive Kalman filter to correct both mass and covariance matrix that is set in the filter, the engine torque is known by the truck's electronic engine controller. Furthermore, he needs to set several thresholds to avoid erroneous estimates. Holm, (2011) works with the EKF and proposes a method that measures the road grade and estimates the mass of trucks on road. The results show that the filter is fast and sufficiently accurate. Lundin and Olsson, (2012) develop two configurations of truck inertial parameters estimation, considering the engine torque measured: (i) one estimates both mass and road grade, while (ii) the other only estimates the mass and uses the road grade provided by a sensor in a model. Although both have shown satisfactory results, the latter is more accurate, robust, and quicker than the former. A hybrid method using a weighted trade-off of RLS and the EKF is presented by Sun *et al.*, (2016), the goal is to increase the results compared with situations where the algorithms are used alone. The estimation demonstrated faster convergence and lower error rate on city bus route.

By analyzing the proposed methods described above, it is possible to highlight some observations.:

- The RLS and EKF estimation methods are the most frequent approaches and present advantages over the others;

- The engine torque can be obtained from CAN-bus as in Paulsson, (2016) or by a model as in Winstead and Kolmanovsky, (2005);
- The vehicles used in most studies are heavy vehicles.

From this scenario, this paper describes the design and implementation of EKF to offline estimation of mass and road grade in a compact passenger car, differing from other approaches in this point and in the fact that only the speed is provided as measurement by the CAN-bus, which means that no additional sensor is used. Furthermore, the engine torque is estimated by the author's method. Once it is an initial feasibility study of this methodology applied to compact cars, it was chosen an offline approach applied to real operating data.

2. MODELLING

2.1 Vehicle Dynamic Model

The car dynamics can be modeled by Newton's second law to rotational and translational systems as in Holm, (2011):

$$\sum F = m\dot{v} \quad (1)$$

$$\sum T = I\dot{\omega} \quad (2)$$

where, $\sum F$ is the sum of all translational forces acting in the car, $\sum T$ is the sum of all torques that acts in the body, m is its mass, \dot{v} is the translational acceleration, I is the rotational moment of inertia, and $\dot{\omega}$ is the rotational acceleration.

The Fig. 1 is the body diagram of the car, thus, the equation that defines the translational forces can be represented by:

$$F_{traction} - F_{air} - F_{gravity} - F_{roll} = m\dot{v} \quad (3)$$

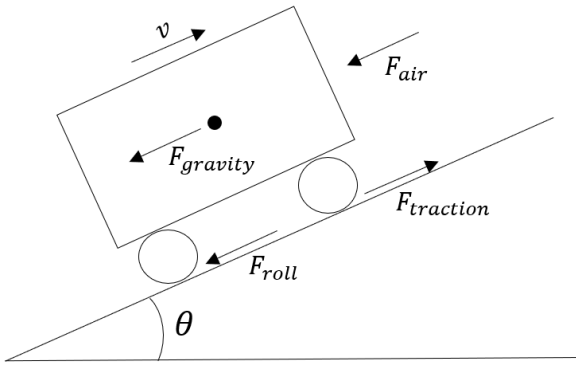


Fig.1. Forces diagram of a vehicle going uphill

The traction force ($F_{traction}$) is the vehicle propulsive force, originated of the combustion within the engine and then transmitted to the wheels and it is defined by Holm (2011) as:

$$F_{traction} = \frac{T_e i_{tf} \eta}{r} - m_r \dot{v}, \quad (4)$$

where, T_e is the engine torque, r is the wheel radius, i_{tf} is the gear ratio of the transmission multiplied by the ratio of the final

drive, η is the mechanical losses and $m_r \dot{v}$ is the rotational inertia of the driveline.

The air resistance (F_{air}) depends on the squared vehicle speed, v , the air density, ρ_{air} , the vehicle aerodynamic drag coefficient C_d , and the car frontal area A_f :

$$F_{air} = \frac{1}{2} \rho_{air} C_d A_f v^2. \quad (5)$$

Another force that acts in a vehicle is the longitudinal component of gravitational force ($F_{gravity}$). It can accelerate if the car is going downhill or decelerate when the car is uphill:

$$F_{gravity} = m g \sin(\theta), \quad (6)$$

where g is the acceleration of gravity and θ is the inclination angle.

Lastly, there is the rolling resistance (F_{roll}), which is an effect of the friction and deformation of the wheels. It depends on the coefficient of rolling resistance (f_r), the vehicle mass (m), the gravitational acceleration (g) and the road slope (θ):

$$F_{roll} = f_r m g \cos(\theta), \quad (7)$$

additionally, f_r is assumed to be independent of speed (Lundin and Olsson, 2012).

Combining the equations (3) to (7), the result is:

$$(m + m_r) \dot{v} = \frac{T_e i_{tf} \eta}{r} - \frac{1}{2} \rho_{air} C_d A_f v^2 - m g (\sin(\theta) + f_r \cos(\theta)) \quad (8)$$

It is necessary to manipulate the equation to separate the variables of interest that are the inclination and the vehicle mass. This can be done by the method available in (Holm, 2011):

$$\sin(\theta) + \cos(\theta) \tan(y) = \frac{\sin(\theta+y)}{\cos(y)} \quad (9)$$

Considering $y = \text{atan}(f_r)$, it is possible to obtain (10) from (8).

$$(m + m_r) \dot{v} = \frac{T_e i_{tf} \eta}{r} - \frac{1}{2} \rho_{air} C_d A_f v^2 - m g \left(\frac{\sin(\theta + \text{atan}(f_r))}{\cos(\text{atan}(f_r))} \right) \quad (10)$$

To simplify the equation, some substitutions are made, namely: $a_1 = \frac{T_e i_{tf} \eta}{r}$, $a_2 = \rho_{air} C_d A_f$, $a_3 = \frac{g}{\cos(\text{atan}(f_r))}$, $\beta = \text{atan}(f_r)$, $\varphi_1 = \frac{1}{m}$, and $\varphi_2 = \sin(\theta + \beta)$.

Therefore, the equation that describes the dynamic behaviour of velocity in a passenger vehicle is:

$$\dot{v} = \frac{\varphi_1 (T_e a_1 - \frac{1}{2} a_2 v^2) - a_3 \varphi_2}{1 + \varphi_1 m_r} \quad (11)$$

The system states are v , φ_1 , φ_2 and the model input is the engine torque T_e . The values of the model parameters used are:

$i_{tf} = 6.39$, $\eta = 0.90$, $r = 0.29$ m, $\rho_{air} = 1.22$ kg/m³, $C_d = 0.35$, $A_f = 2.15$ m², $f_r = 0.01$ and $m_r = 71$ kg

2.2 State Space Model

Once the final system equation is defined (11), the state space model can be established (12). The vehicle mass is considered constant, this means that its variation is considered equal to zero. The road grade is supposed to be constant in a relatively short time interval as in Sun, Y. *et al.* (2016), for this reason the derivative is also equal to zero.

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{v} \\ \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{bmatrix} = \begin{bmatrix} \frac{\varphi_1(T_e a_1 - \frac{1}{2} a_2 v^2) - a_3 \varphi_2}{1 + \varphi_1 m_r} \\ 0 \\ 0 \end{bmatrix} \quad (12)$$

However, to implement the filter, it is adequate to use the model in discrete form. The discretization is carried out using the first-order Euler method and the discrete state space model is given by (13).

$$\mathbf{x}_{k+1} = \begin{bmatrix} v_{k+1} \\ \varphi_{1,k+1} \\ \varphi_{2,k+1} \end{bmatrix} = \begin{bmatrix} v_k + T_s \left(\frac{\varphi_{1,k}(u_k a_1 - \frac{1}{2} a_2 v_k^2) - a_3 \varphi_{2,k}}{1 + \varphi_{1,k} m_r} \right) \\ \varphi_{1,k} \\ \varphi_{2,k} \end{bmatrix}, \quad (13)$$

where T_s is the sample time and the input signal is the engine torque ($u_k = T_e$).

As the speed is the unique measurement used, the measurement equation is:

$$\mathbf{z}_k = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_k \\ \varphi_{1,k} \\ \varphi_{2,k} \end{bmatrix} + \mathbf{e}_k, \quad (14)$$

where \mathbf{e}_k is the measurement noise.

2.3 Model adjustment

In order to correct possible modelling errors in (11), a parameter adjustment was suggested using real speed data, a reference inclination and the knowledge of true mass. So, it was proposed some correction factors to the model parameters a_1 , a_2 and a_3 as follows:

$$\hat{a}_1 = a_1 + p_1 + p_2 v_{k-1} \quad (15)$$

$$\hat{a}_2 = a_2 + p_3 + p_4 v_{k-1} \quad (16)$$

$$\hat{a}_3 = a_3 + p_5 + p_6 v_{k-1} \quad (17)$$

p_1, p_2, p_3, p_4, p_5 and p_6 are the parameters estimated by Least Square Method. It is important to note that the new parameters are now dependent of the measured speed in the previous instant v_{k-1} . This structure was proposed to generate a better use of the dynamics of the unique measurement available.

Another factor was also added (p_7) in the model adjustment. It corresponds to a proportional gain that permits the torque to be estimated by the relation:

$$T_e = p_7 v_{k-1} \quad (18)$$

3. EXTENDED KALMAN FILTER

The Kalman filter uses a set of mathematical equations to estimate the process states of a linear model in a way that minimizes the mean of the squared error (Welch and Bishop, 2006). It solves the problem of estimating the states of a process defined by the linear state space form:

$$\mathbf{x}_k = \mathbf{A}\mathbf{x}_{k-1} + \mathbf{B}u_{k-1} + \mathbf{w}_{k-1}, \quad (19)$$

and measurement equation given by:

$$\mathbf{z}_k = \mathbf{H}\mathbf{x}_k + \mathbf{v}_k, \quad (20)$$

where \mathbf{w}_k and \mathbf{v}_k are the process and measurement noise vectors whose covariance matrices are \mathbf{Q} and \mathbf{R} , respectively. The \mathbf{A} matrix relates the state at the previous step with the states in the current step k . The matrix \mathbf{B} relates the control input \mathbf{u} to the state \mathbf{x} . The matrix \mathbf{H} relates the state to the measurement \mathbf{z}_k .

A common modification of the Kalman filter which linearizes about the current mean and covariance is referred to as an extended Kalman filter or EKF (Welch and Bishop, 2006). The EKF method first linearizes the system around the current state estimate, with a first order Taylor series expansion, then the standard Kalman filter equations are applied (Holm, 2011).

The Kalman filter is divided in two steps, namely: (i) prediction, and (ii) correction. The former predicts the state and error covariance one step of time ahead (\mathbf{x}_k), using the state space model and a priori estimate (\mathbf{x}_{k-1}). The equations that represent this step are:

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}, \mathbf{w}_{k-1}) \quad (21)$$

$$\mathbf{P}_k^- = \mathbf{A}_k \mathbf{P}_{k-1} \mathbf{A}_k^T + \mathbf{Q}_{k-1} \quad (22)$$

(21) is the generic representation of the non-linear state space model, \mathbf{P} is the state error covariance, \mathbf{Q} defines how much the process can be trusted and \mathbf{A} is the process Jacobian matrix that linearizes the model in each iteration and is calculated by:

$$\mathbf{A} = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \dots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \dots & \frac{\partial f_m}{\partial x_n} \end{pmatrix} | \mathbf{x} = \hat{\mathbf{x}}_k^-, u = u_k \quad (23)$$

The correction performs a feedback weighted by a Kalman gain (\mathbf{K}), which incorporates the measurement into the a priori estimate, providing an improved state estimate ($\hat{\mathbf{x}}_k$). It is represented by the equations:

$$\mathbf{K}_k = \mathbf{P}_k^- \mathbf{H}^T (\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}^T + \mathbf{R}_k)^{-1} \quad (24)$$

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + \mathbf{K}_k (\mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_k^-) \quad (25)$$

$$\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_k^- \quad (26)$$

\mathbf{I} is the identity matrix and \mathbf{R} is the matrix that represents the confidence of the measurement.

A representation of the filter flow can be seen in Fig. 2.



Fig.2. The Kalman filter predictor-corrector cycle.

It is important to emphasize that to start the operation of the filter, initial guesses on state and state error covariance (\mathbf{P}) need to be provided together with \mathbf{Q} and \mathbf{R} matrices.

4. ESTIMATION OF VEHICLE MASS WITH EKF

Fig. 3 describes the steps of the EKF application to estimate vehicle mass. In this structure, only the speed is measured and it is provided by CAN-bus. Therefore, vehicle torque, mass and road slope are simultaneously estimated. The torque estimation was performed as an adjustment of the vehicle speed based in Lundin and Olsson (2012) and using the expression (18).

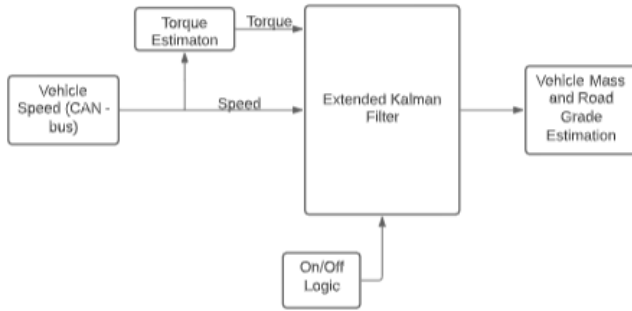


Fig.3. Flowchart of mass estimation

The model equations that compose the filter are those in the discrete state space form (13) and (14). The filter receives the noise covariance matrices, the initial conditions and performs the estimation of the state variables (speed, mass, and road grade) calculating, in each step, the Kalman gain (\mathbf{K}), the state error covariance (\mathbf{P}) and the Jacobian matrix (\mathbf{A}) applied to the model.

Once that the filter estimation is completed, the calculation of the average mass is based in Holm (2011) and is represented in Fig. 4. It is done with respect to the last data from estimation, evaluating the time necessary for the filter to complete the convergence entirely. Once the data is selected, they are separated in groups of the same size. For each group, the standard deviation is calculated and, afterwards, an average standard deviation is obtained using the values calculated for

each group and defined as a limit. The masses of the groups with standard deviation below this limit are, then, used to calculate the mean mass of the estimation. This method was chosen because it is effective in eliminating outliers and selecting the data from convergence.

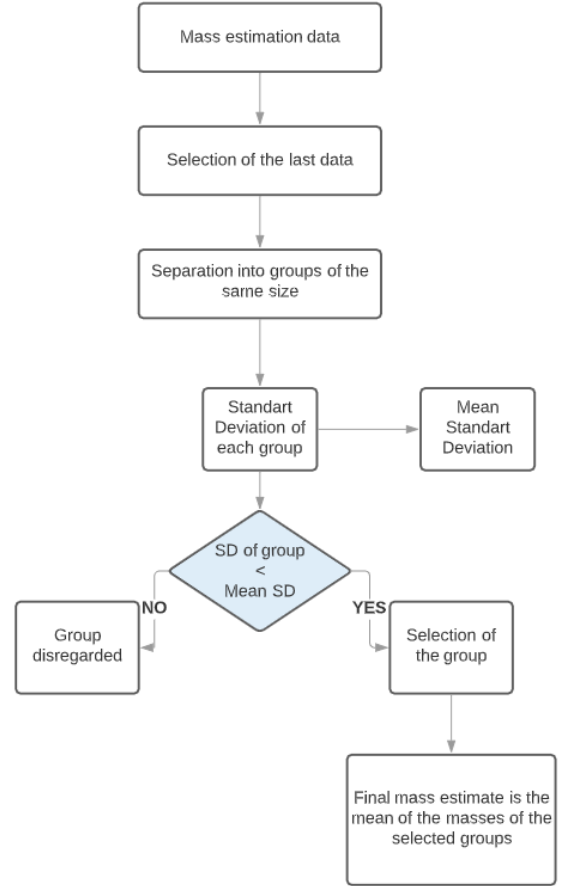


Fig.4. The process of mean mass calculation

Eriksson (2009) shows that during brake and low speeds the estimator does not perform well. In this way, it is possible to add an on/off logic that selects the adequate data to realize the estimation, turning this off and repeating the last estimation in case of non-adequacy. However, if the data is kept avoiding these disturbances, the on/off logics can be dispensed and the data can be used in the estimation.

5. RESULTS AND DISCUSSION

5.1 Experimental Data

Three experiments were conducted 3 times in a compact passenger vehicle, with only the driver. The mass was kept constant at 1092 kg and the experiments were operated under similar circumstances during around 20 minutes. Since there are some limitations in the data that can be provided to the filter, the road selected to do the tests was characterized by few curves and low slopes and also without a traffic jam, to keep the car in movement. The measurement sample time was defined as 7 seconds, once this proved to be satisfactory to represent the car dynamics.

5.2 Model Adjustment

The parameter adjustment of the model was done using the data from Test 01 as a reference and the knowledge of mass and road grade. The values obtained with this test were also used in the others. As can be seen in figures 7, 8 and 9, the model's behaviour is superimposed on the measurement, proving that its adjustment was successful.

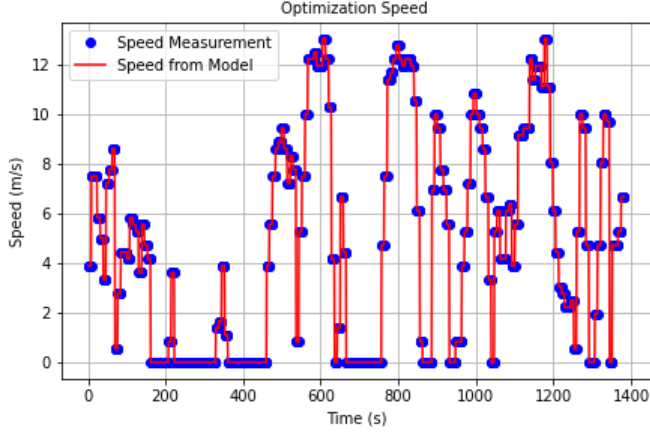


Fig.7. Comparison of measured speed and model speed after optimization in Test 01.

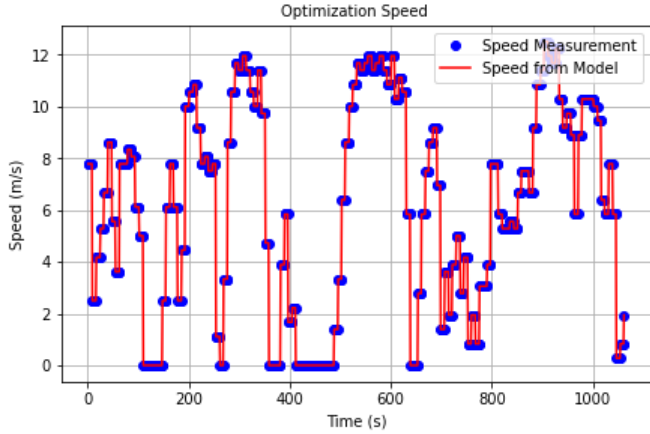


Fig.8. Comparison of measured speed and model speed after optimization in Test 02.

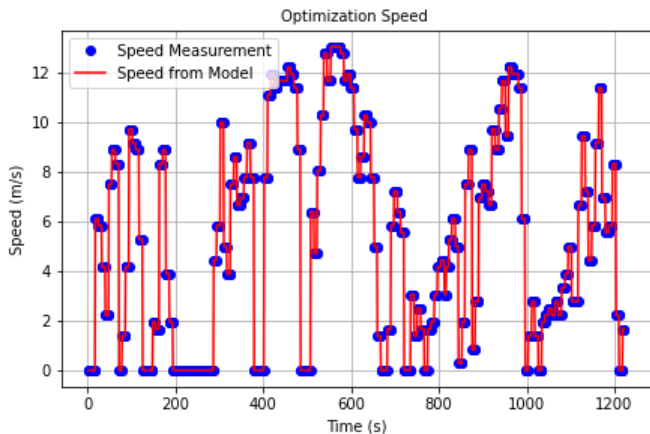


Fig.9. Comparison of measured speed and model speed after optimization in Test 03.

5.3 Mass Estimation

Once that the model is adjusted, the states can be estimated. The initial conditions of the states provided to the EKF are

$$X_0 = [v_0, \frac{1}{1500}, 0.01],$$

so the speed is considered equal to the first measurement (v_0) and the mass initial condition is 1500kg, because it is the maximum mass of the compact vehicle used and it is important to set the mass in a value far from the real to verify the convergence of estimation. To the inclination, once that the state is $\varphi_2 = \sin(\theta + \beta)$, the initial condition of θ is considered 0.005° . The diagonal of the process covariance matrix used is, then

$$Q = [0.1, 10^{-10}, 0.005],$$

the measurement covariance $R = 10^{-4}$ and the state error covariance $P = Q$. The mass estimation for each test together with the respective instant estimation errors can be seen in figures 10 to 15.

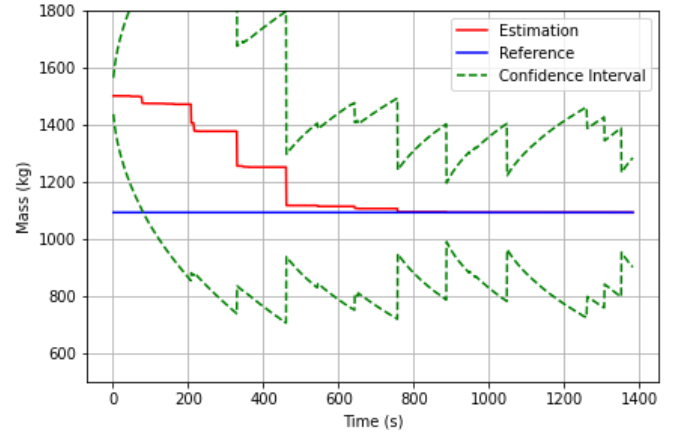


Fig.10. Mass estimation of Test 01

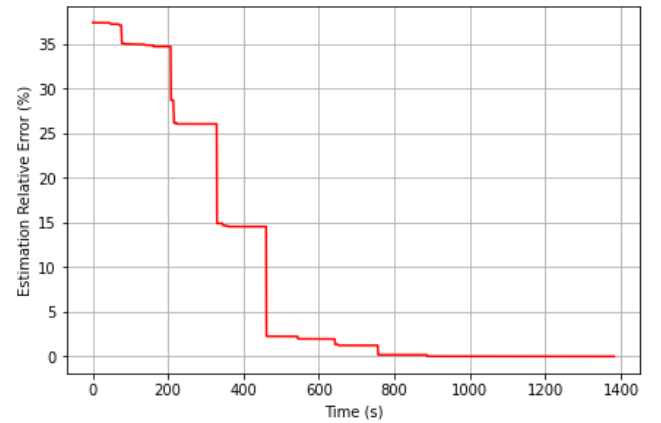


Fig.11. Mass estimation error of Test 0

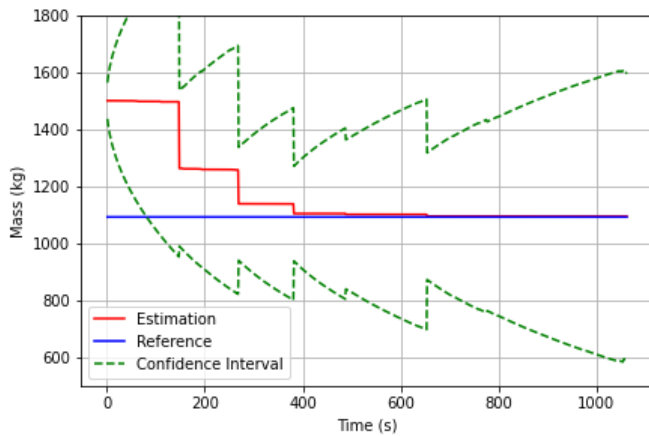


Fig.12 Mass estimation of Test 02

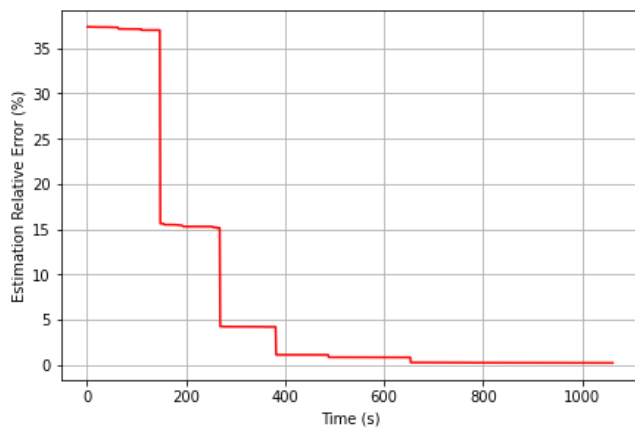


Fig.13 Mass estimation error of Test 02

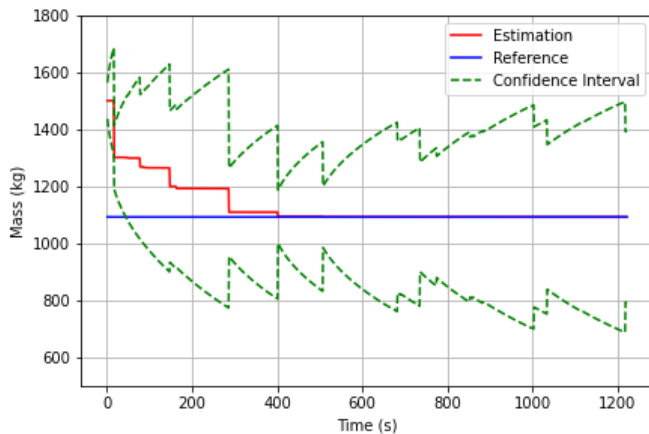


Fig.14. Mass estimation of Test 03

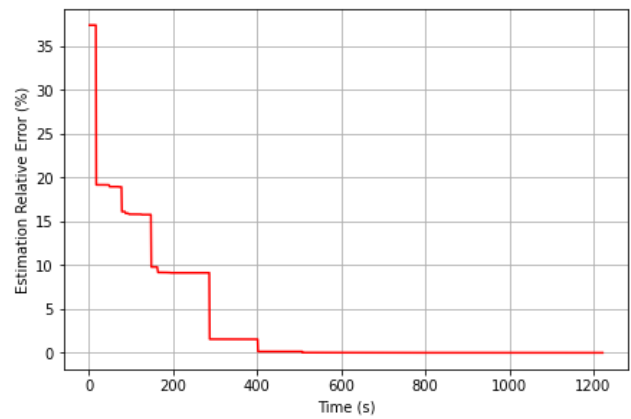


Fig.15. Mass estimation error of Test 03

The results show that the mass estimation, and consequently the relative error, converges in a short time interval of approximately 5 min towards the expected mass of 1092, what represents a quarter of the time of the test. It is possible to highlight the fact that the estimation converges without any disturbance in the path and, once it approaches the true value, it stabilizes so that it does not reach smaller values. As can be seen in Table 1 the errors are smaller to Test 01 and Test 03, but still does not assume significant values in Test 02.

Table 1. Mass estimation results to each test

Experimental Data	Mean Estimated Mass	Relative Mass Error
Test 01	1092,07 kg	0,0069%
Test 02	1095,58 kg	0,33%
Test 03	1092, 02 kg	0,0025%

Another relevant indicator is the confidence interval, represented by the green line in the Figures 10, 12 and 14 , that is associated with the uncertainty of the estimation and represents a range of probable estimates. As can be seen on the charts, this interval is bigger in the beginning, that is when the filter works more to adjust the estimation, and smaller in the end, indicating convergence. Some peaks occur throughout the estimation, due to questions of the measurement dynamics and it decreases temporarily the confidence. In the last 200 data of simulation, that is when the filter estimation is considered convergent, the confidence interval is kept below ± 200 kg to all tests, as can be seen in Table 2.

Table 2. Mass estimation results to each test

Experimental Data	Confidence Interval to last 200 data
Test 01	± 100 kg

Test 02	$\pm 200 \text{ kg}$
Test 03	$\pm 200 \text{ kg}$

6. CONCLUSIONS

This paper proposes a method to estimate the mass of compact passenger vehicles by an EKF with only the speed provided as measurement, compensating the modelling errors by a parameter adjustment and an engine torque estimation, differing from the solutions that already exist in the literature that are focused on heavy vehicles and instant measurement of engine torque from CAN-bus. Such proposal uses a grey box model that follows Newton's second law and is evaluated on a data set collected from a vehicle under controlled conditions to guarantee the filter effectiveness.

As can be seen in the results, the method proposed is satisfactory to the presented proposal and it is necessary only 5 minutes of data to obtain mass estimation errors below 1% with a confidence interval around 20% of the real value. This means that the car algorithms to reach the drivability, performance, safety and comfort can be updated over this time interval, with a satisfactory information of mass. Furthermore, the model adjustment worked successfully in avoiding modelling errors and providing the model input, what helped the filter in their estimations. Thus, the EKF proved to be sufficient and accurate enough to solve this problem.

However, since the mass estimation is an important source of information to increase vehicle performance and the control actions of the vehicle systems need to be done in real time, it would be very advantageous, in future works, to develop some other analyses. Develop an embedded system and apply this tool online is essential to verify the effectiveness in real situations. Furthermore, work with another vehicle masses and under different driving conditions to verify limits of the method and its effectiveness is an important step. Another one is to use different structures of model to improve the results and the confidence interval. Lastly, study the data uncertainty is necessary because the model may have been adjusted to process noise.

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