DOI: 10.48011/asba.v2i1.1578

$\mathcal{H}_\infty$ Robust State Feedback Sampled-Data Control of Interval Linear Systems ⋆

Rafael M. Alves * **André R. Fioravanti** ** **Matheus Souza** *

* *School of Electrical and Computer Engineering, University of Campinas, Campinas, SP, Brazil (e-mails: rafaelalves@dca.fee.unicamp.br; msouza@fee.unicamp.br).*
** *School of Mechanical Engineering, University of Campinas, Campinas, SP, Brazil (e-mail: fioravanti@fem.unicamp.br)*

Abstract:
In this paper, we address the $\mathcal{H}_\infty$ control problem for uncertain sampled-data systems rewritten as hybrid systems. The conditions proposed are formulated as intervals to ensure stability and design controllers that guarantee an upper bound for an associated $\mathcal{H}_\infty$ norm. A numerical example points out the main features of the proposed method.

Keywords: $\mathcal{H}_\infty$ control, robust control, hybrid systems, sampled-data systems, interval uncertainty

1. INTRODUCTION

In systems engineering, many control systems analysis and design techniques are based on time-invariant mathematical models. These models may not exactly represent physical systems because of uncertainties due to the impossibility of accurately estimating their parameters. Thus, control systems must be robust with respect to such uncertainties and guarantee stability and performance even under the effect of external disturbances (Bhattacharyya and Keel, 1995).

In the last decades, a popular performance measure has been employed in optimal control theory: the $\mathcal{H}_\infty$ norm (Zhou et al., 1996). Its importance is twofold: in the frequency domain, the $\mathcal{H}_\infty$ norm represents the peak gain of the Bode plot of the system; in the time domain, the $\mathcal{H}_\infty$ norm represents the worst-case $\mathcal{L}_2$ gain of the system. Based on this performance index, several robust control problems have been tackled (Başar and Bernhard, 2008).

Typical approaches to modelling uncertain systems yield either polytopic (de Oliveira et al., 1999) or interval (Mao and Chu, 2003; Zhang et al., 2006; Hong et al., 2006; Mao, 2004; Lee et al., 2006) dynamic models. Whilst polytopic models typically yield less conservative results, the number of vertices needed to model an uncertain system with several independent uncertainties may become prohibitive. As interval models only store the minimum and the maximum value of each uncertain entry in the system matrices, such models allow us to deal with several uncertainties efficiently.

⋆ This work was in part supported by 'Conselho Nacional de Desenvolvimento Científico e Tecnológico - CNPq' (grant no. 426638/2018-2); Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP/Brazil) grant 2018/04905-1; Coordenação de Aperfeiçoamento de Pessoal de Nivel Superior - Brasil (CAPES) - Finance Code 001.

In this paper, our main goal is to design $\mathcal{H}_\infty$ robust sampled-data controllers for interval systems. $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control techniques for interval systems have been devised in (Alves et al., 2019b,a) for both continuous and discrete-time systems. These results have been extended to design sampled-data controllers for interval systems, therefore the discretised system is equivalent for the $\mathcal{H}_2$ case, and the $\mathcal{H}_\infty$ case is only an approximation. Thus, our approach in this paper is to model the sampled-data control system as a hybrid system (Goebel et al., 2009), which blends continuous and discrete-time behaviour in its dynamics. This approach allows us to deal with the $\mathcal{H}_\infty$ control problem.

1.1 Related Work

The $\mathcal{H}_\infty$ control problem based on frequency and state-space formulations has been considered by several scientists. For example, the optimal solution for precisely known problems can be determined by algebraic Riccati equations (Zhou et al., 1996; Furuta and Phoojaruenchanachai, 1990; Stoustrup, 1993). However, in many practical cases, the system may present uncertainties and, thus, other methods must be sought to ensure robustness to the control system (Bhattacharyya and Keel, 1995). The literature is rich in the area of robust control of interval and (the closely related) polytopic systems. Following the remarkable robust stability result provided by Kharitonov's Theorem (Kharitonov, 1978; Ackermann, 1993), classic references in the area of interval systems (Sezer and Šiljak, 1994; Mansour, 1989; Delgado-Romero et al., 1997; Daoyi, 1985) focused on devising simple tests and conditions for the stability of an interval dynamic system. Stability conditions for interval systems have been revisited and restated in the linear matrix inequality (LMI) (Boyd et al., 1994) framework in (Mao and Chu, 2003; Zhang et al., 2006), in which other properties such as controllability are also investigated; these new results are based on Petersen's

Lemma (Petersen, 1987; Shcherbakov and Topunov, 2008; Ji and Su, 2016) for robust stability under norm-bounded uncertainties. These LMI conditions provide a simple and computationally efficient stability test for the interval system and can be extended to design stabilizing controllers, for instance. Even though these conditions were initially published as necessary and sufficient for quadratic stability and stabilizability of an uncertain system, it was later verified that they are, in fact, only sufficient; the necessity part implication of Petersen's Lemma does not hold for more than one norm-bounded uncertainty (Shcherbakov and Topunov, 2008; Ji and Su, 2016; Yang and Lum, 2005; Mao and Chu, 2006). In the following papers (Hong et al., 2006; Mao, 2004; Lee et al., 2006), robust conditions $\mathcal{H}_\infty/\mathcal{H}_2$ have been proposed in terms of LMIs for continuous interval systems; as the results are based on the Petersen's Lemma these conditions are only sufficient.

Finally, sampled-data control for interval systems was proposed in (Alves et al., 2019b,a), based on equivalent discrete-time systems. These two papers present numerical examples comparing interval and polytopic approaches, demonstrating the interval conditions are numerically more efficient, requiring fewer conditions and variables, mostly for systems with a large number of uncertainties independently. For the precisely known case, stability and performance conditions for the sampled-data control of LTI systems were proposed using hybrid models in (Souza and Geromel, 2015; Geromel and Souza, 2015). These results motivate the development of this paper.

1.2 Contributions of this Paper

In this paper, we rewrite the interval sampled-data control system as a hybrid system, which allows us to devise stability and stabilisation conditions considering also a guaranteed $\mathcal{H}_\infty$ performance. It is important to state that this robust control problem could be addressed using polytopic models and affine Lyapunov functions. The main drawback presented by this approach is the exponential number of vertices: an interval matrix with m uncertain (independent) entries can be represented by a matrix polytope with 2^m vertices, which means more constraints and variables are required to assess its stability. Our conditions, albeit more conservative, only need a constant number of constraints and a polynomial number of variables and, thus, they can be solved more efficiently even for large systems. Moreover, our conditions can be promptly extended to consider time-varying uncertainties.

2. PRELIMINARIES

2.1 Notation

We now introduce some notation that shall be used throughout the paper. The sets of natural, real, and non-negative real numbers are indicated by $\mathbb{N}$, $\mathbb{R}$, and $\mathbb{R}_+$, respectively. The set of real $m \times n$ matrices is denoted by $\mathbb{R}^{m\times n}$ and the set of n-dimensional real column vectors is denoted by $\mathbb{R}^n$. For any matrix $X \in \mathbb{R}^{m\times n}$, $X^\mathsf{T} \in \mathbb{R}^{n\times m}$ denotes its transpose. Additionally, for any matrix $X = (x_{ij}) \in \mathbb{R}^{m\times n}$, $X > 0$ ($X \geq 0$) denotes that $x_{ij} > 0$ ($x_{ij} \geq 0$) for all i, j. For any symmetric matrix $X = X^\mathsf{T} \in \mathbb{R}^{n\times n}$, $X \succ 0$ ($X \succeq 0$) denotes that X is positive definite (semidefinite). The set $\mathbb{S}^n$ is formed by all symmetric matrices of order in $\mathbb{R}^{n\times n}$ and $\mathbb{S}^n_+$ is formed by all symmetric positive definite matrices of order in $\mathbb{R}^{n\times n}$. $\mathbf{He}(X)$ means the sum of a matrix X with its transpose: $\mathbf{He} := X + X^\mathsf{T}$. For any two given matrices $\underline{X}, \overline{X} \in \mathbb{R}^{m\times n}$ such that $\underline{X} \leq \overline{X}$, we define the interval matrix $[\underline{X}, \overline{X}]$ as the set

$$[X] = [\underline{X}, \overline{X}] := \Big\{X \in \mathbb{R}^{m\times n} \ : \ \underline{X} \leq X \leq \overline{X}\Big\}, \qquad (1)$$

whose *center* and *radius* are defined as $X_0 := \frac{1}{2}\underline{X} + \frac{1}{2}\overline{X}$ and as $\Delta X := \frac{1}{2}\overline{X} - \frac{1}{2}\underline{X}$, respectively. We may also write the matrix interval above as

$$[X] := \Big\{X_0 + \sum_{i,j} e_i \delta x_{ij} f_j^\mathsf{T} \ : \ |\delta x_{ij}| \leq \Delta x_{ij}\Big\}, \qquad (2)$$

in which $\Delta x_{ij} = [\Delta X]_{ij}$, e_i and f_j are the i-th and the j-th columns of compatible identity matrices. Uncertain or arbitrary elements in a matrix interval are denoted in bold. In the description of symmetric matrices, we use the symbol $\star$ to denote a block whose symmetric correspondent is already described. Finally, the notation $\xi(t_k^-)$ for $t_k \geq 0$, $k \in \mathbb{N}$, given, indicates the limit of $\xi(t)$ as t goes to t_k from the left.

2.2 Interval System

Let us consider the linear, time-invariant system

$$\mathcal{S} : \begin{cases} \dot{x}(t) = \mathbf{A}x(t) + \begin{bmatrix}\mathbf{B} & \mathbf{E}\end{bmatrix}\begin{bmatrix}u(t)\\ w(t)\end{bmatrix}, \quad x(0) = 0, \\ y(t) = \mathbf{C}x(t) + \mathbf{D}u(t), \end{cases} \qquad (3)$$

which evolves from zero initial condition and in which $x : \mathbb{R}_+ \to \mathbb{R}^{n_x}$ is the state, $w : \mathbb{R}_+ \to \mathbb{R}^{n_w}$ is the disturbance input, $u : \mathbb{R}_+ \to \mathbb{R}^{n_u}$ is the control input and $y : \mathbb{R}_+ \to \mathbb{R}^{n_y}$ is the output. In this article, $\mathcal{S}$ is said to be an *interval system*, meaning that their realization matrices $\mathbf{A} \in [A]$, $\mathbf{B} \in [B]$, $\mathbf{E} \in [E]$, $\mathbf{C} \in [C]$ and $\mathbf{D} \in [D]$ are not precisely known. For simplicity, define the set

$$\mathbb{X} = [A] \times [B] \times [E] \times [C] \times [D], \qquad (4)$$

which allows us to write the uncertainties in compact form $(\mathbf{A}, \mathbf{E}, \mathbf{B}, \mathbf{C}, \mathbf{D}) \in \mathbb{X}$.

2.3 Petersen's Lemma on Quadratic Stability

In this section, we present some relevant auxiliary results for the main theoretical developments of this article. We begin by the important matrix lemma stated by Ian Petersen:

Lemma 1. *[Petersen (1987); Shcherbakov and Topunov (2008)]*
Let $G \in \mathbb{S}^n$, $M \in \mathbb{R}^{n\times p}$, and $N \in \mathbb{R}^{q\times n}$ be given. The inequality

$$G + M\Delta(t)N + N^\mathsf{T}\Delta(t)^\mathsf{T}M^\mathsf{T} \prec 0, \quad \forall t \in \mathbb{R}_+, \qquad (5)$$

holds for all $\Delta : \mathbb{R}_+ \to \mathbb{R}^{p\times q}$ such that $\Delta(t)^\mathsf{T}\Delta(t) \preceq I$ for all $k \in \mathbb{N}$ if, and only if, there exists $\epsilon > 0$ such that

$$G + \epsilon MM^\mathsf{T} + \epsilon^{-1}N^\mathsf{T}N \prec 0. \qquad (6)$$

Unfortunately, this remarkable result cannot be extended for several uncertain matrices without losing necessity; see Ji and Su (2016) for a generalized necessary and sufficient result for two uncertain matrices. The same drawback exists in the S-procedure and, as there is a relationship between Petersen's Lemma and the S-procedure, this might be one way of justifying this fact (Shcherbakov and Topunov, 2008). As a consequence of this loss of necessity, the results developed in Mao and Chu (2003) and in Zhang et al. (2006), which made use of Petersen's Lemma for interval systems, are not necessary, as pointed out by Yang and Lum (2005). Nevertheless, the conditions presented in both papers are very interesting as they are based on only one linear matrix inequality, which means they can tackle even systems with a relatively large number of uncertain parameters. As pointed out by Shcherbakov and Topunov (2008), the conservativeness of such conditions is acceptable in practice.

Remark 1. *Petersen's Lemma (Petersen, 1987) is based on robust stability under norm-bounded uncertainties, it can guarantee the stability for any realization in the interval even for the time-varying system. As our main results are heavily dependent on Petersen's Lemma, the same design conditions presented in this paper can deal with interval time-varying uncertainties with no additional assumptions.*

2.4 Computation Resources

All the code in this paper was run on an HP computer with Windows 10 64 bits operating system, AMD Ryzen 5 2500u, 2.00 GHz, and 16GB memory. Also, interval operations and functions are carried out with INTLAB (Rump, 1999), which is a computational package developed for reliable interval computing that runs on MATLAB.

3. PROBLEM STATEMENT

This paper aims to propose a controller able to stabilise a linear time-invariant uncertain system of the form $\mathcal{S}$, (3), with the sampled-data state-feedback control signal

$$u(t) = Kx(t_k), \quad \forall t \in [t_k, t_{k+1}), k \in \mathbb{N}. \tag{7}$$

We also focus on minimising an upper bound $\mu > 0$ for the worst-case $\mathcal{L}_2$ gain

$$\mathcal{J}_\infty(K) = \sup_{w(t)\in\ell_2\setminus\{0\}} \frac{\int_0^\infty y(t)^\top y(t)\mathrm{d}t}{\int_0^\infty w(t)^\top w(t)\mathrm{d}t}, \tag{8}$$

which is an $\mathcal{H}_\infty$-like performance index for $\mathcal{S}$.

To solve this robust control problem, the LTI system $\mathcal{S}$, given in (3), together with the constraint on u imposed by (7), can be recast as

$$\mathcal{H}: \begin{cases} \dot{\xi}(t) = \begin{bmatrix}\mathbf{A} & \mathbf{B}\\ 0 & 0\end{bmatrix}\xi(t) + \begin{bmatrix}\mathbf{E}\\ 0\end{bmatrix} w(t), \xi(0^-) = 0 \\ y(t) = [\mathbf{C}\ \mathbf{D}]\,\xi(t) \\ \xi(t_k) = \begin{bmatrix} I & 0\\ K & 0\end{bmatrix}\xi(t_k^-) \end{cases} \tag{9}$$

which is valid for all $t \in [t_k, t_{k+1})$, we assume the jump rate is constant, that is $t_{k+1} - t_k = h$. Note that the equivalence between the *hybrid linear system* $\mathcal{H}$ given in (9) and the original one is a result of the particular choice of the augmented state vector $\xi(t) = [x(t)^\top u(t)^\top]^\top$. To simplify the augmented matrices, it can be considered the following representation form

$$\mathcal{H}: \begin{cases} \dot{\xi}(t) = \boldsymbol{\mathscr{A}}\xi(t) + \boldsymbol{\mathscr{E}}w(t), \ \xi(0^-) = 0 \\ y(t) = \boldsymbol{\mathscr{C}}\xi(t) \\ \xi(t_k) = \mathcal{K}\xi(t_k^-). \end{cases} \tag{10}$$

For simplicity, as the uncertainties on $\mathcal{H}$ depend only on the uncertainties of $\mathcal{S}$, we write $(\boldsymbol{\mathscr{A}}, \boldsymbol{\mathscr{E}}, \boldsymbol{\mathscr{C}}) \in \mathbb{X}$. Moreover, as before, we define the $\mathcal{H}_\infty$ cost associated with $\mathcal{H}$ as

$$\mathcal{J}_\infty(\mathcal{H}) = \sup_{w(t)\in\ell_2\setminus\{0\}} \frac{\int_0^\infty y(t)^\top y(t)\mathrm{d}t}{\int_0^\infty w(t)^\top w(t)\mathrm{d}t}. \tag{11}$$

4. HYBRID SYSTEMS STABILITY AND PERFORMANCE

To devise sampled-data control conditions, we must first analyse stability and performance conditions for the hybrid model $\mathcal{H}$. Then, we exploit the equivalence between $\mathcal{S}$ under sampled-data control and $\mathcal{H}$ to provide robust control design conditions.

We begin by stating the following result, which provides stability and guaranteed $\mathcal{H}_\infty$ performance for $\mathcal{H}$. As we shall see, this result, which is based on the developments of (Souza et al., 2014), is still not computationally viable.

Lemma 2. *Let $\mathcal{H}$ be an interval hybrid system with $(\boldsymbol{\mathscr{A}}, \boldsymbol{\mathscr{E}}, \boldsymbol{\mathscr{C}}) \in \mathbb{X}$ and let the scalar $\mu > 0$ and the matrix $\mathcal{K}$ be given. $\mathcal{H}$ is globally asymptotically stable and the performance index defined in* (11) *verifies the bound $\mathcal{J}_\infty(\mathcal{H}) < \mu$ for all $(\boldsymbol{\mathscr{A}}, \boldsymbol{\mathscr{E}}, \boldsymbol{\mathscr{C}}) \in \mathbb{X}$ if there exists a matrix $X(t) \in \mathbb{S}_+^{n_\xi}$ such that*

$$\begin{bmatrix} \dot{X} + \mathbf{He}(X(t)\boldsymbol{\mathscr{A}}) & \star & \star \\ \boldsymbol{\mathscr{E}}^\top X(t) & -\mu I & \star \\ \boldsymbol{\mathscr{C}} & 0 & -I \end{bmatrix} \prec 0, \tag{12}$$

$$\begin{bmatrix} X(h) & \star \\ X(0)\mathcal{K} & X(0) \end{bmatrix} \succ 0 \tag{13}$$

hold for all $(\boldsymbol{\mathscr{A}}, \boldsymbol{\mathscr{E}}, \boldsymbol{\mathscr{C}}) \in \mathbb{X}$ for $t \in [0, h]$.

Proof. Let us consider the Lyapunov candidate function v given by $v(t) = \xi(t)^\top P(t)\xi(t)$, $t \in \mathbb{R}_+$, in which $P(t) = X(t - t_k)$ for $t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$. Let us first provide a sketch of the proof of the stability part. For this, we assume that the system evolves from a given $\xi(0^-)$ and that $w \equiv 0$. For any $t \in (t_k, t_{k+1})$, it follows that

$$\dot{v}(t) = \xi(t)^\top \left(\dot{P}(t) + \boldsymbol{\mathscr{A}}^\top P(t) + P(t)\boldsymbol{\mathscr{A}}\right)\xi(t) < 0, \tag{14}$$

from the first block of (12). Moreover, from (13), it follows that

$$v(t_k) < v(t_k^-) \tag{15}$$

holds for all $k \in \mathbb{N}$. As pointed out in (Amorim et al., 2018), these conditions ensure global exponential stability for the dynamics of $\mathcal{H}$.

Now, let us move our attention to the $\mathcal{H}_\infty$ guaranteed cost. Schur complement allows us to conclude that (12) is equivalent to

$$\begin{bmatrix} \dot{P}(t) + \mathbf{He}(P(t)\boldsymbol{\mathscr{A}}) + \boldsymbol{\mathscr{C}}^\top\boldsymbol{\mathscr{C}} & \star \\ \boldsymbol{\mathscr{E}}^\top P(t) & -\mu I \end{bmatrix} \prec 0 \tag{16}$$

Thus, this inequality implies that

$$\dot{v}(t) < -y(t)^\top y(t) + \mu w(t)^\top w(t), \tag{17}$$

holds for all $t \in (t_k, t_{k+1})$. Integrating this inequality in this interval and remembering that $v(t_k^+) = v(t_k)$ and that $v(t_k^-) > v(t_k)$, it follows that

$$\int_{t_k}^{t_{k+1}} \big(y(t)^\mathsf{T} y(t) - \mu w(t)^\mathsf{T} w(t)\big)\mathrm{d}t < v(t_k^-) - v(t_{k+1}^-), \quad (18)$$

is valid for all $k \in \mathbb{N}$. Thus, as $\mathcal{H}$ is globally asymptotically stable,

$$\int_0^\infty \big(y(t)^\mathsf{T} y(t) - \mu w(t)^\mathsf{T} w(t)\big)\mathrm{d}t < v(0^-) = 0, \quad (19)$$

which implies that μ is an upper bound for the $\mathcal{H}_\infty$ guaranteed cost. The proof is complete. □

The conditions stated in this lemma involve linear matrix inequalities that depend on all realization matrices in $\mathbb{X}$. The following theorem provides a set of computationally feasible conditions for the robust control design problem.

Theorem 1. *Let $\mathcal{H}$ be an interval hybrid system with $(\boldsymbol{\mathscr{A}},\boldsymbol{\mathscr{E}},\boldsymbol{\mathscr{C}}) \in \mathbb{X}$ and let the scalar $\mu > 0$ and the matrix $\mathcal{K}$ be given. $\mathcal{H}$ is globally asymptotically stable and the performance index defined in (11) verifies the bound $\mathcal{J}_\infty < \mu$ for all $(\boldsymbol{\mathscr{A}},\boldsymbol{\mathscr{E}},\boldsymbol{\mathscr{C}}) \in \mathbb{X}$ if there exist a matrix $X(t) \in \mathbb{S}_+^{n_\xi}$ and positive scalars α_{ij}, $i,j \in \{1,\cdots,n_\xi\}$, ϵ_{ij}, $i \in \{1,\cdots,n_\xi\}$ and $j \in \{1,\cdots,n_w\}$, γ_{ij}, $i \in \{1,\cdots,n_y\}$ and $j \in \{1,\cdots,n_\xi\}$, such that (13) and*

$$\begin{bmatrix} \dot X(t) + \mathbf{He}(X(t)\mathcal{A}_0) + M & \star & \star & \star & \star & \star \\ \mathcal{E}_0^\mathsf{T} X(t) & -\mu I + T & \star & \star & \star & \star \\ \mathcal{C}_0 & 0 & -I + U & \star & \star & \star \\ \mathcal{X}_a & 0 & 0 & -\mathbb{A} & \star & \star \\ \mathcal{X}_e & 0 & 0 & 0 & -\mathbb{E} & \star \\ \mathcal{I}_c & 0 & 0 & 0 & 0 & -\mathbb{C} \end{bmatrix} \prec 0, \quad (20)$$

hold for $t \in [0,h]$, in which

$$M = \sum_{i,j} \Delta a_{ij}^2 \alpha_{ij} e_j e_j^\mathsf{T}, \quad T = \sum_{i,j} \Delta e_{ij}^2 \epsilon_{ij} f_j f_j^\mathsf{T}, \quad (21)$$

$$U = \sum_{i,j} \Delta c_{ij}^2 \gamma_{ij} g_i g_i^\mathsf{T}, \quad (22)$$

$$\mathcal{X}_a = \big[\underbrace{X(t) \ \cdots \ X(t)}_{n_\xi}\big]^\mathsf{T}, \quad \mathcal{X}_e = \big[\underbrace{X(t) \ \cdots \ X(t)}_{n_w}\big]^\mathsf{T}, \quad (23)$$

$$\mathcal{I}_c = \big[\underbrace{I_{n_\xi} \ \cdots \ I_{n_\xi}}_{n_y}\big]^\mathsf{T}, \quad (24)$$

$$\mathbb{A} = \mathbf{diag}\big(\alpha_{11},\cdots,\alpha_{1n_\xi},\cdots,\alpha_{n_\xi 1},\cdots,\alpha_{n_\xi n_\xi}\big), \quad (25)$$

$$\mathbb{E} = \mathbf{diag}\big(\epsilon_{11},\cdots,\epsilon_{1n_w},\cdots,\epsilon_{n_\xi 1},\cdots,\epsilon_{n_\xi n_w}\big), \quad (26)$$

$$\mathbb{C} = \mathbf{diag}\big(\gamma_{11},\cdots,\gamma_{1n_\xi},\cdots,\gamma_{n_y 1},\cdots,\gamma_{n_y n_\xi}\big), \quad (27)$$

vectors e_j, f_j, g_i are the j-th and i-th columns of identity matrices of compatible dimensions and $\mathcal{A}_0$, $\mathcal{E}_0$ and $\mathcal{C}_0$ are the center matrices of $\mathbb{X}$.

Proof. Applying Schur Complement to (20), we obtain the equivalent inequality

$$\begin{aligned} &\begin{bmatrix} \dot X(t) + \mathbf{He}(X(t)\mathcal{A}_0) + M & \star & \star \\ \mathcal{E}_0^\mathsf{T} X(t) & -\mu I + T & \star \\ \mathcal{C}_0 & 0 & -I - U \end{bmatrix} + \\ &+ \sum_i^{n_\xi}\sum_j^{n_\xi} \left(\alpha_{ij}^{-1} \begin{bmatrix} X(t)e_i \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} e_i^\mathsf{T} X(t) & 0 & 0 \end{bmatrix} \right) + \\ &+ \sum_i^{n_\xi}\sum_j^{n_w} \left(\epsilon_{ij}^{-1} \begin{bmatrix} X(t)e_i \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} e_i^\mathsf{T} X(t) & 0 & 0 \end{bmatrix} \right) + \\ &+ \sum_i^{n_y}\sum_j^{n_\xi} \left(\gamma_{ij}^{-1} \begin{bmatrix} e_j \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} e_j^\mathsf{T} & 0 & 0 \end{bmatrix} \right) \prec 0. \end{aligned} \quad (28)$$

This inequality can be reorganized as follows:

$$\begin{aligned} &\begin{bmatrix} \dot X(t) + \mathbf{He}(X(t)\mathcal{A}_0) & \star & \star \\ \mathcal{E}_0^\mathsf{T} X(t) & -\mu I & \star \\ \mathcal{C}_0 & 0 & -I \end{bmatrix} + \\ &+ \sum_i^{n_\xi}\sum_j^{n_\xi} \left(\Delta a_{ij}^2 \alpha_{ij} \begin{bmatrix} e_j \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} e_j \\ 0 \\ 0 \end{bmatrix}^\mathsf{T} + \alpha_{ij}^{-1} \begin{bmatrix} X(t)e_i \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} X(t)e_j \\ 0 \\ 0 \end{bmatrix}^\mathsf{T} \right) + \\ &+ \sum_i^{n_\xi}\sum_j^{n_w} \left(\Delta e_{ij}^2 \epsilon_{ij} \begin{bmatrix} 0 \\ f_j \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ f_j \\ 0 \end{bmatrix}^\mathsf{T} + \epsilon_{ij}^{-1} \begin{bmatrix} X(t)e_j \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} X(t)e_j \\ 0 \\ 0 \end{bmatrix}^\mathsf{T} \right) + \\ &+ \sum_i^{n_y}\sum_j^{n_\xi} \left(\Delta c_{ij}^2 \gamma_{ij} \begin{bmatrix} 0 \\ 0 \\ g_i \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ g_i \end{bmatrix}^\mathsf{T} + \gamma_{ij}^{-1} \begin{bmatrix} e_j \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} e_j \\ 0 \\ 0 \end{bmatrix}^\mathsf{T} \right) \prec 0. \end{aligned} \quad (29)$$

Thus, by Petersen's Lemma (Lemma 1), it follows that (29) implies that

$$\begin{aligned} &\begin{bmatrix} \dot X(t) + \mathbf{He}(X(t)\mathcal{A}_0) & \star & \star \\ \mathcal{E}_0^\mathsf{T} X(t) & -\mu I & \star \\ \mathcal{C}_0 & 0 & -I \end{bmatrix} + \\ &+ \sum_i^{n_\xi}\sum_j^{n_\xi} \mathbf{He}\left(\begin{bmatrix} e_j \\ 0 \\ 0 \end{bmatrix} \delta a_{ij} \begin{bmatrix} e_i^\mathsf{T} X(t) & 0 & 0 \end{bmatrix} \right) + \\ &+ \sum_i^{n_\xi}\sum_j^{n_w} \mathbf{He}\left(\begin{bmatrix} 0 \\ f_j \\ 0 \end{bmatrix} \delta e_{ij} \begin{bmatrix} e_i^\mathsf{T} X(t) & 0 & 0 \end{bmatrix} \right) + \\ &+ \sum_i^{n_y}\sum_j^{n_\xi} \mathbf{He}\left(\begin{bmatrix} 0 \\ 0 \\ g_i \end{bmatrix} \delta c_{ij} \begin{bmatrix} e_j^\mathsf{T} & 0 & 0 \end{bmatrix} \right) \prec 0, \end{aligned} \quad (30)$$

holds for all $|\delta a_{ij}| \leqslant \Delta a_{ij}$, i, $j = 1,\ldots,n_\xi$; $|\delta e_{ij}| \leqslant \Delta e_{ij}$, $i = 1,\ldots,n_\xi$, $j = 1,\ldots,n_w$; $|\delta c_{ij}| \leqslant \Delta c_{ij}$, $i = 1,\ldots,n_y$, $j = 1,\ldots,n_\xi$. Regrouping the terms in (30), we can rewrite it in interval form as (12), which implies that $\mathcal{H}$ is globally asymptotically stable and the $\mathcal{H}_\infty$ performance bound holds. The proof is complete. □

Remark 2. *The conditions in Theorem 1, as they are stated, are still not ready for optimisation, as they depend continuously on $t \in [0,h]$. Nevertheless, their computational implementation as convex conditions is rather simple using piecewise linear functions as in (Allerhand and Shaked, 2010) or as a Sum of Squares problem (Parrilo, 2000).*

5. SAMPLED-DATA CONTROL OF INTERVAL SYSTEMS

We now exploit the analysis results devised in the previous section to propose sampled-data robust controllers for interval systems with $\mathcal{H}_\infty$ guaranteed performance. We begin with the following lemma.

Lemma 3. *Let $\mathcal{S}$ be an interval hybrid system with $(\mathbf{A},\mathbf{E},\mathbf{B},\mathbf{C},\mathbf{D}) \in \mathbb{X}$ and let the scalar $\mu > 0$ be given. There exists a feedback gain $\hat{K} \in \mathbb{R}^{n_u \times n_\xi}$ such that the sampled-data feedback law (7) quadratically stabilises $\mathcal{S}$ and ensures that $\mathcal{J}_\infty(K) < \mu$ for all $(\mathbf{A},\mathbf{E},\mathbf{B},\mathbf{C},\mathbf{D}) \in \mathbb{X}$ if there exist matrices $S(t) \in \mathbb{S}_+^{n_x}$, $Z(t) \in \mathbb{S}_+^{n_u}$, $L(t) \in \mathbb{R}^{n_x \times n_u}$ and $W \in \mathbb{S}_+^{n_x}$, such that the linear matrix inequalities*

$$\begin{bmatrix} -\begin{bmatrix} \dot{S}(t) & \star \\ \dot{L}(t)^\mathsf{T}(t) & \dot{Z}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{He}(\mathbf{A}S(t)+\mathbf{B}L(t)^\mathsf{T}) & \star \\ L^\mathsf{T}(t)\mathbf{A}^\mathsf{T} + Z^\mathsf{T}(t)\mathbf{B}^\mathsf{T} & 0 \end{bmatrix} & \star & \star \\ \begin{bmatrix} \mathbf{E}^\mathsf{T} & 0 \end{bmatrix} & -\mu I & \star \\ \begin{bmatrix} \mathbf{C}S(t) + \mathbf{D}L^\mathsf{T}(t) & \mathbf{C}L(t) + \mathbf{D}Z(t) \end{bmatrix} & 0 & -I \end{bmatrix} \prec 0, \tag{31}$$

$$\begin{bmatrix} S(h) & \star & \star \\ L(h)^\mathsf{T} & Z(h) & \star \\ S(h) & L(h) & W \end{bmatrix} \succ 0, \tag{32}$$

$$\begin{bmatrix} W & \star & \star \\ W & S(0) & \star \\ \hat{K} & L(0)^\mathsf{T} & Z(0) \end{bmatrix} \succ 0, \tag{33}$$

$$\begin{bmatrix} S(t) & \star \\ L(t)^\mathsf{T} & Z(t) \end{bmatrix} \succ 0, \tag{34}$$

hold for all $(\mathbf{A},\mathbf{E},\mathbf{B},\mathbf{C},\mathbf{D}) \in \mathbb{X}$ and $t \in [0,h]$. Furthermore, the quadratically stabilizing state-feedback gain is given by $K = \hat{K}W^{-1}$.

Proof. This result is based on Lemma 1. First, applying a congruence transformation in (12) with the matrix

$$\begin{bmatrix} X(t)^{-1} & \star & \star \\ 0 & I & \star \\ 0 & 0 & I \end{bmatrix} \tag{35}$$

and defining $Y = X^{-1}$, it follows that (12) is equivalent to

$$\begin{bmatrix} -\dot{Y}(t) + \mathbf{He}(\mathscr{A}Y(t)) & \star & \star \\ \mathscr{E}^\mathsf{T} & -\mu I & \star \\ \mathscr{C}Y(t) & 0 & -I \end{bmatrix} \prec 0. \tag{36}$$

Now, let us define the following partition of Y:

$$Y(t) = \begin{bmatrix} S(t) & \star \\ L(t)^\mathsf{T} & Z(t) \end{bmatrix}. \tag{37}$$

This partition allows us to rewrite (36) as (31). As (34) holds, $Y(t) \succ 0$ for all $t \in [0,h]$ and this implies that $X(t) \succ 0$ on the same interval.

Now, take (33) and (34). Both can be rewritten equivalently as

$$Y(h) \succ Y(h) \begin{bmatrix} I \\ 0 \end{bmatrix} W^{-1} \begin{bmatrix} I & 0 \end{bmatrix} Y(h) \tag{38}$$

$$W^{-1} \succ \begin{bmatrix} I & K^\mathsf{T} \end{bmatrix} Y(0)^{-1} \begin{bmatrix} I \\ K \end{bmatrix}, \tag{39}$$

respectively. Hence, combining both inequalities, we may eliminate W and obtain (13), completing the proof. □

As before, we now present computationally feasible conditions for the design of our state-feedback controller.

Theorem 2. *Let $\mathcal{S}$ be an interval hybrid system with $(\mathbf{A},\mathbf{E},\mathbf{B},\mathbf{C},\mathbf{D}) \in \mathbb{X}$ and let the scalar $\mu > 0$ be given. There exists a feedback gain $\hat{K} \in \mathbb{R}^{n_u \times n_\xi}$ such that the sampled-data feedback law (7) quadratically stabilises $\mathcal{S}$ and ensures that $\mathcal{J}_\infty(K) < \mu$ for all $(\mathbf{A},\mathbf{E},\mathbf{B},\mathbf{C},\mathbf{D}) \in \mathbb{X}$ if there exists matrices $S(t) \in \mathbb{S}_+^{n_x}$, $Z(t) \in \mathbb{S}_+^{n_u}$, $L(t) \in \mathbb{R}_+^{n_x \times n_u}$, $W \in \mathbb{S}_+^{n_x}$ and positive scalars α_{ij}, $i,j \in \{1,\cdots,n_\xi\}$, ϵ_{ij}, $i \in \{1,\cdots,n_\xi\}$ and $j \in \{1,\cdots,n_w\}$, βij, $i \in \{1,\cdots,n_\xi\}$ and $j \in \{1,\cdots,n_u\}$, γ_{ij}, $i \in \{1,\cdots,n_y\}$ and $j \in \{1,\cdots,n_\xi\}$, δij, $i \in \{1,\cdots,n_y\}$ and $j \in \{1,\cdots,n_u\}$, such the linear matrix inequalities (32), (33), (34) and*

$$\begin{bmatrix}
-\begin{bmatrix} \dot{S}(t) & \star \\ \dot{L}^\mathsf{T}(t) & \dot{Z}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{He}(A_0S(t)+B_0L^\mathsf{T}(t))+M & \star \\ L^\mathsf{T}(t)A_0^\mathsf{T} + Z(t)B_0^\mathsf{T} & 0 \end{bmatrix} & \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \star \\
\begin{bmatrix} E_0^\mathsf{T} & 0 \end{bmatrix} & -\mu I + T & \star & \star & \star & \star & \star & \star & \star & \star & \star & \star \\
\begin{bmatrix} C_0S(t) + D_0L^\mathsf{T}(t) & C_0L(t) + D_0Z(t) \end{bmatrix} & 0 & -I + U & \star & \star & \star & \star & \star & \star & \star & \star & \star \\
\begin{bmatrix} \mathcal{S}_{n_x} & 0 \end{bmatrix} & 0 & 0 & -\mathbb{A} & \star & \star & \star & \star & \star & \star & \star & \star \\
\begin{bmatrix} \mathcal{L}_{\mathsf{T}n_x} & 0 \end{bmatrix} & 0 & 0 & 0 & -\mathbb{B} & \star & \star & \star & \star & \star & \star & \star \\
\begin{bmatrix} 0 & \mathcal{L}_{n_x} \end{bmatrix} & 0 & 0 & 0 & 0 & -\mathbb{A} & \star & \star & \star & \star & \star & \star \\
\begin{bmatrix} 0 & \mathcal{Z}_{n_x} \end{bmatrix} & 0 & 0 & 0 & 0 & 0 & -\mathbb{B} & \star & \star & \star & \star & \star \\
\begin{bmatrix} \mathcal{S}_{n_y} & 0 \end{bmatrix} & 0 & 0 & 0 & 0 & 0 & 0 & -\mathbb{C} & \star & \star & \star & \star \\
\begin{bmatrix} \mathcal{L}_{\mathsf{T}n_y} & 0 \end{bmatrix} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathbb{D} & \star & \star & \star \\
\begin{bmatrix} 0 & \mathcal{L}_{n_y} \end{bmatrix} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathbb{C} & \star & \star \\
\begin{bmatrix} 0 & \mathcal{Z}_{n_y} \end{bmatrix} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathbb{D} & \star \\
\begin{bmatrix} \mathcal{I}_{n_w} & 0 \end{bmatrix} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathbb{E}
\end{bmatrix} \prec 0, \tag{40}$$

in which

$$M = \sum_{i,j} 2(\Delta a_{ij}^2 \alpha_{ij} h_i h_i^\mathsf{T}) + \sum_{i,j} 2(\Delta b_{ij}^2 \beta_{ij} h_i h_i^\mathsf{T}), \tag{41}$$

$$T = \sum_{i,j} (\Delta e_{ij}^2 \epsilon_{ij} f_j f_j^\mathsf{T}), \tag{42}$$

$$U = \sum_{i,j} 2(\Delta c_{ij}^2 \gamma_{ij} g_i g_i^\mathsf{T}) + \sum_{i,j} 2(\Delta d_{ij}^2 \delta_{ij} g_i g_i^\mathsf{T}), \tag{43}$$

$$\mathcal{S}_{n_x} = \underbrace{\begin{bmatrix} S(t) & \cdots & S(t) \end{bmatrix}}_{n_x}^\mathsf{T}, \quad \mathcal{L}_{\mathsf{T}n_x} = \underbrace{\begin{bmatrix} L(t) & \cdots & L(t) \end{bmatrix}}_{n_x}^\mathsf{T} \tag{44}$$

$$\mathcal{L}_{n_x} = \underbrace{\begin{bmatrix} L^\mathsf{T}(t) & \cdots & L^\mathsf{T}(t) \end{bmatrix}}_{n_x}^\mathsf{T}, \tag{45}$$

$$\mathcal{Z}_{n_x} = \underbrace{\begin{bmatrix} Z(t) & \cdots & Z(t) \end{bmatrix}}_{n_x}^\mathsf{T}, \quad \mathcal{L}_{n_y} = \underbrace{\begin{bmatrix} L(t)^\mathsf{T} & \cdots & L(t)^\mathsf{T} \end{bmatrix}}_{n_y}^\mathsf{T}, \tag{46}$$

$$\mathcal{L}_{\mathsf{T}n_y} = \underbrace{\begin{bmatrix} L(t) & \cdots & L(t) \end{bmatrix}}_{n_y}^\mathsf{T}, \tag{47}$$

$$\mathcal{S}_{n_y} = \underbrace{[S(t) \ \cdots \ S(t)]}_{n_y}^{\mathsf{T}}, \quad \mathcal{Z}_{n_y} = \underbrace{[Z(t) \ \cdots \ Z(t)]}_{n_y}^{\mathsf{T}}, \quad (48)$$

$$\mathcal{I}_{n_w} = \underbrace{[I_{n_x} \ \cdots \ I_{n_x}]}_{n_w}^{\mathsf{T}} \quad (49)$$

$$\begin{aligned}
\mathbb{A} =& \mathbf{diag}\big(\alpha_{11}, \cdots, a_{1n_x}, \cdots, \alpha_{n_x 1}, \cdots, \alpha_{n_x n_x}\big), && (50)\\
\mathbb{E} =& \mathbf{diag}\big(\epsilon_{11}, \cdots, \epsilon_{1n_w}, \cdots, \epsilon_{n_x 1}, \cdots, \epsilon_{n_x n_w}\big), && (51)\\
\mathbb{B} =& \mathbf{diag}\big(\beta_{11}, \cdots, \beta_{1n_u}, \cdots, \beta_{n_x 1}, \cdots, \beta_{n_x n_u}\big), && (52)\\
\mathbb{C} =& \mathbf{diag}\big(\gamma_{11}, \cdots, \gamma_{1n_x}, \cdots, \gamma_{n_y 1}, \cdots, \gamma_{n_y n_x}\big), && (53)\\
\mathbb{D} =& \mathbf{diag}\big(\delta_{11}, \cdots, \delta_{1n_u}, \cdots, \delta_{n_y 1}, \cdots, \delta_{n_y n_u}\big), && (54)
\end{aligned}$$

hold for all $t \in [0, h]$; vectors h_i, f_j, g_i are the i-th columns of identity matrices of compatible dimensions. The matrices A_0, E_0, B_0, C_0 and D_0 are the center matrices of $\mathbb{X}$. Futhermore, the quadratically stabilizing state-feedback gain is given by $K = \hat{K}W^{-1}$.

Proof. First, we apply Schur Complement and some algebraic manipulations on (40). By Petersen's Lemma (Lemma 1), we can rewrite it in interval form as (31), implying that $u(t) = Kx(t_k)$, with $K = \hat{K}W^{-1}$ makes $\mathcal{H}$ quadratically stable and verifies the guaranteed performance $\mathcal{J}_\infty < \mu$. For more details, the proof of Theorem 1 presents methods similar to those used in Theorem. □

6. NUMERICAL EXAMPLE

Consider the uncertain system (3) with realization matrices taken from Example 2 of Wang and Michel (1994):

$$[A] = \begin{bmatrix} 1 \pm 0.1 & -1 \pm 0.1 \\ 0 \pm 0.05 & 4 \pm 0.2 \end{bmatrix}, \quad E = B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

The realization matrices C and D are given by

$$\underline{C} = \overline{C} = \begin{bmatrix} I \\ 0 \end{bmatrix}, \underline{D} = \overline{D} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

To design a sampled-data control law of the form (7) with sampling period of $h = 0.1$s, we implement the conditions of Theorem 2 approximating derivative as in (Allerhand and Shaked, 2012), with 20 interior points, that is, $X(t)$ is a piecewise linear function with 20 evenly spaced breakpoints in the interval $[0, 0.1]$. This procedure yields the robust gain

$$K = \begin{bmatrix} 8.9735 & 20.8907 \end{bmatrix}.$$

This design procedure required 1751 lines of inequalities and 146 optimization variables and took 9.6489s to terminate.

Table 1 shows J_∞ guaranteed costs provided by Theorem 2 and by the analysis condition provided by Theorem 1. We also validate these bounds by a Monte Carlo procedure. In this example, we generate 50 thousand possible realizations and compute the closed-loop $J_\infty(\mu)$ performance. For each system realization, we compute their norm using Lemma 2, which is necessary and sufficient in the precisely known case. We also simulate, for each realization, the closed-loop response to a unit pulse input of 1 second using the interval controller. Results obtained are displayed by Table 2 and by Figure 1.

Table 1. Comparative table of closed-loop J_∞ for the restriction of synthesis and analysis.

	$J_\infty(\mu)$
Guaranteed Synthesis	10.0300 (Theorem 2)
Guaranteed Analysis	5.5003 (Theorem 1)
Worst Case	5.0240 (Lemma 2)
Average	4.3718 (Lemma 2)
Std. Deviation	0.0022 (Lemma 2)

Table 2. Comparative table of closed-loop J_∞ performance simulation achieved.

	$J_\infty(\mu)$
Worst Case	2.3932
Average Norm	2.1108
Std. Deviation	0.0004

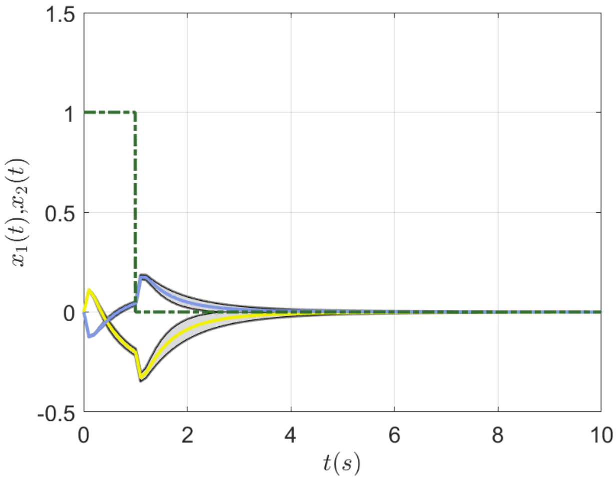

Figure 1. Closed-loop response to a step input of 1 second using the interval controller bounded $\mathcal{H}_\infty$ norm. The yellow and the blue lines are the means of the states $x_1(t)$ and $x_2(t)$, respectively. The black lines bounded the means by their standard deviations.

7. CONCLUSION

In this paper, we provide novel analysis and synthesis techniques to verify stability and to design stabilizing sampled-data controllers for uncertain linear systems. Our approach is based on hybrid systems, allowing us to take into account $\mathcal{H}_\infty$ performance, which was not possible with equivalent models. A numerical example points out the main features of the proposed methods. The prospects for future accomplishments are to implement robust sampled-data conditions for analysis and synthesis, considering performances $\mathcal{H}_\infty/\mathcal{H}_2$: state feedback (for $\mathcal{H}_2$), filtering, and output feedback.

REFERENCES

Ackermann, J. (1993). *Robust control: Systems with uncertain physical parameters.* Springer Science & Business Media.

Allerhand, L.I. and Shaked, U. (2010). Robust stability and stabilization of linear switched systems with dwell time. *IEEE Transactions on Automatic Control*, 56(2), 381–386.

Allerhand, L.I. and Shaked, U. (2012). Robust state-dependent switching of linear systems with dwell time. *IEEE Transactions on Automatic Control*, 58(4), 994–1001.

Alves, R.M., Fioravanti, A.R., and Souza, M. (2019a). Sampled-data control of interval systems with guaranteed LQ/$\mathcal{H}_2$ performance. In *2018 14 Simpósio Brasileiro de Automação Inteligente (SBAI)*, 2292–2297.

Alves, R.M., Fioravanti, A.R., and Souza, M. (2019b). Sampled-data control of interval systems with guaranteed $\mathcal{H}_\infty$ performance. In *2019 18th European Control Conference (ECC)*, 2826–2831. IEEE.

Amorim, M.F., Gonçalves, A.P., Fioravanti, A.R., and Souza, M. (2018). Optimal $\mathcal{H}_2$ output-feedback control of sampled-data systems. *International Journal of Control*, 93, 2228–2238.

Başar, T. and Bernhard, P. (2008). *$\mathcal{H}_\infty$ optimal control and related minimax design problems: a dynamic game approach.* Springer Science & Business Media.

Bhattacharyya, S.P. and Keel, L.H. (1995). Robust control: the parametric approach. In *Advances in control education 1994*, 49–52. Elsevier.

Boyd, S., El Ghaoui, L., Feron, E., and Balakrishnan, V. (1994). *Linear matrix inequalities in system and control theory*, volume 15. Siam.

Daoyi, X. (1985). Simple criteria for stability of interval matrices. *International Journal of Control*, 41(1), 289–295.

de Oliveira, M.C., Bernussou, J., and Geromel, J.C. (1999). A new discrete-time robust stability condition. *Systems & Control Letters*, 37(4), 261–265.

Delgado-Romero, J., Suarez-Meza, H., Acosta-Villarreal, G., and Hernandez-Morales, E. (1997). Robust stability of interval matrices: simple tests. In *Circuits and Systems, 1997. Proceedings of the 40th Midwest Symposium on*, volume 1, 533–536. IEEE.

Furuta, K. and Phoojaruenchanachai, S. (1990). An algebraic approach to discrete-time $\mathcal{H}_\infty$ control problems. In *American Control Conference, 1990*, 3067–3072. IEEE.

Geromel, J.C. and Souza, M. (2015). On an LMI approach to optimal sampled-data state feedback control design. *International Journal of Control*, 88(11), 2369–2379.

Goebel, R., Sanfelice, R.G., and Teel, A.R. (2009). Hybrid dynamical systems. *IEEE control systems magazine*, 29(2), 28–93.

Hong, L., Lai, C.T., Kau, S.W., Lee, C.H., and Fang, C.H. (2006). Robust $\mathcal{H}_2$ control analysis and design for continuous-time interval state-space systems. *Journal of the Chinese Institute of Engineers*, 29(2), 221–228.

Ji, X. and Su, H. (2016). An extension of Petersen's lemma on matrix uncertainty. *IEEE Transactions on Automatic Control*, 61(6), 1655–1657.

Kharitonov, V. (1978). Asymptotic stability of an equilibrium position of a family of systems of differential equations. *Differential equations*, 14, 1483.

Lee, H.J., Xie, W.J., Chen, H.W., Kau, S.W., and Fang, C.H. (2006). Robust $\mathcal{H}_\infty$ control for interval descriptor systems. In *SMC*, 2926–2931.

Mansour, M. (1989). Robust stability of interval matrices. In *Decision and Control, 1989., Proceedings of the 28th IEEE Conference on*, 46–51. IEEE.

Mao, W.J. and Chu, J. (2006). Correction to "quadratic stability and stabilization of dynamic interval systems". *IEEE Trans. Autom. Control*, 51(8), 1404–1405.

Mao, W.J. (2004). Necessary and sufficient condition for robust $\mathcal{H}_\infty$ control of dynamic interval systems. In *Intelligent Control and Automation, 2004. WCICA 2004. Fifth World Congress on*, volume 1, 733–737. IEEE.

Mao, W.J. and Chu, J. (2003). Quadratic stability and stabilization of dynamic interval systems. *IEEE Transactions on Automatic Control*, 48(6), 1007–1012.

Parrilo, P.A. (2000). *Structured semidefinite programs and semialgebraic geometry methods in robustness and optimization.* Ph.D. thesis, California Institute of Technology.

Petersen, I.R. (1987). A stabilization algorithm for a class of uncertain linear systems. *Systems & Control Letters*, 8(4), 351–357.

Rump, S. (1999). INTLAB - INTerval LABoratory. In T. Csendes (ed.), *Developments in Reliable Computing*, 77–104. Kluwer Academic Publishers, Dordrecht.

Sezer, M.E. and Šiljak, D.D. (1994). On stability of interval matrices. *IEEE Trans. on Autom. Contr.*, 39(2), 368–371.

Shcherbakov, P. and Topunov, M. (2008). Extensions of Petersen's lemma on matrix uncertainty. *IFAC Proceedings Volumes*, 41(2), 11385–11390.

Souza, M. and Geromel, J.C. (2015). On a convex characterisation of stability and performance for hybrid linear systems. In *2015 54th IEEE Conference on Decision and Control (CDC)*, 6623–6628. IEEE.

Souza, M., Vital, G.W., and Geromel, J.C. (2014). Optimal sampled–data state feedback control of linear systems. *IFAC Proceedings Volumes*, 47(3), 5556–5561.

Stoustrup, J. (1993). The $\mathcal{H}_\infty$ control problem: A state space approach: Anton a. stoorvogel. *Automatica*, 29(4), 1149–1150.

Wang, K. and Michel, A.N. (1994). Necessary and sufficient conditions for the controllability and observability of a class of linear, time-invariant systems with interval plants. *IEEE Transactions on Automatic Control*, 39(7), 1443–1447.

Yang, G.H. and Lum, K.Y. (2005). Comments on "quadratic stability and stabilization of dynamic interval systems". *IEEE Transactions on Automatic Control*, 50(2), 276–277.

Zhang, D.Q., Zhang, Q.L., and Chen, Y.P. (2006). Controllability and quadratic stability quadratic stabilization of discrete-time interval systems —- an LMI approach. *IMA Journal of mathematical control and information*, 23(4), 413–431.

Zhou, K., Doyle, J.C., Glover, K., et al. (1996). *Robust and optimal control*, volume 40. Prentice hall New Jersey.