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# Safe Adaptive Cruise Control with Control Barrier Function and Smith Predictor ⋆

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**Abstract:** This work presents the development of Adaptive Cruise Control (ACC) applied to a vehicle. The ACC tracks a predefined controlled vehicle cruise speed, however when a leading vehicle with lower speed is encountered, the ACC must adapt the controlled vehicle speed to maintain a safe distance between the vehicles. The control strategy applied combines Control Lyapunov Function (CLF), related to performance/stability objectives and Control Barrier Function (CBF), related to safety conditions represented by a safe set. CLF and CBF are integrated with Quadratic Programming (QP) and a relaxation is used to make performance/stability objectives as a soft constraint and safety conditions as a hard constraint. The system model is based on a vehicle available at EPUSP and presents an input time-delay, that can degrade performance and stability. The input delay is compensated with a Smith Predictor. The initial results were obtained through numerical simulations and, in the future, the scheme will be implemented in the vehicle. The numerical simulations indicate that the proposed controller respect the performance/stability objectives and the safety conditions.

*Keywords:* Adaptive Cruise Control, Control Barrier Function, Control Lyapunov Function, Quadratic Programming, Smith Predictor.

## 1. INTRODUCTION

Autonomous driving is a research area of automotive engineering with many topics related to control. One of these topics is Advanced Driver Assistance Systems (ADAS). The ADAS are technologies developed to help driving easily and safely (Xiao and Gao (2010)). One of these technologies is Cruise Control (CC). CC tracks a predefined controlled vehicle cruise speed. In 1995, Adaptive Cruise Control (ACC) was improved in Japan (Vahidi and Eskandarian (2003), Ioannou and Chien (1993)). ACC must adapt a controlled vehicle speed to maintain a safe distance if a leading vehicle with lower speed is encountered (Liang and Peng (2000)). In order to detect the leading vehicle and obstacles, ACC considers sensors like radar or lidar.

The ACC problem involves performance/stability objectives (track a desired cruise speed) and safety conditions (maintain a safe distance to a leading vehicle). The performance objectives and the safety conditions can conflict in certain situations and the control must set priority for safety conditions.

Several control strategies presented in the literature have been applied to ACC, such as intelligent control (Kuyumcu and Sengor (2016)), sliding mode control (Ganji et al. (2014)) and model predictive control (MPC) (Li et al. (2017)), (Magdici and Althoff (2017)). Among these controllers, the MPC has been one of the most discussed. This is due to its capability to insert constraints to increase the performance and the robustness (Brugnolli et al. (2019b)).

The control strategy applied in this work for solving the ACC problem is proposed for Ames et al. (2019) and Ames et al. (2017). It combines Control Lyapunov Function (CLF), related to performance/stability objectives and Control Barrier Function (CBF), related to safety conditions represented by a safe set. CLF and CBF can be integrated with Quadratic Programming (QP) to satisfy both objectives and a relaxation is used to make the performance/stability objectives as a soft constraint and the safety conditions as a hard constraint (Ames et al. (2017)).

Several applications using this control strategy are proposed in the literature such as bipedal walking robot (Nguyen et al. (2016)), robotic manipulator (Rauscher et al. (2016)), two-wheeled inverted pendulum (Gurriet et al. (2018)), quadrotors (Wu and Sreenath (2016)) and multi-robot systems (Wang et al. (2017)).

This control strategy has already been applied to ACC problem. In Ames et al. (2017), the control strategy was applied to ACC and lane keeping problems separately and the results were presented through numerical simulations. In Xu et al. (2017), the control strategy was applied to ACC and lane keeping problems simultaneously and the results were obtained experimentally on robot testbeds. In Mehra et al. (2015), the control strategy was applied to ACC problem and the results were obtained experimen-

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tally on scale-model cars. However, in all those cases, the models have not transport delays associated to them.

The system model presented in this work is based on a vehicle available at EPUSP. This same vehicle was controlled with ACC in Brugnolli et al. (2019b) and Brugnolli et al. (2019a) using Dahlin Control and MPC, respectively. The vehicle dynamics presents an input time-delay, that can degrade performance and stability. The input delay is compensated with a Smith Predictor. The main works presented in the literature related to this control strategy typically deal with no input delay systems. The initial results were obtained through numerical simulations and, in the future, the technique will be implemented in the vehicle.

In section 2, the concepts of CLF, CBF and the QP applied to integrate CLF and CBF are presented. The system modeling is described in section 3. The Smith predictor is presented in section 4. In section 5, the ACC is detailed. Simulation results and conclusions are presented in section 6 and 7, respectively.

## 2. CONTROL STRATEGY

This section presents the concepts of CLF, CBF and the QP applied to integrate CLF and CBF in the proposed control strategy.

Throughout this work, we will assume an affine control system:

$$\dot{x} = f(x) + g(x)u, \tag{1}$$

with $f$ and $g$ locally Lipschitz, $x \in D \subset \mathbb{R}^n$ and $u \in U \subset \mathbb{R}^m$ is the set of admissible inputs.

### *2.1 Control Lyapunov Functions*

CLFs utilize Lyapunov functions together with inequality constraints on the derivative to establish entire classes of controllers that stabilize a given system (Ames et al. (2014)).

Suppose we have the control objective of (asymptotically) stabilizing the nonlinear control system (1) to a point $x^* = 0$. This can be achieved by finding a feedback control law $u = k(x)$ that drives a positive definite Lyapunov function $V(x)$ to zero (Ames et al. (2019)). That is, if

$$\exists\, u = k \;\; s.t. \;\; \dot{V} \leq -cV, \tag{2}$$

where

$$\dot{V} = L_f V + L_g V k, \tag{3}$$

then the system is stabilizable. The term $L_f V$ stands for $\frac{\partial V}{\partial x} f$, the term $L_g V$ stands for $\frac{\partial V}{\partial x} g$ and $c$ is a positive constant.

The function $V$ is a CLF if it is positive definite and satisfies (Ames et al. (2017)):

$$\inf_{u \in U} \left[ L_f V + L_g V u + cV \right] \leq 0. \tag{4}$$

The importance of this definition is that it allows us to consider the set of all stabilizing controllers for every point $x \in D$ (Ames et al. (2017), Sontag (1983)), such that:

$$K_{clf}(x) := \{ u \in U : L_f V + L_g V u + cV \leq 0 \}. \tag{5}$$

Freeman and Kokotovic introduced the min-norm controller, $u^*(x)$, defined pointwise as the element of $K_{clf}$ having minimum Euclidean norm (Freeman and Kokotovic (1996)). The min-norm controller can be interpreted as the solution of the QP:

$$\begin{aligned} u^*(x) = \underset{u \in \mathbb{R}^m}{\arg\min}\ & \frac{1}{2} u^T u \\ s.t. \;\; & L_f V + L_g V u + cV \leq 0. \end{aligned} \tag{6}$$

### *2.2 Control Barrier Functions*

Two important concepts related to control systems are liveness and safety. Liveness requires that “good” things eventually happen, such as asymptotic stability or tracking. Liveness is mathematically related to CLFs. Safety requires that “bad” things do not happen, such as invariance of a set $C$. Any trajectory starting inside an invariant set will never reach the complement of the set (Ames et al. (2019)). Safety is mathematically related to CBFs.

The concept of Barrier functions were first utilized in optimization (Boyd and Vandenberghe (2004)). Barrier functions are directly related to Lyapunov-like functions (Tee et al. (2009),Wieland and Allgower (2007)), multi-objective control and it guarantees invariance of sets.

There are two types of barrier functions. The reciprocal barrier function $B(x)$ is unbounded on the set $C$ boundary, i.e., $B \to \infty$ as $x \to \partial C$. The zeroing barrier function $h(x)$ vanishes on the set $C$ boundary, i.e., $h \to 0$ as $x \to \partial C$. In each case, if $B$ or $h$ satisfy Lyapunov-like conditions, then the forward invariance of $C$ is guaranteed (Ames et al. (2019)). The natural extension of a barrier function to a system with control inputs is a CBF (Wieland and Allgower (2007)). As shown for CLFs in (5), in CBFs we impose inequality constraints on the derivative to obtain entire classes of controllers that render a given set forward invariant.

As previously mentioned, safety can be related to invariance of a set, i.e., not leaving a safe set. In particular, we consider a set $C$ defined as the superlevel safe set of a continuously differentiable function $h : D \subset \mathbb{R}^n \to \mathbb{R}$ yielding (Ames et al. (2019)):

$$\begin{aligned} C &= \{ x \in D \subset \mathbb{R}^n : h \geq 0 \}, \\ \partial C &= \{ x \in D \subset \mathbb{R}^n : h = 0 \}, \\ \mathrm{Int}(C) &= \{ x \in D \subset \mathbb{R}^n : h > 0 \}. \end{aligned} \tag{7}$$

The definition of safety is given by:

*Definition 1.* Let $u = k(x)$ be a feedback controller such that (1) is locally Lipschitz. For any initial condition $x_0 \in D$ there exists a maximum interval of existence $I(x_0)$ such that $x(t)$ is the unique solution to (1) on $I(x_0)$. The set $C$ is forward invariant if for every $x_0 \in C$, $x(t) \in C$ for $x(0) = x_0$ and all $t \in I(x_0)$. The system (1) is safe with respect to the set $C$ if the set $C$ is forward invariant (Ames et al. (2019)).

After the set $C$ and safety has been defined, we can define Reciprocal Control Barrier Function (RCBF) $B$ and Zeroing Control Barrier Function (ZCBF) $h$.

*Definition 2.* Consider the control system (1) and the set $C \subset \mathbb{R}^n$ defined by (7). A continuously differentiable function $B : \mathrm{Int}(C) \to \mathbb{R}$ is called a RCBF if there exist class $\kappa$ functions $\alpha_1, \alpha_2, \alpha_3$, such that, for all $x \in \mathrm{Int}(C)$ (Ames et al. (2017)),

$$\frac{1}{\alpha_1(h)} \leq B \leq \frac{1}{\alpha_2(h)}, \tag{8}$$

$$\inf_{u \in U} \left[L_f B + L_g B u - \alpha_3(h)\right] \leq 0. \tag{9}$$

Given a RCBF $B$, for all $x \in \mathrm{Int}(C)$, define the set (Ames et al. (2017))

$$K_{rcbf}(x) = \left\{u \in U : L_f B + L_g B u - \alpha_3(h) \leq 0\right\}. \tag{10}$$

Considering control values in this set, the forward invariance of $C$ is guaranteed by the following corollary:

*Corollary 1.* Consider a set $C \subset \mathbb{R}^n$ defined by (7) and let $B$ be an associated RCBF for the system (1). Then any locally Lipschitz continuous controller $u : \mathrm{Int}(C) \to U$ such that $u \in K_{rcbf}$ will render the set $\mathrm{Int}(C)$ forward invariant (Ames et al. (2017)).

*Definition 3.* Consider the control system (1) and the set $C \subset \mathbb{R}^n$ defined by (7) for a continuously differentiable function $h : \mathbb{R}^n \to \mathbb{R}$. The function $h$ is called a ZCBF defined on set $D$ with $C \subseteq D \subset \mathbb{R}^n$, if there exists an extended class $\kappa$ functions $\alpha$ such that

$$\sup_{u \in U} \left[L_f h + L_g h u + \alpha(h)\right] \geq 0. \tag{11}$$

Given a ZCBF $h$, for all $x \in D$, define the set (Ames et al. (2017))

$$K_{zcbf}(x) = \left\{u \in U : L_f h + L_g h u + \alpha(h) \geq 0\right\}. \tag{12}$$

With this definition, we have the following corollary:

*Corollary 2.* Consider a set $C \subset \mathbb{R}^n$ defined by (7) and let $h$ be an associated ZCBF for the system (1). Then any locally Lipschitz continuous controller $u : D \to U$ such that $u \in K_{zcbf}$ will render the set $C$ forward invariant (Ames et al. (2017)).

In this paper, we will consider the following relationship between RCBF and ZCBF:

$$B = \frac{1}{h}. \tag{13}$$

### *2.3 Integrating CLFs and CBFs Through QP*

Using QP, we can integrate performance/stability objectives (represented by CLFs) and safety conditions (represented by CBFs). By relaxing the constraint represented by the CLF condition (4), and adjusting the weight on the relaxation parameter $\delta$, the QP can mediate the tradeoff between performance/stability and safety, making the performance/stability objectives as a soft constraint and the safety conditions as a hard constraint (Ames et al. (2017)).

Given a RCBF $B$ associated with a set $C$ defined by (7) and a CLF $V$, they can be integrated into a single controller through a QP such as (Ames et al. (2017)):

$$\begin{aligned} \mathbf{u}^*(x) = &\underset{\mathbf{u}=(u,\delta) \in \mathbb{R}^m \times \mathbb{R}}{\arg\min} \ \frac{1}{2}\mathbf{u}^T H(x)\mathbf{u} + F(x)^T \mathbf{u} \\ s.t.\ \ & L_f V + L_g V u + cV - \delta \leq 0 \\ & L_f B + L_g B u - \alpha(h) \leq 0, \end{aligned} \tag{14}$$

where $c$ was defined in (2), $\alpha$ was defined in (11), $H(x) \in \mathbb{R}^{(m+1)\times(m+1)}$ is positive definite and $F(x) \in \mathbb{R}^{m+1}$.

## 3. SYSTEM MODELING

In this work, we present the control strategy applied to a system model based on a vehicle available at EPUSP. The vehicle considered is a Volkswagen Polo Sedan model with spark-ignition engine 2.0 L. The vehicle is controlled by an open-source electronic control unit (ECU) and was tested on an inertial dynamometer from NAPRO company (Brugnolli et al. (2019b)).

Although the vehicle has a customized ECU, it does not have an electronic brake system. Thus, the brake pedal was discarded as a control input. The control input $u(t)$ chosen for the vehicle is the accelerator pedal, which range varies from 0% (not pushed) to 100% (fully pushed) (Brugnolli et al. (2019b)). The system output $y(t)$ considered for the ACC is the translational vehicle speed.

The system identification, obtained in Brugnolli et al. (2019b), is given by:

$$\frac{Y(s)}{U(s)} = \frac{0.085s + 0.1788}{s + 0.2041} e^{-0.5s}. \tag{15}$$

The vehicle dynamics presents an input time-delay of 0.5 s, represented by $e^{-0.5s}$, that can degrade performance and stability. To compensate the input delay, it is applied a Smith Predictor, that will be described in next section.

The state space representation without considering delay is given by:

$$\begin{aligned} \dot{x}(t) &= -0.2014x(t) + 0.1615u(t) \\ y(t) &= x(t) + 0.085u(t), \end{aligned} \tag{16}$$

where $x$ is the system state.

## 4. SMITH PREDICTOR

Control systems with time-delay is a topic analyzed by engineers and scientists for decades. Time-delay is commonly seen in engineering applications, such as thermic and chemical. The delay can be found in system states, control input or system output and can be generated by sensors and actuators in control loop. Time-delay degrades performance and stability of control systems.

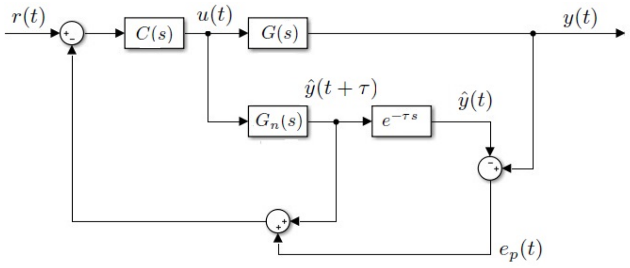



Fig. 1. Block diagram of the Smith predictor

A common alternative to project control systems with time-delay is to use Padé approximation to represent time-delay. This can generate considerable increase in system order and sensibility to perturbations. One of the most traditional structures and widely used in industry to compensate time-delay is the Smith predictor, proposed by O. J. Smith (Smith (1957)).

Smith predictor is a control structure that shifts the delay outside the control loop, so that the controller acts on the process as if the closed loop dynamics are not delayed (Normey-Rico and Camacho (2008)). When Smith predictor was developed, was supposed a constant time-delay and an exact system model. Therefore, Smith predictor is very sensitive to model uncertainties. Besides that, it is only applied to SISO (Single-Input-Single-Output) and stable systems.

Fig. 1 shows a block diagram of Smith predictor. $C(s)$ is the controller, $G_n(s)$ is the nominal system model without time-delay, $e^{-\tau s}$ represents the time-delay $\tau$ and $G(s)$ is the real system model. $r(t)$ is the reference input, $u(t)$ is the control input, $y(t)$ is the system output, $\hat{y}(t+\tau)$ and $\hat{y}(t)$ are the predicted values for system output.

The closed loop transfer function is given by:

$$\frac{Y(s)}{R(s)} = \frac{C(s)R(s)}{1 + C(s)\left[G(s) - G_n(s)e^{-\tau s} + G_n(s)\right]}. \tag{17}$$

If there is no error between the nominal system model and the real system model, i.e., $G_n(s)e^{-\tau s} = G(s)$, the prediction error $e_p(t)$ will be zero. Therefore, (17) becomes:

$$\frac{Y(s)}{R(s)} = \frac{C(s)G(s)}{1 + C(s)G_n(s)} \tag{18}$$

and the delay is eliminated. Thus, the feedback signal will be a prediction of the process output and the controller $C(s)$ can be designed considering the process without time-delay.

In this work, the system model (15) presents a delay represented by $e^{-0.5s}$. Using Smith predictor, the controller design can be done disregarding time-delay. The design procedure is done considering the system in structure (1), so it was used the system model (16).

## 5. ADAPTIVE CRUISE CONTROL

The ACC tracks a predefined controlled vehicle cruise speed $v_c$ $(r(t))$, that must be maintained, as in common CC systems. This problem can be expressed as a performance/stability objective and related to a CLF. However when a leading vehicle with lower speed $v_l$ is encountered, the ACC must adapt controlled vehicle speed, represented as $y$ in (16), to maintain a safe distance $D$ between the vehicles. This distance is determined with sensors such as radar or lidar. This problem can be expressed as a safety condition and related to a CBF. If the leading car increases its speed or leaves the lane and there is no conflict between the safe distance and the desired cruise speed, ACC automatically increases controlled vehicle speed (Ames et al. (2017)). So the QP-based controller (14) can be applied. The weight on the relaxation parameter $\delta$ must be chosen so that the CLF becomes a soft constraint, the CBF becomes a hard constraint and the system tracks the cruise speed $v_c$ while maintain a safe distance $D$ from the leading vehicle. The constraints do not need to be simultaneously satisfiable. Fig. 2 shows a block diagram of the control strategy applied.

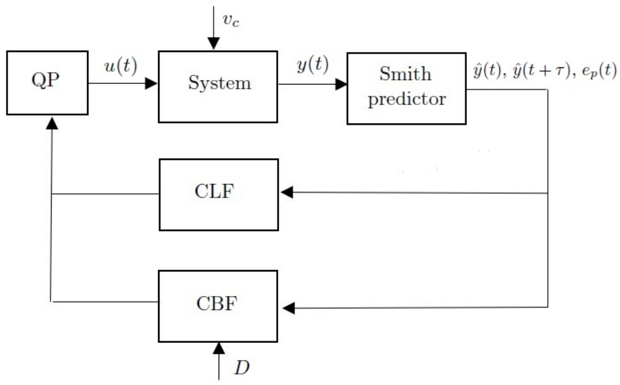



Fig. 2. Block diagram of the control strategy applied

The CLF $V$ that characterizes the soft constraint is given by:

$$V = \left[(\hat{y}(t+\tau) + e_p(t)) - v_c\right]^2, \tag{19}$$

where $\hat{y}(t+\tau)$, $e_p(t)$ and $v_c = r(t)$ are shown in Fig. 1.

The CBF $h$ that characterizes the hard constraint is given by:

$$h = D - \tau_d(\hat{y}(t+\tau) + e_p(t)), \tag{20}$$

where $D = (x_l - x_c)$ is the distance between the leading vehicle and the controlled vehicle, and $\tau_d$ is the desired time headway, which is an estimation of a human driver reaction time.

## 6. SIMULATION RESULTS

For the evaluation of the ACC with CBF and Smith predictor, numerical simulations results were obtained with Matlab/Simulink. It is important to highlight that before applying the control with Smith predictor, the delay time $e^{-0.5s}$ presented in system model (15) was estimated using Padé approximation and the control was applied without Smith predictor. The system became unstable for all simulation tests using the control strategy proposed. Besides that, as described in Orosz and Aames (2019), when time-delays are introduced in system dynamics they may still render the system unsafe, even when safety is ensured for the non-delayed system. One way to deal with that is the use of Smith predictor.

We consider three simulation experiments, shown in Figs. 3, 4 and 5. As mentioned anteriorly, $y$ is the controlled vehicle speed or the system output, $v_c$ is the predefined vehicle cruise speed or the reference input $r$ as shown in Fig. 1 and $v_l$ is the leading vehicle speed. The objective is that $y$ tracks the reference $v_c$ when the distance $D$ between the vehicles guarantees a safety behaviour and $y$ has its value reduced otherwise. The control input $u$, related to the vehicle accelerator pedal, cannot be greater than 100% or smaller than 0%.

For this problem, the QP (14) was represented as(Ames et al. (2017)):

$$\begin{aligned} \mathbf{u}^*(x) = & \underset{\mathbf{u}=(u,\delta)^T\in\mathbb{R}^2}{\arg\min}\ \frac{1}{2}\mathbf{u}^T H_{acc}\mathbf{u} + F_{acc}^T\mathbf{u} \\ s.t.\ \ & A_{clf}\mathbf{u} \le b_{clf} \\ & A_{cbf}\mathbf{u} \le b_{cbf}, \end{aligned} \tag{21}$$

where

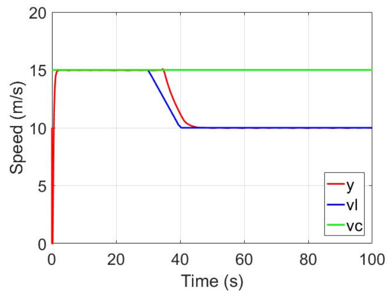
$$A_{clf} = [L_gV, -1]\,,\ b_{clf} = -L_fV - cV, \tag{22}$$

and

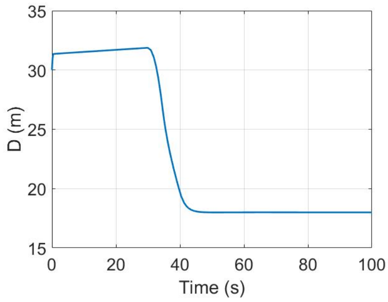
$$A_{cbf} = [L_gB, 0]\,,\ b_{clf} = -L_fB + \alpha(h). \tag{23}$$

The QP (21) was solved using a closed-form expression demonstrated in Ames et al. (2017). Besides that, the QP can be solved numerically using Matlab function quadprog or Hildreth's QP procedure (Hildreth (1957)). It was chosen $\alpha(h) = \frac{\gamma}{B}$, where $\gamma > 0$ and $B$ was related to $h$ using (13). The numerical values of the parameters used in all simulations were $\tau_d = 1.8$ (Ames et al. (2017)), $\gamma = 10000$, $c = 1000$, $F_{acc} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $H_{acc} = \begin{bmatrix} 1 & 0 \\ 0 & p_\delta \end{bmatrix}$, where $p_\delta = 100$ is the weight on the relaxation parameter $\delta$.

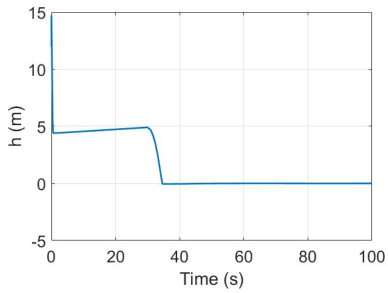
In the simulation 1, the initial conditions adopted were $x_{0l} = 30\ m$ (leading vehicle initial position), $x_{0acc} = 0\ m$ (controlled vehicle initial position) and $y_0 = 10\ m/s$ (controlled vehicle initial speed). The cruise speed was $v_c = 15\ m/s$ and the leading vehicle speed $v_l$ is shown in Fig. 3. The simulation results show that the controlled vehicle reaches the cruise speed, respecting the CLF constraint, and when the leading vehicle speed reduces, the CBF constraint acts in the QP and reduces the controlled vehicle speed, so that the safety requirements are satisfied. It is also important to highlight the influence of the weight on the relaxation parameter $\delta$ so that the CLF becomes a soft constraint and the CBF a hard constraint. The parameter $c$ exerts influence on the convergence rate of the cruise speed when CBF is not active and the parameter $\gamma$ exerts influence on the convergence rate to the safe set when CBF is active. The ZCBF $h$ always satisfies the condition (7) for the safe set and the distance $D$ between the vehicles is guaranteed. The control effort $u$ was great



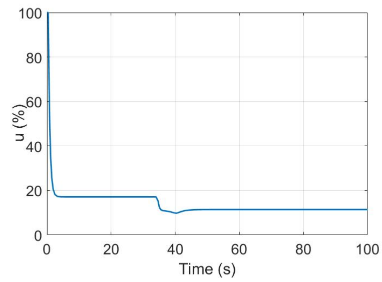

(a)




(b)




(c)




(d)

Fig. 3. Simulation 1

in the beginning due to the convergence rate $c$, however it is reduced afterwards.

In the simulation 2, the initial conditions were the same for the simulation 1, except that $x_{0l} = 20\ m$, the cruise speed was the same and the leading vehicle speed is shown in Fig. 4. The simulation results show that the controlled vehicle speed increases progressively to reach the cruise speed due to the CLF constraint. However, the controlled vehicle speed never exceeds the leading vehicle speed due to the CBF constraint. When the leading vehicle speed reaches the cruise speed, the controlled vehicle speed does not increase, because it cannot exceed the cruise speed.

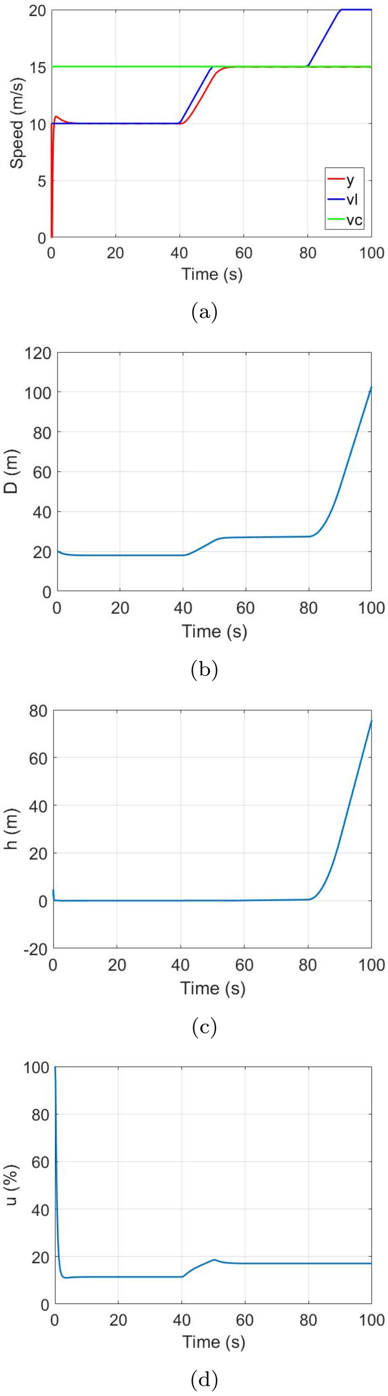



Fig. 4. Simulation 2

The CBF $h$ and the variables $D$ and $u$ follow the same considerations of the simulation 1.

In the simulation 3, the initial conditions adopted were $x_{0l} = 150\ m$, $x_{0acc} = 0\ m$ and $y_0 = 18\ m/s$. The cruise speed was $v_c = 22\ m/s$ and the leading vehicle speed $v_l$ is depicted in Fig. 5. The simulation results show that, between 0 $s$ and 10 $s$, the controlled vehicle reaches the cruise speed due to the CLF constraint. Between 10 $s$ and 20 $s$, the controlled vehicle speed decreases in order to satisfy the CBF constraint. Between 20 $s$ and 80 $s$, the controlled vehicle reaches the cruise speed again, but does not exceed it. Finally, after 80 $s$, the controlled vehicle

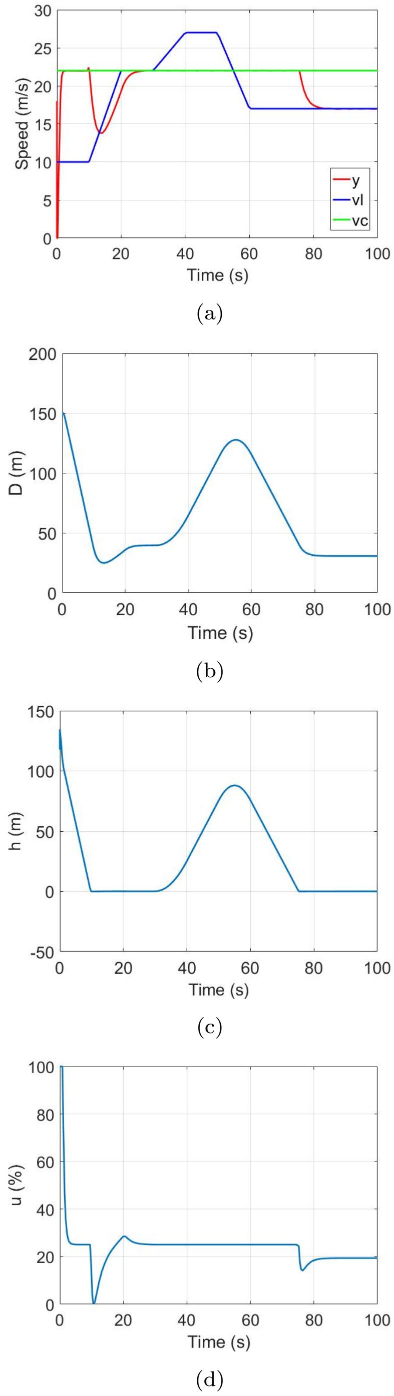



Fig. 5. Simulation 3

speed decreases again to satisfy the CBF constraint. In the simulation 1, the delay in the speed decrease is lower because $D$ is lower. In the simulation 3, the delay in the speed decrease is higher because $D$ is higher. The CBF $h$ and the variables $D$ and $u$ follow the same considerations of other simulations.

## 7. CONCLUSIONS

This work presents the development of ACC applied to a vehicle using a control strategy that combines CLF, related to performance/stability objectives and CBF, related to safety conditions represented by a safe set. CLF and CBF are integrated with QP and a relaxation is used to make the performance/stability objectives becomes a soft constraint and the safety conditions a hard constraint. The system model is based on a vehicle available at EPUSP and presents an input time-delay, that can degrade performance and stability. The input delay is compensated by a Smith Predictor. The numerical simulations indicate that the proposed controller respect the performance/stability objectives and the safety conditions. In all simulation experiments the controlled vehicle reaches the cruise speed with a convergence rate determined by $c$, due to the CLF constraint, and when the leading vehicle speed decreases, the CBF constraint acts on the QP and reduces the controlled vehicle speed, so that the safety requirements are satisfied. The weight on the relaxation parameter $\delta$ makes the CLF a soft constraint and the CBF a hard constraint and the parameter $\gamma$ exerts influence on the convergence rate to the safe set when the CBF is active. In all cases, the ZCBF $h$ satisfies the safety condition and the distance $D$ between the vehicles is guaranteed. As future works, this control scheme will be implemented in the real vehicle.

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