

Continuous-Time PDC-LMI Fuzzy Applied to 3SSC Boost Converter Output Voltage Control

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Abstract: The application of DC-DC converters is widely approached in several studies to ensure its performance and robustness. This paper proposes the voltage control of a three-state switching cell (3SSC) boost converter in the continuous time. On this occasion, the output voltage is controlled by a Tagaki-Sugeno-Kang (TSK) Fuzzy with integral action, which the gains are obtained using Parallel Distributed Compensation via Linear Matrix Inequalities (PDC-LMI) combined through linear membership functions and activated by the duty cycle and inductor current. This approach aims to reject problems originated by load and input voltage variations and improve the stability response of the converter output voltage. The obtained results of this paper evidences the effectiveness of the proposed controller in the continuous time, reducing the effects of external disturbances and maintaining the stability of the output voltage conforming to the control design characteristics.

Keywords: Boost converter; TSK Fuzzy; PDC; LMI; 3SSC.

1. INTRODUCTION

Studies involving robust control of DC-DC converters are important due to the challenge to ensure the performance and robustness under disturbances originated by variations of load and input voltage Olalla et al. (2012). The boost converter, original or equivalent mathematical model, is used for the control design because it is useful in industry such as power supplies, voltage pre-regulators, alternative topologies and rectifiers according to Erickson and Maksimovic (2001) and Mohan (1995).

There are several control techniques known in the literature applied to ensure the closed-loop stability of DC-DC Converters. Among these approaches, there is the design presented by Fernandes et al. (2014), that proposes the state feedback control of a DC-DC boost converter. Also, Rego et al. (2019) and Moreira et al. (2019) use model based predictive control and generalized predictive control in these studies, both using linear matrix inequalities (LMIs) procedures.

The study of Guesmi et al. (2005) propose a PID fuzzy controller to regulate the output voltage of a boost converter. Further, Bizon and Oproescu (2005), Chen et al. (2014) and Darvill et al. (2015) applies the gain-scheduling fuzzy-logic control (FLC) in order to guarantee the performance of system and Raj et al. (2017) which propose a combination of the Takagi-Sugeno-Kang (TSK) Fuzzy adapted

controller with neural network to control a boost DC-DC converter.

The association of FLC, Parallel Distributed Compensation (PDC) structure, Lyapunov's stability analysis and LMIs allow to perform an approximated representation of a nonlinear dynamic systems (Aguirre et al., 2007). All this formulation is able to improve the control design efficiency applied to voltage output of boost converter affected by external disturbances such as input voltage and load. Besides, the high efficiency of the DC-DC boost converter depends of the control method ability to reject the issues of these external variables, that evidence the relevance due to the scope of application and for the current academic research in this field.

The input voltage problems more known are the control signal saturation and the non-minimal phase negative effects (Linares-Flores et al., 2014). Complications originated by load already are widely studied (Erickson and Maksimovic, 2001; Mohan, 1995). However, notice that these external variables can be modeled using FLC and PDC-LMI approach at continuous time considering the state-space feedback robust design. In addition, the problem of the external variables also affects the 3SSC boost converter topology for output voltage control (Costa et al., 2017).

Based on the aforementioned background literature, this study proposes a robust control technique using PDC-LMI

procedure adopting the FLC methodology considering the TSK formulation (Aguirre et al., 2007) applied to the equivalent mode of 3SSC boost converter. Further, it is adopted the augmented average state-space for output voltage control (Reis et al., 2011; Costa et al., 2017). The TSK FLC methodology utilized in this paper deals the input voltage and load as membership functions, which those variations are modeled using polytopes under the LMI context.

This paper provides for time response analysis tests involving disturbances variations of load and input voltages, worth pointing out that the control technique uses only the measurable variables of the 3SSC. As consequence, the responses evidences the proposed approach effectiveness. It is demonstrated through the stability of the output voltage and soft action of the signal control.

Therefore, contributions presented in this study are:

- (1) PDC-LMI TSK Fuzzy controller with integral action in continuous time;
- (2) the gain-scheduled external variables (input voltage and load) modeled by membership functions;
- (3) the control technique uses the converter variables available without require adding new ones.
- (4) the efficiency of rejection disturbances using the FLC associated with state feedback control law.

Hence, the obtained result of the control strategy evidences the possibility of closed-loop output voltage stability based on the PDC-LMI fuzzy gain-scheduled control design.

This study is organized as follows: In Section 2, the mathematical model of the 3SSC boost converter is presented in the average state-space. In Section 3, the control strategy is defined and it is given basic concepts about the design of fuzzy regulators stabilization with LMIs. In Section 4, the results obtained from the mathematical model of the boost converter and fuzzy control rules with integral action are presented and discussed. Finally, Section 5, presents the conclusions of the study and suggestions for future research.

2. 3SSC BOOST CONVERTER

The 3SSC boost converter model is illustrated in Figure 1, that was modeled by Middlebrook and Cuk (1976), Bascope and Barbi (2000), implemented theoretical and empirically by Costa et al. (2017).

According to Middlebrook and Cuk (1976), the boost converter model in average state-space and operating in the Continuous Conduction Mode (CCM) is expressed by (1)-(5) .

$$\begin{aligned}\dot{x} &= A_t x + B_t u \\ y &= C_t x + D_t u\end{aligned}\quad (1)$$

$$A_t = \begin{bmatrix} -\frac{(1-D_{cycle})(R_{co}||R_o)}{L} & -\frac{(1-D_{cycle})R_o}{L(R_o+R_{co})} \\ \frac{(1-D_{cycle})R_o}{C_o(R_o+R_{co})} & -\frac{1}{C_o(R_o+R_{co})} \end{bmatrix}, \quad (2)$$

$$B_t = \frac{V_G}{R'} \begin{bmatrix} \frac{R_o}{L} \frac{(1-D_{cycle})R_o+R_{co}}{(R_o+R_{co})} \\ -\frac{R_o}{(R_o+R_{co})} \end{bmatrix}, \quad (3)$$

$$C_t = \left[(1-D_{cycle})(R_{co}||R_o) \quad \frac{R_o}{(R_o+R_{co})} \right], \quad (4)$$

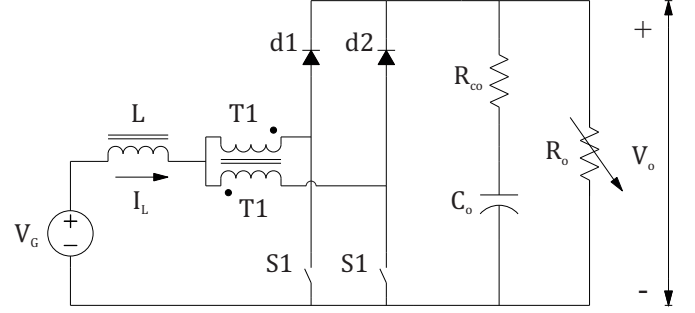


Figure 1. Boost Converter with Three-State Switching Cell (3SSC).

$$D_t = -V_G \frac{(R_{co}||R_o)}{R'} \quad (5)$$

which $R' = (1-D_{cycle})^2 R_o + D_{cycle}(1-D_{cycle})(R_{co}||R_o)$. The state systems $x = [i_L \quad v_c]^T$ are, respectively, the inductor current (L) and the capacitor voltage (C_o). D_{cycle} is the duty cycle, u control signal and y the output voltage (V_o).

3. PDC-LMI TSK FUZZY

The TSK fuzzy model is the modeling of nonlinear systems by combining a number of time invariant linear models, which describe the system performance at different points in its average state-space (Teixeira et al., 2000).

The implementation of fuzzy regulators applied to nonlinear systems described by TSK models considers the concept of PDC and it is designed a linear controller for each rule of the fuzzy model. Then, the combination of these controllers becomes a global fuzzy regulator capable of keeping the system stable (Aguirre et al., 2007).

3.1 TSK Fuzzy Regulator

The fuzzy regulator is defined by the expression of the control law in (6), which aims to determine feedback gains from local states F_i (Aguirre et al., 2007).

$$u(t) = -\sum_{i=1}^r \alpha_i(z(t)) F_i x(t) = -F(a)x(t) \quad (6)$$

According to Tanaka et al. (1998), the expression that describes the system states for a control law defined as (6), it is given by (7).

$$\dot{x}(t) = \sum_{i=1}^r \alpha_i^2(z(t)) G_{ii} x(t) + 2 \sum_{i < j}^r \alpha_i(z(t)) \alpha_j(z(t)) \left\{ \frac{G_{ij} + G_{ji}}{2} \right\} x(t) \quad (7)$$

Which

$$G_{ij} = A_i - B_i F_j \quad (8)$$

3.2 Stability of Fuzzy Regulators via LMIs

Issues related to control stability are simplified by using the LMI approach, because it has the advantage to be solved numerically using mathematical programming software and dedicated to LMIs solutions. Thus, the implementation of TSK Fuzzy regulators minimizes certain

problems of precision loss during the numerical implementation process using LMIs (Boyd et al., 1994).

In this study, the stability conditions for fuzzy systems are obtained through the Lyapunov quadratic functions given by $V(x_{(t)}) = x_{(t)}^T P x_{(t)}$, which $P = P^T > 0$ is a positive symmetric matrix (Tanaka and Sugeno, 1992).

In addition, the system stability is achieved by the feedback gains F_i , from $i = 1, \dots, r$, that r is the number of model rules defined by the LMIs in (9)-(12). The stability problem is solved by getting the symmetric matrices X , T_{ijh} , R_{ij} , and the matrices S_{ijh} and M_i , satisfying $X > 0$, $T_{ijh} \geq 0$ to $(i, j, h = 1, 2, \dots, r, i < j)$ Teixeira et al. (2003).

$$\begin{bmatrix} C_{11} - Z_{1h} & C_{12h} & \cdots & C_{1rh} \\ C_{21h} & C_{22} - Z_{2h} & \cdots & C_{2rh} \\ \vdots & \vdots & \ddots & \vdots \\ C_{r1h} & C_{r2h} & \cdots & C_{rr} - Z_{rh} \end{bmatrix} \leq 0 \quad (9)$$

for $i, j, h = 1, \dots, r$,

$$C_{ii} = A_i X - B_i M_i + X A_i^T - M_i^T B_i^T \quad (10)$$

$$Z_{jh} = \begin{cases} R_{jh}, & \text{if } j < h, \\ R_{hj}, & \text{if } j > h, \\ 0, & \text{if } j = h, \end{cases} \quad (11)$$

$$C_{ijh} = \begin{cases} \frac{1}{2} (A_i X - B_i M_j + A_j X - B_j M_i + \\ X A_i^T - M_j^T B_i^T + X A_j^T - M_i^T B_j^T) + \\ T_{ijh} + (S_{ijh} - S_{jih}^T) + \frac{1}{2} W_{ijh}, & \text{if } i < j, \\ \frac{1}{2} (A_j X - B_j M_i + A_i X - B_i M_j + \\ X A_j^T - M_i^T B_j^T + X A_i^T - M_j^T B_i^T) + \\ T_{ijh} + (S_{jih} - S_{ijh}^T) + \frac{1}{2} W_{jih}, & \text{if } i > j, \\ W_{ikh} = \begin{cases} R_{ik}, & \text{if } l = h \text{ or } k = h \\ 0, & \text{if } l \neq h \text{ and } kl \neq h, \end{cases} \end{cases} \quad (12)$$

Since, the set of LMIs are achievable and they are governed by membership functions. The feedback gains are F_i and a matrix P is defined by $P = X^{-1}$, and $F_i = M_i X^{-1}$.

4. SIMULATIONS AND RESULTS

In this section, it is presented the fuzzy model rules reached from the boost converter numerical model, the block diagram of the integral action and the numerical results are discussed.

4.1 Boost Converter Numerical Model and Fuzzy Rules

The results obtained by the application of TSK Fuzzy Regulator, using the parameters applied by Rego et al. (2019), to control the boost converter output voltage in Figure 1 are shown in Table 1.

In order to obtain the numerical model of boost converter, it was considered uncertainty polytopes in the variations of the input voltage and output power. Then, each vertex of the uncertainty polytopes was considered a rule for the global model applied to the fuzzy regulator described by (9)-(12).

$-f(36V, 1000W)$

Table 1. Boost Implementation Parameters

Parameters	Values
Input Voltage (V_G)	26 – 36 (V)
Output Voltage (V_o)	48 (V)
Duty Cicle (D_{cycle})	0.25 – 0.46
Switching Frequency (f_s)	22 (kHz)
Filter Inductor (L)	36 (μH)
Inductive Resistance (R_L)	0 (Ω)
Output Capacitor (C_o)	4,400 (μF)
Series Resistance (R_{co})	26.7 (m Ω)
Output Power (P_o)	380 – 1000 (W)

$$A_1 = \begin{bmatrix} -549.8777 & -2.06 \times 10^4 \\ 168.5019 & -97.5126 \end{bmatrix}, B_1 = \begin{bmatrix} 1.34 \times 10^6 \\ -27.3551 \end{bmatrix}, \\ C_1 = [0.0198 \ 0.9885], D_1 = -0.7304,$$

$-f(26V, 1000W)$

$$A_2 = \begin{bmatrix} -397.1339 & -1.49 \times 10^4 \\ 121.6958 & -97.5126 \end{bmatrix}, B_2 = \begin{bmatrix} 1.34 \times 10^6 \\ -37.6559 \end{bmatrix}, \\ C_2 = [0.0143 \ 0.9885], D_2 = -1.0054,$$

$-f(36V, 380W)$

$$A_3 = \begin{bmatrix} -553.8112 & -2.08 \times 10^4 \\ 169.7072 & -37.3199 \end{bmatrix}, B_3 = \begin{bmatrix} 1.34 \times 10^6 \\ -10.4939 \end{bmatrix}, \\ C_3 = [0.0199 \ 0.9956], D_3 = -0.2802,$$

$-f(26V, 380W)$

$$A_4 = \begin{bmatrix} -399.9748 & -1.50 \times 10^4 \\ 122.5663 & -37.3199 \end{bmatrix}, B_4 = \begin{bmatrix} 1.34 \times 10^6 \\ -14.4975 \end{bmatrix}, \\ C_4 = [0.0144 \ 0.9956], D_4 = -0.3871,$$

4.2 Integral Action Modeling

Here, the integral action used to implement the proposed control strategy and illustrated by the block diagram in Figure 2 are illustrated. It was initially proposed by Costa et al. (2017) based on (Fadali, 2009; Levine, 1999).

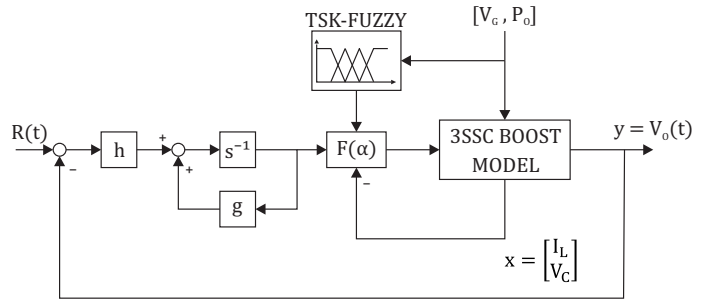


Figure 2. Block diagram of the boost converter with integral action

According to Fadali (2009), the expressions of the increased model in open loop are given by:

$$\begin{aligned} \dot{x} &= \bar{A}x + \bar{B}u \\ y &= \bar{C}x + \bar{D}u \end{aligned} \quad (13)$$

Which

$$\bar{A} = \begin{bmatrix} A & 0 \\ -hC & g \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ -hD \end{bmatrix}, \bar{C} = [C \ 0] \quad (14)$$

The closed loop model for a set of gains $F = -[K \ -K_i]$ is characterized by:

$$\begin{bmatrix} \dot{x}(t) \\ v(t) \end{bmatrix} = \begin{bmatrix} A - BK & BK_i \\ -h(C - DK) & g - hDK_i \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} + \begin{bmatrix} 0 \\ h \end{bmatrix} r(t), \quad (15)$$

$$y(t) = [(C - DK) \quad DK_i] \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} \quad (16)$$

which $(r(t))$ is the reference, $(v(t))$ is the integral action and the matrices (g) and (h) that correspond to the degree of freedom of the system. These matrices can be varied in order to improve the control response in a permanent regime. Therefore, it is a parameter of the designer.

4.3 Simulation Results

The numerical implementation of this project considers the fuzzy model rules and the augmented feedback gains as presented in subsection 4.1. The integral action are discussed in 4.2 with $g = h = 1$ and initial condition given by $x(0) = [38.5 \ 26]^T$. The numerical simulation applies the 4th Order Runge Kutta Method for the nonlinear boost model, considering the range of 300 ms in the time domain.

For the control law $u(t) = -F(\alpha)x(t) = -\sum_{\alpha_i} \frac{\alpha_i F_i}{\sum \alpha_i} x(t)$, its augmented gains are:

$$F_1 = -1 \times 10^{-3} [0.4170 \ 3.7772 \ 1.2598] \quad (17)$$

$$F_2 = -1 \times 10^{-3} [0.3013 \ 2.7299 \ 0.9096] \quad (18)$$

$$F_3 = -1 \times 10^{-3} [0.4203 \ 3.8004 \ 1.2692] \quad (19)$$

$$F_4 = -1 \times 10^{-3} [0.3034 \ 2.7453 \ 0.9166] \quad (20)$$

Figure 3 illustrates the membership functions applied to fuzzy gain-scheduling control law. The functions M_1 and M_2 are activated by the duty cycle, which it is related with V_g . Besides, M_3 and M_4 are related with load variation through the I_L .

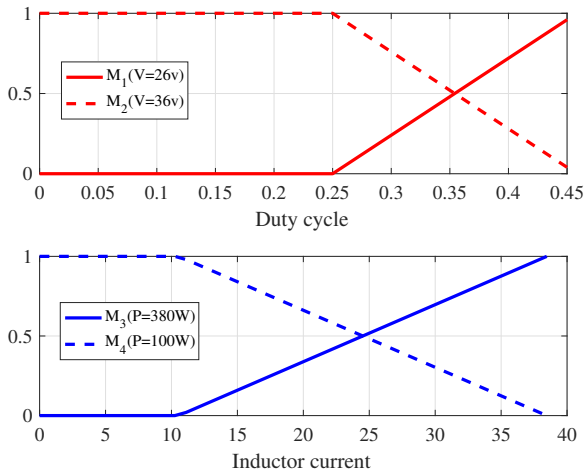


Figure 3. Membership Functions

Figure 4 presents the variation the response of the disturbance variables. The first curve represents the output power (a) and the second response is the input voltage (b).

In Figure 5, the control action holds the boost converter output voltage tracking the reference voltage described previously in Moreira et al. (2019). In addition, it is noticed that the low intensity voltage peak occurs in the same instant time of the input voltage variation and the high intensity voltage peak occurs changing the power

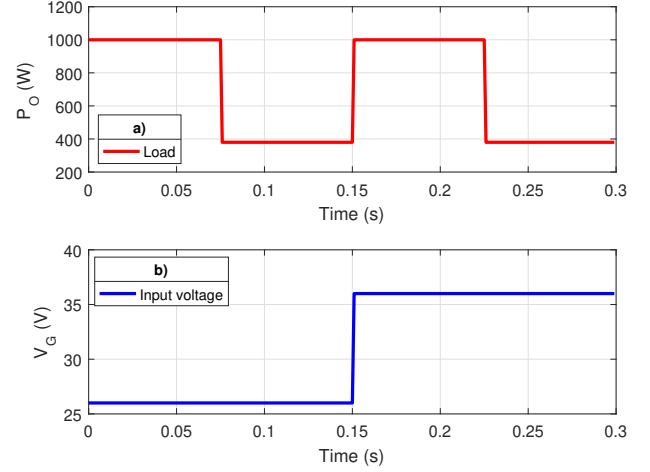


Figure 4. a) Output Power (P_o) and b) Input Voltage (V_G)

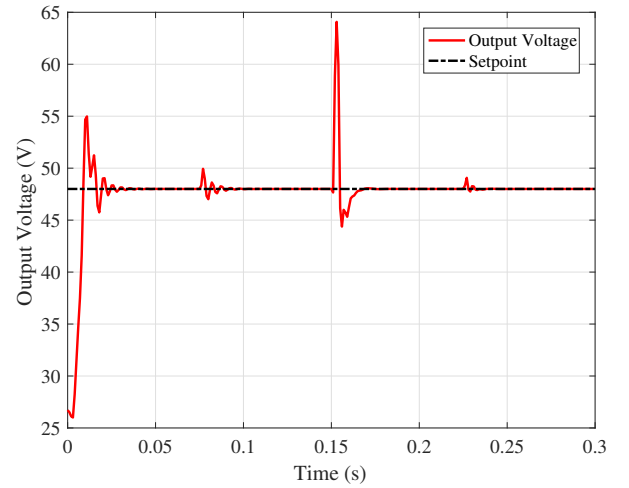


Figure 5. Output Voltage (V_o)

demand by the load variation. Those V_o overshoots are an inherent problem of the boost converter.

The evidences of the control signal behavior and softness are presented in Figure 6. In Figure 7, the capacitor voltage and the inductor current are shown during the simulation process.

5. CONCLUSION

In this paper, the control technique TSK Fuzzy was proposed and applied to the 3SSC boost converter in the continuous time. The objective of the study was to guarantee the stability of the plant considering uncertainties caused by variations in input voltage and load. When analyzing the results of the Section 4, it appears that the control techniques proposed in Section 3, associated with the PDC-LMI techniques and linear membership functions, ensure the system stability with smooth signal control action.

The results achieved in this study grant the possibility for deeper research in fuzzy LMI techniques applied to the converters in continuous time. Moreover, there is

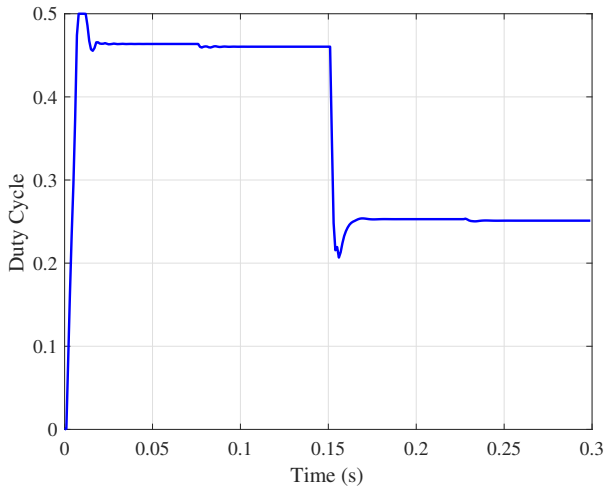


Figure 6. Duty Cycle (D_{cycle})

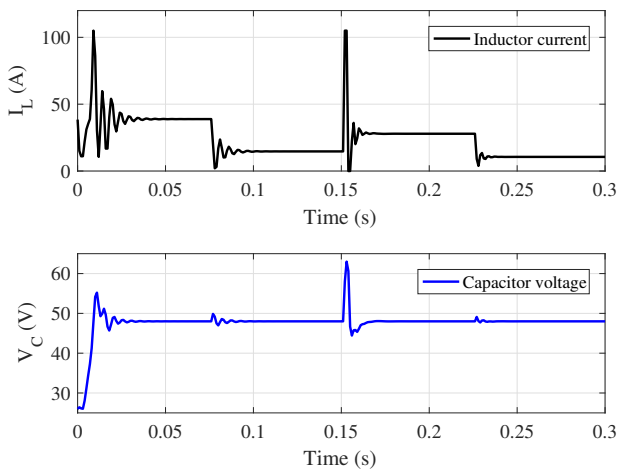


Figure 7. a) Inductor Current (x_1) and b) Capacitor Voltage (x_2)

the alternative application with Anti-Windup and Linear Parameters Variant (LPV) systems for the boost 3SSC converter as future works. Further, a feedforward design added to the control strategy in order to reject high output voltage overshoot as registered in Figure 5.

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REFERENCES

Aguirre, L., Bruciapaglia, A., and Miyagi, P.E e Piqueira, J. (2007). *Enciclopédia de automática: controle e automação*. Blucher.

- Bascope, G. and Barbi, I. (2000). Generation of a family of non-isolated dc-dc pwm converters using new three-state switching cells. *2000 IEEE 31st Annual Power Electronics Specialists Conference. Conference Proceedings (Cat. No.00CH37018)*, 2, 858–863 vol.2.
- Bizon, N. and Oproescu, M. (2005). Hysteretic fuzzy control of the boost converter. In *International conference on Electronics, Computers and Artificial Intelligence 2005 (ECAI'05)*.
- Boyd, S., El Ghaoui, L., Feron, E., and Balakrishnan, V. (1994). *Linear Matrix Inequalities in System and Control Theory*, volume 15 of *Studies in Applied Mathematics*. SIAM, Philadelphia, PA.
- Chen, C.C., Chen, C.L., Chang, J.X., and Yang, C.F. (2014). Lpv gain-scheduling control for a phase-shifted pwm full-bridge soft switched converter. *IFAC Proceedings Volumes (IFAC-PapersOnline)*, 19, 6135–6140.
- Costa, M., Reis, F., Campos, J., Nogueira, F., and Almeida, O. (2017). Robust MPC-LMI controller applied to three state switching cell boost converter. *Eletrônica de Potência*, 22, 81–90. doi:10.18618/REP.2017.1.2652.
- Darvill, J., Tisan, A., and Cirstea, M. (2015). An anfsi-pi based boost converter control scheme. In *2015 IEEE 13th International Conference on Industrial Informatics (INDIN)*, 632–639.
- Erickson, R.W. and Maksimovic, D. (2001). *Fundamentals of Power Electronics*. Springer, 2ed edition.
- Fadali, M. (2009). *Digital control engineering analysis and design*. Elsevier/Academic Press, Amsterdam Boston.
- Fernandes, D., Vieira, M., Vitorino, M., Costa, F., and Ribeiro, P. (2014). Modeling and state-space feedback control of a dc-dc converter for photovoltaic systems. In *2014 IEEE Energy Conversion Congress and Exposition, ECCE 2014*, 1119–1128. doi:10.1109/ECCE.2014.6953525.
- Guesmi, K., Essounbouli, N., Hamzaoui, A., Manamanni, N., and Zaytoon, J. (2005). A fuzzy controller synthesis for a boost converter. *IFAC Proceedings Volumes*, 38(1), 181 – 186. doi:https://doi.org/10.3182/20050703-6-CZ-1902.01759. 16th IFAC World Congress.
- Levine, S. W. (1999). *The Control Handbook*. CRC Press, INC, N W Boca Raton, USA, Florida.
- Linares-Flores, J., Hernández Méndez, A., García-Rodríguez, C., and Sira-Ramírez, H. (2014). Robust nonlinear adaptive control of a “boost” converter via algebraic parameter identification. *IEEE Transactions on Industrial Electronics*, 61(8), 4105–4114.
- Middlebrook, R.D. and Cuk, S. (1976). A general unified approach to modelling switching-converter power stages. In *1976 IEEE Power Electronics Specialists Conference*, 18–34.
- Mohan, N. (1995). *Power electronics : converters, applications, and design*. Wiley, New York.
- Moreira, T., Rego, R., Rios, C., Nogueira, F., and Costa, M. (2019). Análise comparativa entre controlador gpc-t e mpc-lmi com anti-windup lmi aplicado em conversor boost ccte. In *12th Seminar on Power Electronics and Control, At Natal - RN*.
- Olalla, C., Leyva, R., Queinnec, I., and Maksimovic, D. (2012). Robust gain-scheduled control of switched-mode dc-dc converters. *IEEE Transactions on Power Electronics*, 27(6), 3006–3019.

- Raj, R.N., Purushothaman, K.V., and Singh, N.A. (2017). Adaptive tsk-type neural fuzzy controller for boost dc-dc converter. In *2017 IEEE International Conference on Circuits and Systems (ICCS)*, 441–446.
- Rego, R., Moreira, T., Nogueira, F., and Costa, M. (2019). Tube mpc applied to 3ssc boost converter with time-varying parameters disturbance. In *The 12th Seminar on Power Electronics and Control*.
- Reis, F.E.U., Torrico-Bascope, R.P., and Costa, M.V.S. (2011). LQR control with integral action applied to a high gain step-up dc-dc converter. In *11th Brazilian Power Electronics Conference (COBEP)*, 256–261. IEEE, RN- Natal. doi:10.1109/COBEP.2011.6085262.
- Tanaka, K., Ikeda, T., and Wang, H.O. (1998). Fuzzy regulators and fuzzy observers: relaxed stability conditions and lmi-based designs. *IEEE Transactions on Fuzzy Systems*, 6(2), 250–265.
- Tanaka, K. and Sugeno, M. (1992). Stability analysis and design of fuzzy control systems. *Fuzzy Sets and Systems*, 45(2), 135 – 156. doi:https://doi.org/10.1016/0165-0114(92)90113-I.
- Teixeira, M.C.M., Assuncao, E., and Avellar, R.G. (2003). On relaxed lmi-based designs for fuzzy regulators and fuzzy observers. *IEEE Transactions on Fuzzy Systems*, 11(5), 613–623.
- Teixeira, M., Pietrobom, H., and Assunção, E. (2000). Novos resultados sobre estabilidade e controle de sistemas não-lineares utilizando modelos fuzzy e lmi. *Controle e Automação*, 11, 37–48.