Optimal Control Concerns Regarding the COVID-19 (SARS-CoV-2) Pandemic in Bahia and Santa Catarina, Brazil [⋆]

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Abstract: The COVID-19 pandemic is the profoundest crisis of the 21[rst] century. The SARS-CoV-2 virus arrived in Brazil around March, 2020, and its social and economical backlashes have since been catastrophic. In this paper, we investigate how Model Predictive Control (MPC) can be used to plan appropriate social distancing policies that mitigate the pandemic effects in Bahia and Santa Catarina (Brazil), two states of very different regions, cultures and population demography. In addition, the parameters of Susceptible-Infected-Recovered-Deceased (SIRD) models for these two states are identified using an optimization procedure. The control input to the process is a social isolation guideline passed to the population. Two MPC strategies are designed: a) a centralized MPC, which coordinates a single control policy for both states; and b) a distributed strategy, for which a single optimization is solved for each state. Simulation results are shown to illustrate and compare both control strategies. Discussions are drawn regarding the effectiveness of MPC to guide social distancing measures in future pandemics.

Resumo: A pandemia COVID-19 apresenta-se como a mais profunda crise do século 21. O vírus SARS-CoV-2 chegou ao Brasil em Março de 2020 e seus efeitos sociais e econômicos têm sido catastróficos desde então. Neste artigo, investigamos como Controle Preditivo baseado em modelo (CPM) pode pautar políticas públicas eficientes para o distanciamento social, visando mitigar os efeitos da pandemia, no que diz respeito aos estados brasileiros da Bahia e de Santa Catarina. Abordamos o problema através da identificação dos parâmetros do modelo SIRD via otimização. A variável manipulada é a meta de isolamento social passada à população; duas estratégias de CPM são desenvolvidas: a) um CPM centralizado, que coordena o isolamento em ambos os estados através do mesmo sinal de controle; e b) um CPM distribuído, no qual dois problemas de otimização são resolvidos, resultando em uma lei de controle individual para cada estado. Resultados de simulação ilustram, comparam e discutem ambas estratégias.

Keywords: Model Predictive Control; COVID-19; Social isolation; SIRD Model; Identification.
Palavras-chaves: CPM; COVID-19; Isolamento social; Modelo SIRD; Identificação.

1. INTRODUCTION

The COVID-19 pandemic is the definite crisis of the 21[rst] century. The SARS-CoV-2 virus was first registered in humans in China, by the end of 2019. This virus causes a severe acute respiratory syndrome, which may lead to death. The spread of the contagion has been very rapid, around the globe; by mid-Jun, this disease had caused the death of over 410000 people. Vaccines are currently being developed, but are previewed to be ready only by mid-2021 (Lurie et al., 2020). Therefore, in order to address

and mitigate the pandemic effects, global scientific efforts are being provided (Bedford et al., 2019), while countries have adopted social distancing measures, seeking to avoid the spread of the virus (Adam, 2020).

Much more than presenting drastic effects on health systems, this pandemic has also caused social and economical backlashes, especially in countries with larger social inequalities. In Brazil, one of the leading countries in numbers of COVID-19 cases and deaths, catastrophic outcomes are already felt (The Lancet, 2020). The effects of the virus on populations with poorer access to health systems

and sanitation facilities [1] are strikingly stronger (San Lau et al., 2020).

In this paper, the Brazilian context is taken into account (Werneck and Carvalho, 2020): Brazil is a continent-sized country with 26 federated states, which have been choosing different social distancing measures since mid-March. The federal government is reluctant to implement nation-wide policies, claiming that the negative economic effects are too steep and that social distancing is an erroneous choice (The Lancet, 2020). The government suggests that the economy cannot stop and that herd immunity is the only viable solution to this pandemic. However, the expectations and scenarios previewed on recent literature are ruinous (Rocha Filho et al., 2020; Morato et al., 2020b). In order to account for locations which have been following very different paths regarding COVID-19, the data [2] from two states is considered: from (*i*) Bahia (BA), which lies on the northeast sea-side and is larger than Spain (in total surface), and from (*ii*) Santa Catarina (SC), which is in the south of the country and is three times larger than Belgium. We highlight that these states have very distinct social, historical and cultural background, and have exhibited different behaviours facing COVID-19.

The main concept behind social distance is to prevent health systems from becoming saturated due to large amounts of COVID-19 patients being treated at the same time. With social distancing policies, hospital bed shortages do not occur, since the large original peak of infections becomes distributed over time. Even though a strong public health system is available in Brazil, many states were already exhibiting a near-collapse situation by May, with over 95 % of Intense Care Unit (ICU) hospital beds occupied with COVID-19 patients. Since BA and SC have very different demographic conditions; by the end of June, BA had roughly 70 % of ICU occupancy, whereas SC had a lower rate, estimated at 22 %, although both states applied strict social isolation rules at an initial stage (around March). A fundamental issue regarding social distancing is to perform these interventions at the correct moments and for the correct duration. Well-designed distancing policies should help to mitigate the contagion spread, avoiding the saturation of the heath systems, altogether minimizing social and economic side-effects.

Motivated by the previous discussion, the problem of defining optimal social distancing policies is investigated in this work, regarding the application for BA and SC. For such, the Model Predictive Control (MPC) (Camacho and Bordons, 2013) framework is used, since it can conveniently consider the effect of lockdown/quarantine measures as the constraints of a minimization problem (regarding the number of infected individuals). Furthermore, we compare and discuss the differences regarding the use of a central-ized MPC scheme, which generates a single control law for both states, and of a distributed MPC, which solves two separate procedures, with individual laws for each state.

Based on a Susceptible-Infected-Recovered-Deceased (SIRD) model, adjusted for the COVID-19 pandemic (Bastos and Cajueiro, 2020), which embeds the effects of social distancing measures (Section 2), the main contributions of this paper are the following:

- An optimization procedure is developed in order to minimize a Least-Squares criterion and estimate the parameters of the virus infection/spread model, considering both states (BA, SC). Uncertainty in the available datasets is considered (Section 3).
- Based on the obtained models, two different MPC strategies are designed in order to determine when to apply (or not) social distancing measures (Section 4).
- Simulation results illustrate the results obtained with both strategies; discussions are drawn in order to evaluate how optimal control can be used to guide social distancing in pandemic situations (Sections 5 and 6). The discussion is formally based on comparisons regarding the COVID-19 spread in both states of BA and SC and how the different control strategies can address the goal of mitigating the viral spread.

2. SARS-COV-2 PANDEMIC SPREAD MODEL

Recent literature (Peng et al., 2020; Kucharski et al., 2020) demonstrates that the infection rate and evolution dynamics of the SARS-CoV-2 virus can be adequately described by Susceptible-Infected-Recovered-Deceased (SIRD) models. In this Section, the SIRD model from (Keeling et al., 2008) is detailed. An additional dynamic variable models the population's response to isolation policies, as proposed in (Bastos and Cajueiro, 2020; Morato et al., 2020b).

The SIRD models, as in Eq. (1), describes a contagion spread in a population which is split into four non-intersecting classes:

- Susceptible people [3] $S(t)$, who are prone to contract the virus;
- Infected individuals $I(t)$, which are currently sick;
- Recovered people $R(t)$, who have already recovered from the SARS-CoV-2;
- and Deceased individuals $D(t)$, who have died due to the contagion.

$$[\textbf{SIRD}] \begin{cases} \dfrac{dS(t)}{dt} = -(1-\psi(t))\dfrac{\beta I(t)S(t)}{N(t)}, \\[2mm] \dfrac{dI(t)}{dt} = (1-\psi(t))\dfrac{\beta I(t)S(t)}{N(t)} - \dfrac{\gamma I(t)}{1-\rho}, \\[2mm] \dfrac{dR(t)}{dt} = \gamma I(t), \\[2mm] \dfrac{dD(t)}{dt} = \dfrac{\rho}{1-\rho}\gamma I(t), \end{cases} \quad (1)$$

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[1] A very illustrative example of these differences can be seen in the city of São Paulo: the city hall released a technical note by the end of April stating that the observed mortality rate is 10 times larger in neighborhoods of the city with worse social conditions and precarious housing. See `https://www.prefeitura.sp.gov.br/cidade/secretarias/upload/saude/PMSP_SMS_COVID19_Boletim\%20Quinzenal_20200430.pdf`.

[2] We note that this paper was written in June, 2020, with the available data until then.

[3] In this paper, we do not consider the effects of demographic variations. Despite recent discussion regarding the possibilities of reinfection (Del Rio and Malani, 2020), we assume that the recovered individuals will not be reinfected (at least for simplicity purposes), i.e. an individual does not contract the disease twice.

In Eq. (1), β stands for the probability of disease transmission per individual; γ stands for the recovery rate, impacting on the amount of individuals that "leave" from the infected class; ρ denotes the observed mortality rate of the virus. Social distancing measures are expressed through $\psi(t)$, which denotes the average amount of people circulating freely, i.e. $\psi = 1$ stands for a complete isolation condition ($100\,\%$ quarantine, when contacts are reduced to zero), whereas $\psi = 0$ means no social distancing.

The size of the total population exposed is denoted $N(t)$; it holds that $N(t) = N_0 - D(t)$, being N_0 is the initial population size (prior to the contagion). In this work, for simplicity, we assume a constant $N(t)$, considering that the natural deaths balances the amount newborns. We note that the term $\beta I(t)/N(t)$ gives the average number of contacts sufficient for viral transmission to one susceptible individual, per unit of time, while $(\beta I(t)/N(t))S(t)$ gives the number of new cases w.r.t. the amount of susceptible individuals (those that are "available for infection"), per unit of time.

An essential concept in epidemiology theory is the effective reproduction number, usually denoted by $R_t(t)$. This index is able to measure the average effective potential of transmission of a given disease at a given moment t. In practical analysis, it represents how many cases are expected to be generated by a single primary case, in a population which all invidious are susceptible. From the systems theory viewpoint, R_t represents the epidemic velocity. If $R_t > 1$ the infection is spreading and the number of infected people increases, which typically happens at the beginning of the epidemic; otherwise, if $R_t < 1$, it means that more individuals "leave" from the infected class, either recovering or dying and, thereby, the epidemic is ceasing. This effective reproduction number R_t is affected by different factors, including biological characteristics from the virus itself, and governments policies to control the number of susceptible people, which can be reduced by social distancing.

In order to calculate $R_t(t)$, we assumed that, at the beginning of the pandemic, $S \approx N$. Considering the parameters β, γ and ρ from Eq. 1, $R_t(t)$ is, then, approximately given as follows, being ψ the observed social distancing factor:

$$R_t(t) \approx \frac{(1 - \psi(t))\beta(1 - \rho)}{\gamma}. \quad (2)$$

The following inequality can be directly checked to verify if the disease is spreading in a given moment t: $\frac{(1-\psi(t))\beta(1-\rho)}{\gamma} < 1$. The social distancing ratio $\psi(t)$ directly affects the contagion spread, which is a fundamental aspect of the control strategy proposed herein.

In order to take into account the effect of public health policies, enacted by local governments to mitigate the effects of the COVID-19 pandemic, a model is included for the dynamics of $\psi(t)$, which concatenates to the SIRD dynamics. This time-varying parameter ψ models not only social isolation, but also incentives to use of masks and other measures which contain the contagion spread. The dynamics for ψ are obtained according to the first-order heuristics proposed and validated in (Bastos and Cajueiro, 2020; Morato et al., 2020b):

$$\frac{d\psi(t)}{dt} = \frac{1}{\varrho}(u(t)\psi_\infty - \psi(t)), \quad (3)$$

for which $u(t)$ is the control variable defined within $[0\,,\,1]$ that sets the social distancing goal. Note that $\psi(t)$ converges to ψ_∞ with a settling period of 3ϱ, with $u = 1$. It follows that ψ_∞ is a factor that represents the maximal observed effect of social distancing in a given place. For larger values of ψ_∞ (closer to 1), when hard quarantine measures are enacted (u closer to 1), the SIRD model dynamics (with $\psi(t) \to \psi_\infty$) are slowed down, exhibiting a smaller peak of infections and number of deaths. Furthermore, larger ψ values directly influence the transmission spread factor R_t. The main control goal is to ensure $\frac{(1-\psi(t))\beta(1-\rho)}{\gamma} < 1$, in such way that the contagion ceases.

3. IDENTIFICATION PROCEDURE

For the sake of practical purposes, model parameters are estimated using real data employed in a similar technique to the estimation scheme presented by (Bastos and Cajueiro, 2020).

An Ordinary Least Square (OLS) method is applied to estimate parameters β, γ and ρ from Eq. 1. For the considered application, the OLS procedure is performed over the official data provided by Brazilian Ministry of Health, considering the first confirmed case in each state (06/03/20 for BA and 13/03/2020 for SC) until the last data point of 16/06/2020. The complete data-set can be found in the open-source repository $Brasil.IO$[4]. We note that the official data (disclosed daily) corresponds the total (cumulative) number of infections, recoveries and deaths; in order to compute the number of daily (active) infections, we use $I(t) = I_c(t) - R(t) - D(t)$, being $I_c(t)$ the cumulative number of infections. This is the same procedure as done in (Morato et al., 2020b; Bastos and Cajueiro, 2020).

Firstly, we stress that the SIRD model is identified considering the estimated social distancing indexes observed in the different states of Brazil, as presented in (Jorge et al., 2020). The initial condition for ψ is taken as 0.2, which is a baseline/natural social distancing factor that corresponds to minimal measures by the population, such as the use of masks, recurrent hand sanitation and so on. A thorough discussion on this matter is presented in (Morato et al., 2020a). Furthermore, for the sake of simplicity of the optimization algorithm formulation, the differential equations $dI(t)/dt$ and $dD(t)/dt$ are modified to yield linear dependence w.r.t. the identified parameters, as follows:

$$\frac{dI(t)}{dt} = \frac{(1 - \psi(t))\beta I(t)S(t)}{N(t)} - \gamma I(t) - \alpha I(t), \quad (4)$$

$$\frac{dD(t)}{dt} = \alpha I(t), \quad (5)$$

being $\alpha = \rho\gamma/(1 - \rho)$.

Then, an optimization problem is formulated taking into account the minimum square error between real data (as disclosed by the Brazilian Ministry of Health) and the estimated SIRD dynamic model, w.r.t. parameters β, γ

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[4] Refer to https://brasil.io/dataset/covid19/.

and α. For each variable $I(t)$, $R(t)$ and $D(t)$, the following estimation error is established:

$$E_I = (I(t) - \hat{I}(t, \beta, \gamma, \alpha))^2, \qquad (6)$$

$$E_R = (R(t) - \hat{R}(t, \beta, \gamma, \alpha))^2, \qquad (7)$$

$$E_D = (D(t) - \hat{D}(t, \beta, \gamma, \alpha))^2, \qquad (8)$$

being $\hat{I}$, $\hat{R}$ and $\hat{D}$ the variables estimated with the SIRD model. With these variables, the complete OLS optimization problem is formulated as follows:

$$\min_{\beta, \gamma, \alpha} \sum_{i=t_i}^{i=t_i+t_{opt}} (k_1 E_I(i) + k_2 E_R(i) + k_3 E_D(i)), \qquad (9)$$

$$\text{s.t.:} \qquad 0 \leq \beta \leq 0.65, \qquad (10)$$

$$0 \leq \gamma \leq 0.7, \qquad (11)$$

$$0 \leq \alpha \leq 0.2. \qquad (12)$$

This optimization begins with initial conditions $\beta = 0.5$, $\gamma = 0.5$ and $\alpha = 0.1$. The tuning parameters k_1, k_2 and k_3 are taken as positive weighting values, used to normalize the total optimization cost w.r.t. E_I, E_R and E_D.

Then, in order to solve this problem, we consider a sliding optimization horizon of fixed size $(t_{opt} - t_i)$, since the available data may not represent the real trend of the epidemic dynamics. This is due to the fact that uncertainties are present in the data available regarding the reported cases and, hence, this data corruption may deteriorate the overall parameters estimation and the obtained model prediction. Moreover, the initial data points usually embed substantial variations in the number of cases reported due to the absence of testing when the viral spread starts. Also, as a natural consequence of pandemic, infections, recovered and mortality rates start with strong variations at the beginning of the spread, until convergence to a steadier behavior is observed.

The horizon window $(t_{opt} - t_i)$ is smaller than the total available amount of data and rolls along the complete number of daily samples. We have found that the best model-data fitting results are achieved with a windows between 5 and 10 days [5], which is coherent with the viral dynamics, since the average incubation period of the SARS-CoV-2 virus is of 5 days (and, at most, 14 days). Hence, we identify piece-wise constant parameters values β, γ and α, for each window $(t_{opt} - t_i)$. The procedure starts with $t_i = 1$ as the first day of available and follows with $t_i^+ = t_i + t_{opt} + 1$; the optimal parameters identified on the previous window (β_{opt}, γ_{opt} and α_{opt}) are set as initial conditions for following loop, as in any moving-horizon optimization strategy.

4. AN OPTIMAL SOCIAL DISTANCING METHOD

Based on the SIRD model detailed in Sec. 2 and the parameter estimates found through the optimization procedure from Sec. 3, two different optimal control procedures are proposed, aiming to guide social distancing policies in SC and BA. These procedures are set within an MPC

[5] In practice, we use a fixed horizon size of 6 days, as discussed in Sec. 5.

framework, in centralized and distributed paradigms, as detailed in the sequel.

We recall that MPC operates in a discrete-time paradigm. Therefore, since new measurements of infections and deaths are available every day, and the contagion dynamics are slow (in the order of days), the SIRD model and the social distancing dynamics from Eqs. (1) and (3). are Euler-discretized with $T_s = 1$ day. The discrete sampling instants are denoted as $k = t/T_s$.

The MPC procedures are designed through a minimization problem, where performance goals are delimited as quadratic maps. Regarding the COVID-19 situation, the control objective is evident: minimize the number of active infections $(I(k))$ while altogether reducing the social distancing efforts $(u(k))$. We stress that long-term rigid social distancing provokes devastating economic and psychological effects and, thus, such measures should be kept for the smallest duration possible, e.g. (Eichenbaum et al., 2020).

One cannot expect to increase of decrease social isolation instantaneously. As observed in practice, the population takes some time to respond to new social isolation measures, adapting to the enacted paradigm. Therefore, in consonance with the dynamic Eq. (3) and with real isolation policies put in practice in Brazil, we consider that the control action u can vary ± 0.05 per day, which means that actual isolation factor will increase/decrease with a rate of, at most, $5\%/\text{day}$.

We note that this is a preliminary assumption, since the actual implemented social isolation policy should be a "translation" of the control signal u into a feasible set of actions. These actions could represent different guidelines, such as: total isolation, with no one leaving their homes (for $u = 1$), a partial isolation, with people allowed to leave only for short periods, with masks (for $u = 0.9$), and so forth, until a total "relaxed" condition (for $u = 0$).

Bearing in mind the problem and constraints detailed above, the first control procedure proposed is set as a single, centralized MPC (CMPC) algorithm which takes into account the evolution of the contagion in both states, BA and SC, and, thereby, determines a single control action u which guides the social isolation policies. Such optimization, with cost function J_{CMPC}, is expressed as follows:

$$\min_{U(k)} \quad \sum_j \sum_{i=1}^{N_p} \frac{\left(I_j(k+i)^T q_I I_j(k+i)\right)}{n_j^2}$$

$$+ \sum_{i=1}^{N_p} \left(u(k+i-1)^T q_u u(k+i-1)\right), \qquad (13)$$

$$\text{s.t.:} \quad \text{Discrete SIRD Models } \forall i \in \mathbb{N}_{[1, N_p]}, \qquad (14)$$

$$0 \leq u(k+i-1) \leq 1, \qquad (15)$$

$$-0.05 \leq u(k+i) - u(k+i-1) \leq 0.05, \qquad (16)$$

$$0_{2 \times 1} \leq \begin{bmatrix} I_j(k+i) \\ D_j(k+i) \end{bmatrix} \leq \begin{bmatrix} n_j^{ICU} \\ n_j \end{bmatrix} \forall j, \qquad (17)$$

where N_p is a prediction horizon; the sub-script j indicates the state (i.e. I_{BA} stands for the infections in Bahia), n_j stands for the total population size of the j-th state, n_j^{ICU} represents the total ICU beds available in the state and

$U(k)$ represents the sequence of control actions inside the prediction horizon, i.e. $U(k) = \text{col}\{u(k)\,u(k+1)\,\ldots\,u(k+N_p-1)\}$. The weights q_I and q_u determine the trade-off between conflicting objectives of minimizing the spread and reducing social isolation efforts.

A distributed MPC (DMPC) formulation is much like the one in Eq. (13), but considers the SIRD model for just a single state, and, thereby, finds an individual control law for the referenced state. This optimization procedure, with cost function J_{DMPC}, is:

$$\min_{U(k)} \quad \sum_{i=1}^{N_p} \frac{\left(I_j(k+i)^T q_I I_j(k+i)\right)}{n_j^2}$$

$$+ \sum_{i=1}^{N_p} \left(u(k+i-1)^T q_u u(k+i-1)\right), \quad (18)$$

$$\text{s.t.: Discrete SIRD Model } (j) \ \forall i \in \mathbb{N}_{[1,\,N_p]}, \quad (19)$$

$$0 \leq u(k+i-1) \leq 1, \quad (20)$$

$$-0.05 \leq u(k+i) - u(k+i-1) \leq 0.05, \quad (21)$$

$$0_{2\times1} \leq \begin{bmatrix} I_j(k+i) \\ D_j(k+i) \end{bmatrix} \leq \begin{bmatrix} n_j^{ICU} \\ n_j \end{bmatrix}. \quad (22)$$

The CMPC approach, regarding Brazil, would stand for a single social isolation guide to all 26 states, where a DMPC design would represent social isolation defined through individual state guidelines. For simplicity, although the whole approach presented here can be extrapolated for other scenarios, this work considers only the states of Bahia and Santa Catarina in the following results.

Remark 1. In recent papers (Morato et al., 2020b; Köhler et al., 2020), the issue of MPC regarding COVID-19 has also been discussed. Notice that the differences between the CMPC/DMPC formulations presented in this paper to those in the references are the following: *(i)* the control signal presented by (Köhler et al., 2020) is a factor that multiplies the contagion transmission factors β and γ, while in this paper and in work of (Morato et al., 2020b), it goes through a dynamic model regarding ψ; *(ii)* both previous papers consider uncertainty and approach the problem using a robust design procedure, which is out of the scope of this paper, since we do not have sufficient data for consistent robust parameters estimates; and *(iii)* the DMPC/DMPC approaches consider slew rate constraints on the control signal u, which had not yet been tested (the previous references considered that the social isolation reference could vary arbitrarily at each future sample $k+i$).

5. MAIN RESULTS

We proceed by depicting the results concerning the identification procedure and the obtained control results. The following results were obtained with the aid of Matlab software, Yalmip toolbox and fmincon solver.

The SIRD identification procedure is performed through the optimization given in Eq. (9), with the weights presented in Table 1. The identification is performed considering a moving-horizon window of 6 days for both states.

Remark 2. At the beginning of the pandemic in SC, there was a considerable lack of reported recovered cases (until 05/05/20), inconsistent with regard to the active infections and R_t. However, this does not affect the overall forecasts due to the moving-horizon optimization strategy and, thereby, does not affect the obtained control results.

Table 1. Optimization Weights.

Parameter	k_1	k_2	k_3
Value	1	10	2

The obtained model parameters for the last data-set window, from 10/06 until 16/06/20, are presented in Table 2. Furthermore, Figs. 1, 2 and 3 depict the model-data fitting results for the whole data-sets, regarding I, D and R_t. Evidently, the identification procedure yields quite good parameter estimates, since the simulated SIRD model with parameter estimated with 6 day windows represents the official data with small mismatches. We stress that the coefficient of determination R_{cd} of the obtained identification results is very close to 1.

It is worth mentioning that different pandemic stages are seen in each state. From Figs. 1 and 3, it can be seen that SC has reduced the number of active infections, which is also reflected in $R_t < 1$, representing that, at least at that moment, the disease reached its peak of active cases. Anyhow, the state of BA shows an increasing viral spread trend with $R_t > 1$, which means that the number of cases are still expected to grow.

Table 2. Model Parameters.

States		β	γ	α
BA	From June 10th	0.181	0.053	0.017
SC	to June 16th	0.087	0.737	0.010

We note that the SIRD model parameters used for control are those for the last available window, as given in Table 2. Since a window of 6 days is shown to be sufficient to estimate the SIRD model parameters with model-fitting efficiency (R_{cd} coefficient close to 1), the most adequate control procedure is to adjust the model of the MPC controller in an iterative fashion, as time progresses. This kind of procedure allows one to incorporate the variability of the SIRD parameters, which is inherent to the SARS-CoV-2 viral spread dynamics. We cannot proceed with such paradigm since we consider the control action being deployed through the future (for which we have no data). Thus, we simply keep the last available SIRD parameters as those used for control purposes.
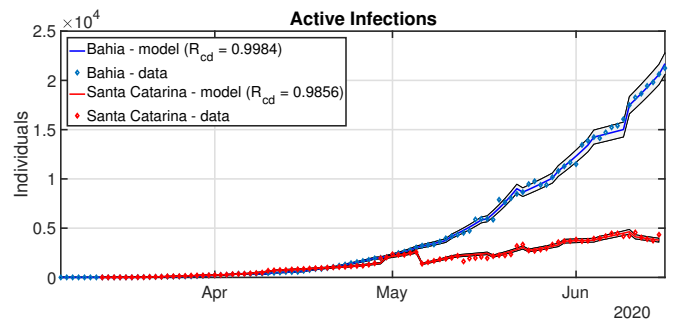

Figure 1. Identification Procedure: Active Infections.

Considering the given parameters for the SIRD model, the control results are presented. The values for the social isolation response dynamics of Eq. (3) are borrowed
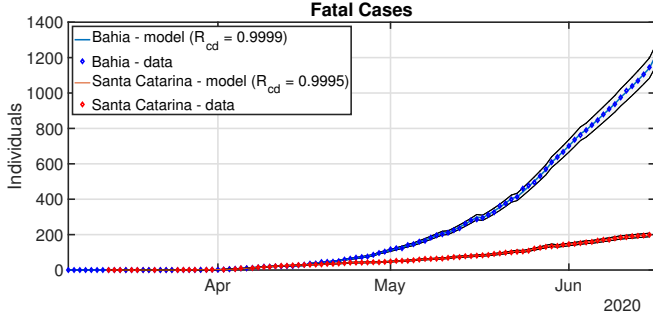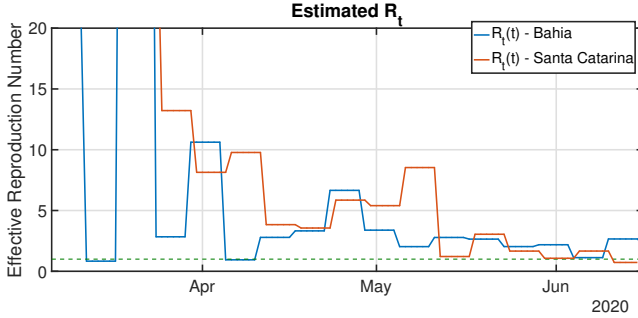
Figure 2. Identification Procedure: Fatal Cases.

Figure 3. Identification Procedure: R_t.

from (Morato et al., 2020b). The maximal social isolation factors, as presented in Table 3, were retrieved from recent technical notes from these states.

Table 3. Social Isolation Response.

State	ϱ	ψ_∞
BA	1.66 days	0.563
SC	1.66 days	0.514

The MPC strategies are synthesized with a prediction horizon of $N_p = 30$ days. This is coherent since the incubation period of the SARS-CoV-2 virus is of, at most, 14 days. The weights q_I and q_u are chosen, respectively, as 0.5 each, so that the MPC tries to find a "balance" between minimizing infections and relaxing quarantine measures.

The achieved results are obtained considering an initial condition w.r.t. the available data at 11/06/2020. We note, as of this date, BA has many infections (the ICU beds at BA are almost full), while SC has already passed through the infection peak. The control strategy is assumed to act from 12/06 to mitigate the backlashes. The results indicate what **could still be done** to avoid the expected catastrophic results if no stronger health policy is employed. Through the sequel, NC denotes the results with "no control", i.e. with $u = 0$ and, thus, with no social isolation, i.e. $\psi \to 0$.

In Fig. 4, the derived control laws are presented and compared to the social isolation factors ψ. The CMPC strategy is presented on the upper subplot, while the DMPC is shown below. Both strategies have similar behaviours w.r.t. BA, trying to suppress the spread of the virus by increasing the quarantine "strength" as the infections increase, and relaxing it afterwards; the forecast to the end of social isolation policies in BA is for June, 2021. Since SC shows an already decaying infection curve, the DMPC takes into account this specificity and indicates a

relaxation much before, around August 2020. We note that the CMPC, since it considers both states, must determine a stronger policy to SC due to the elevated infection rate at BA, while the DMPC approach is able to individually plan the isolation, as expected.

It must be stressed that we analyze the SIRD models as if there is no coupling effects between them. Anyhow, in practice, the 26 states in Brazil cannot pursue individual social isolation laws (as the DMPC approach) since their borders are not closed. The DMPC results only indicate that local conditions should be taken into account, but a centralized coordination (like the CMPC) is forcefully necessary to reduce the infections all over Brazil. It seems to us much more prudent if the federal government dispatch a coordinated social distancing health policy (following a CMPC method), while each state figuring out their possible relaxations according to a DMPC approach and taken into account the infection level in the frontiers states. It does not seem reasonable to relax social isolation in SC by August 2020 and expect that there is no migration/transit between people from neighbouring states (as Paraná or Rio Grande do Sul), which show much greater infection levels (and are previewed to relax quarantine much later).

W.r.t. the depicted control laws, Figures 5 and 6 show, respectively, the evolution of the active infections and the total number of deaths due to COVID-19 in both states, over time. The results indicate that **over** 100000 **lives could still be saved** in BA and 70 lives in SC, w.r.t. a NC condition. The amount of deaths in a NC scenario for BA are astounding. Of course, each life matters and this catastrophe is a lot to bear. Psychological and social traumas will mark the country. A hard isolation and a coordinated social distancing action could still be able to save many lives.

As a final (yet strikingly important) comment, we must discuss that this work only sketches preliminary results on how optimal control can be formalized for pandemic scenarios. An actual application of the proposed method (either CMPC or DMCP) depends on how the control signal can be translated into actual public health policies to be put into practice. This can be understood as some kind of actuator filtering of the control signal, since abrupt daily variations on u make no sense regarding health policies. As an example, one cannot expect to determine relaxations (allow public transport) in one day to revert it in the following. The paradigm to consider only two states (total lockdown or total release) has been previously studied (Morato et al., 2020b) and also offers an elegant solution, but it seems that the preferable way to follow is to determine discretized values for u, which can be converted directly into practicable health policies. This kind of control signal is to be considered in future works, yet an easy route is to adequately filter the control signal generated with the proposed methods in this paper.

6. DISCUSSIONS AND CONCLUSIONS

In this brief article, we investigated how predictive control and optimization-based procedures could be used to formulate social isolation guidelines for the COVID-19 pandemic in Brazil, taking into account the spread of the virus in the states of Bahia and Santa Catarina. Centralized and
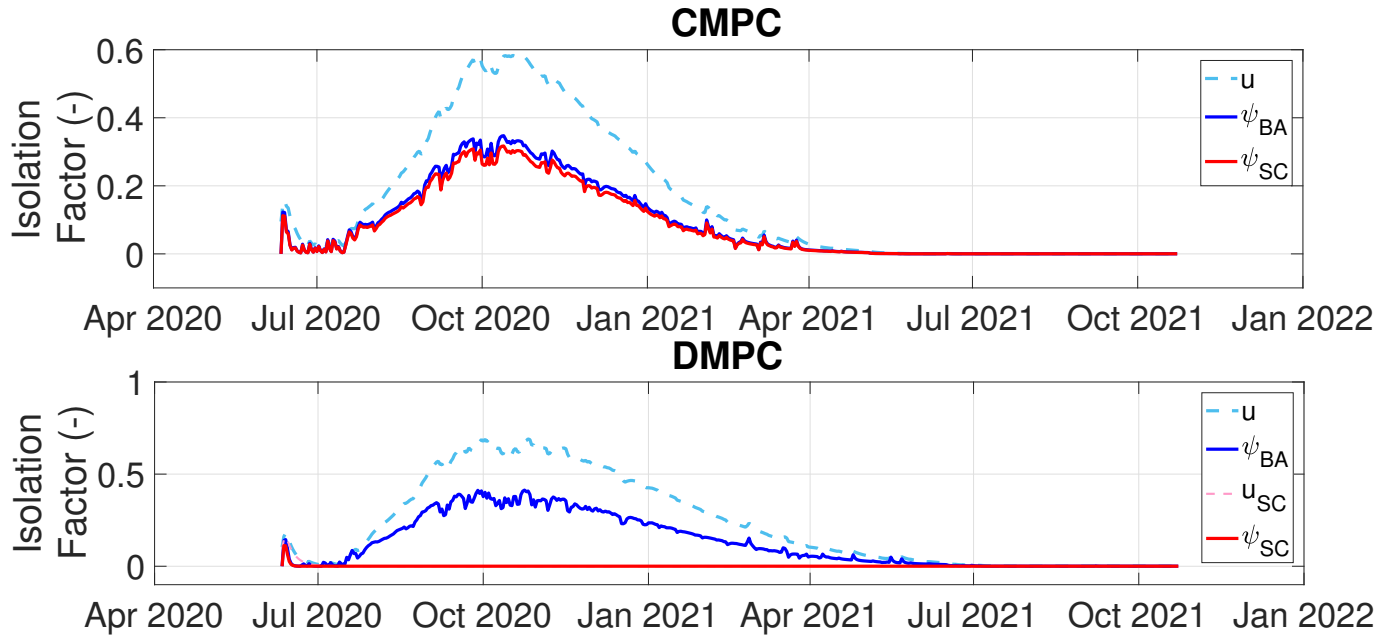
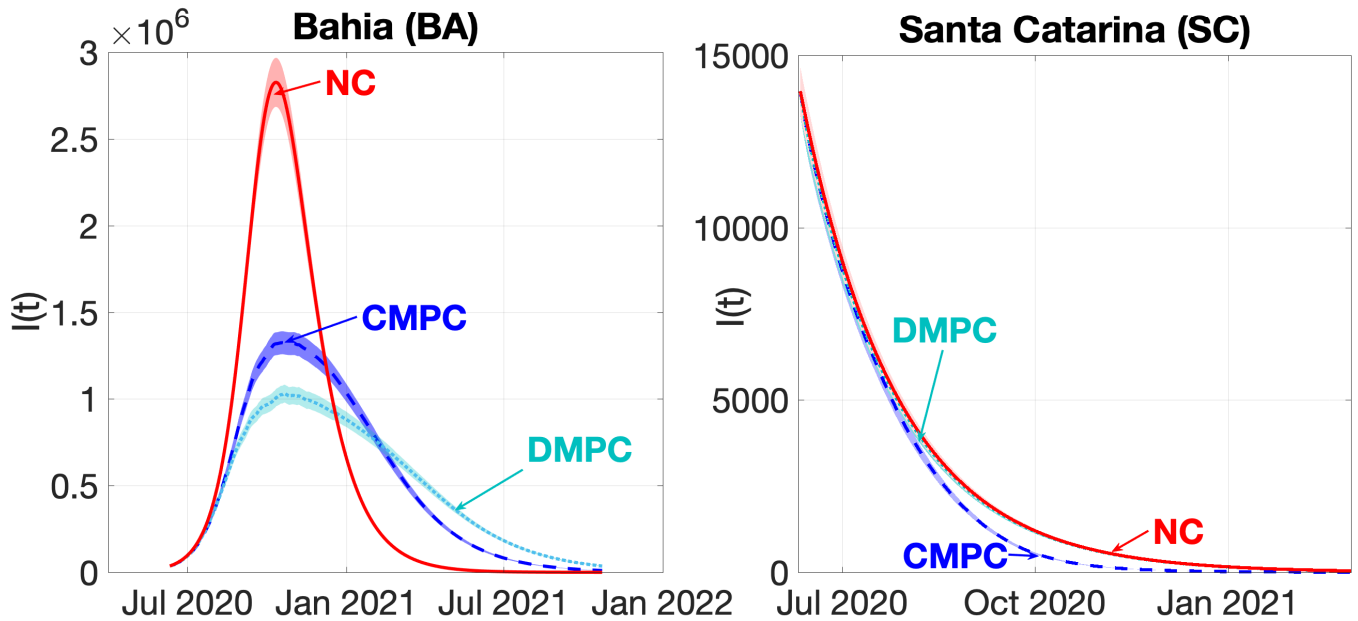Figure 4. Control Action and Social Response.

Figure 5. Control Results: Active Infections.

distributed MPC approaches based on SIRD models with parameters identified via Least-Square optimization were proposed in this work. The results indicate that strong quarantine/lockdown measures still have to be enacted for some months before any relaxations can be though of.

Below, we summarize some key points:

- The results corroborate the hypothesis formulated in (Hellewell et al., 2020) and previously discussed in (Morato et al., 2020b), which indicate that herd immunity cannot be considered as plausible solution for Brazil, offering great risk and leading to elevated

fatality due to multiple social-economical issues of the country.

- The control results show that a centralized, coordinated federal government action is necessary to set guidelines to the states, which can performed individual optimization procedures to determine when to relax quarantine measures. A forecast is presented which indicates that a coordinated social isolation public policy could save over 100000 lives in just in these two states.

- The SARS-CoV-2 contagion is an inherently complex phenomenon and is influenced by many factors and exact prediction of the future dynamics is not
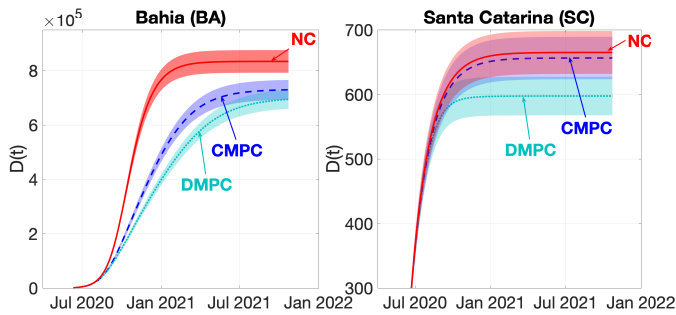
Figure 6. Control Results: Deceased Individuals.

possible and, therefore, the quantitative results presented herein cannot be account for without taking into account the uncertainty margins. Anyhow, the qualitative results are strong. The most correct control procedure should be based on a recurrent model tuning and re-calculation of the control law. Since the country as been experiencing an unwillingness to formally start harder social isolation measures (The Lancet, 2020), the social and economic costs of the pandemic might be brutal.

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