AN INTELLIGENT SLIDING MODE CONTROLLER FOR UNDERACTUATED MECHANICAL SYSTEMS

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Abstract— This paper presents an intelligent controller for uncertain underactuated nonlinear systems. The adopted approach is based on sliding mode control and enhanced by a fuzzy scheme to cope with modeling inaccuracies and external disturbances that can arise. The sliding surfaces are defined as a linear combination of both actuated and unactuated variables. A fuzzy inference system is added to compensate the performance drop when, in order to avoid the chattering phenomenon, the sign function is substituted by a saturation function in the conventional sliding mode controller. An application of the proposed scheme is introduced for the cart pole problem, in order to illustrate the controller design method. Numerical results are presented to demonstrate the improved performance of the resulting intelligent controller.

Keywords— Cart-pole system, Fuzzy logic, Intelligent control, Sliding modes, Underactuated mechanical systems.

Resumo— Este artigo apresenta um controlador inteligente para sistemas não-lineares subatuados e incertos. A abordagem adotada baseia-se no controle por modos deslizantes, mas é aprimorada por um esquema fuzzy para lidar com imprecisões de modelagem e eventuais perturbações externas. As superfícies de deslizamento são definidas como uma combinação de ambas variáveis atuadas e não-atuadas. Um sistema de inferência fuzzy é adicionado para compensar a perda de performance quando, no intuito de evitar o chaveamento de alta-frequência, a função sinal é substituída pela função saturação no controlador convencional por modos deslizantes. Uma aplicação do esquema proposto é apresentada para o problema do pêndulo invertido montado em um carro. Resultados numéricos são apresentados para demonstrar o aumento de performance proporcionado pelo controlador inteligente.

Palavras-chave— Controle inteligente, Lógica Fuzzy, Modos deslizantes, Pêndulo invertido, Sistemas mecânicos subatuados.

1 Introduction

A mechanical system could be defined as underactuated if it has more degrees of freedom to be controlled than independent control inputs/actuators. Underactuated mechanical systems (UMS) play an essential role in several branches of industrial activity and their application scope ranges from robotic manipulators (Lai et al., 2009; Xin and Liu, 2013) and overhead cranes (Rapp et al., 2012; Sun et al., 2013) to aerospace vehicles (Consolini et al., 2010; Tsiotras and Luo, 2000) and watercrafts (Do and Pan, 2009; Serrano et al., 2014).

Basically, underactuation could arise due to the following main reasons (Seifried, 2013):

- Design issues, as for instance in the case of ships, overhead cranes, and helicopters;
- Non-rigid body dynamics, for example if one or more flexible links are considered within a robotic manipulator;
- Actuator failure, as is the case with aerial and underwater vehicles.

Despite this broad spectrum of applications, the problem of designing accurate controllers for underactuated systems is unfortunately much more tricky than for fully actuated ones. Moreover, the dynamic behavior of an UMS is frequently uncertain and highly nonlinear, which in fact makes the design of control schemes for such systems a challenge for conventional and well established methodologies.

Therefore, much effort has been made in order to improve both set-point regulation and trajectory tracking of underactuated mechanical systems. The most common strategies are partial feedback linearization (Seifried, 2013; Spong, 1994), feedforward control by model inversion (Seifried, 2012b; Seifried, 2012a), adaptive approaches (Pucci et al., 2015; Nguyen and Dankowicz, 2015), sliding mode control (Ashrafiuon and Erwin, 2008; Xu and Özgüner, 2008; Sankaranarayanan and Mahindrakar, 2009; Qian et al., 2009; Muske et al., 2010), backstepping (Chen and Huang, 2012; Xu and Hu, 2013; Rudra et al., 2014), controlled Lagrangians (Bloch et al., 2000; Bloch et al., 2001), and passivity-based methods (Ortega et al., 2002; Gómez-Estern and der

Schaft, 2004; Ryalat and Laila, 2016). However, it should be highlighted that the control of uncertain UMS remains hard to be accomplished, specially if a high level of uncertainty is involved (Liu and Yu, 2013).

Intelligent control, on the other hand, has proven to be a very attractive approach to cope with uncertain nonlinear systems (Bessa and Barrêto, 2010; Bessa et al., 2012; Tanaka et al., 2013). By combining nonlinear control techniques, such as feedback linearization or sliding modes, with adaptive intelligent algorithms, for example fuzzy logic or artificial neural networks, the resulting intelligent control strategies can deal with the nonlinear characteristics as well as with modeling imprecisions and external disturbances that can arise.

Due to its ability to undertake assignments in an environment of imprecision and imperfect information, fuzzy logic has been widely employed to both control and identification of dynamical systems. In spite of the simplicity of fuzzy's heuristic approach, in some situations a more rigorous mathematical treatment of the problem is required. Recently, much effort has been made to combine fuzzy logic with nonlinear control methodologies.

As a matter of fact, sliding mode control is an appealing technique because of its robustness against both structured and unstructured uncertainties as well as external disturbances. Nevertheless, the discontinuities in the control law must be smoothed out to avoid the undesirable chattering effects. The adoption of properly designed boundary layers have proven effective in completely eliminating chattering, however, leading to an inferior tracking performance, as demonstrated by Bessa (2009).

In this context, considering that fuzzy logic can perform universal approximation (Kosko, 1994), Bessa and Barrêto (2010) showed that adaptive fuzzy inference systems can be successfully applied for uncertainty and disturbance compensation within the boundary layer of smooth sliding mode controllers.

In this work, a sliding mode controller with a fuzzy compensation scheme is proposed for uncertain underactuated mechanical systems. On this basis, a smooth sliding mode controller is considered to confer robustness against modeling imprecisions and an adaptive fuzzy inference system is embedded in the boundary layer to cope with unmodeled dynamical effects. The convergence properties of the proposed controller are proved by means of the Lyapunov stability theory. Numerical simulations are carried out in order to illustrate the overall control system performance.

2 Intelligent sliding mode control of underactuated systems

The equations of motion of a mechanical system with n degrees of freedom (DOF) and m actuator inputs are usually expressed in the following vector form (Seifried, 2013):

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{k}(\boldsymbol{q},\dot{\boldsymbol{q}}) = \boldsymbol{g}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{B}(\boldsymbol{q})\boldsymbol{u}\,, \qquad (1)$$

where $\boldsymbol{q} \in \mathbb{R}^n$ is the vector of generalized coordinates, $\boldsymbol{u} \in \mathbb{R}^m$ the actuator input vector, $\boldsymbol{M}(\boldsymbol{q}) \in \mathbb{R}^{n\times n}$ the positive definite and symmetric inertia matrix, $\boldsymbol{k}(\boldsymbol{q},\dot{\boldsymbol{q}}) \in \mathbb{R}^n$ takes the Coriolis and centrifugal effects into account, $\boldsymbol{g}(\boldsymbol{q},\dot{\boldsymbol{q}}) \in \mathbb{R}^n$ represents the generalized applied forces, and $\boldsymbol{B}(\boldsymbol{q}) \in \mathbb{R}^{n\times m}$ is the input matrix.

Definition 1 *The mechanical system described in equation (1) is called fully-actuated if* $m = \operatorname{rank}(B) = n$, *or underactuated if* $m < n$.

Considering an UMS, the vector of generalized coordinates can be partitioned as $\boldsymbol{q} = [\boldsymbol{q}_a^\top\ \boldsymbol{q}_u^\top]^\top$, where $\boldsymbol{q}_a \in \mathbb{R}^m$ and $\boldsymbol{q}_u \in \mathbb{R}^{n-m}$ denote, respectively, actuated and unactuated coordinates. In these cases, the input matrix is also conveniently assumed to be $\boldsymbol{B}(\boldsymbol{q}) = [\boldsymbol{B}_a\ \boldsymbol{B}_u]^\top = [\boldsymbol{I}\ \boldsymbol{0}]^\top$, where $\boldsymbol{I} \in \mathbb{R}^{n\times n}$ is the identity matrix.

Therefore, for control purposes, equation (1) may be rewritten as (Ashrafiuon and Erwin, 2008; Seifried and Blajer, 2013):

$$\begin{bmatrix} \boldsymbol{M}_{aa} & \boldsymbol{M}_{au} \\ \boldsymbol{M}_{au}^\top & \boldsymbol{M}_{uu} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{q}}_a \\ \ddot{\boldsymbol{q}}_u \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_a + \boldsymbol{u} \\ \boldsymbol{f}_u \end{bmatrix}, \qquad (2)$$

where $\boldsymbol{f}_a = \boldsymbol{g}_a - \boldsymbol{k}_a$ and $\boldsymbol{f}_u = \boldsymbol{g}_u - \boldsymbol{k}_u$.

As described in (Ashrafiuon and Erwin, 2008), equation (2) can be solved for the accelerations:

$$\ddot{\boldsymbol{q}}_a = \boldsymbol{M}_{aa}^{'-1}(\boldsymbol{f}_a^{'} + \boldsymbol{u})\,, \qquad (3)$$

$$\ddot{\boldsymbol{q}}_u = \boldsymbol{M}_{uu}^{'-1}(\boldsymbol{f}_u^{'} - \boldsymbol{M}_{au}^\top \boldsymbol{M}_{aa}^{-1}\boldsymbol{u})\,, \qquad (4)$$

where $\boldsymbol{M}_{aa}^{'} = \boldsymbol{M}_{aa} - \boldsymbol{M}_{au}\boldsymbol{M}_{uu}^{-1}\boldsymbol{M}_{au}^\top$, $\boldsymbol{M}_{uu}^{'} = \boldsymbol{M}_{uu} - \boldsymbol{M}_{au}^\top\boldsymbol{M}_{aa}^{-1}\boldsymbol{M}_{au}$, $\boldsymbol{f}_a^{'} = \boldsymbol{f}_a - \boldsymbol{M}_{au}\boldsymbol{M}_{uu}^{-1}\boldsymbol{f}_u$, and $\boldsymbol{f}_u^{'} = \boldsymbol{f}_u - \boldsymbol{M}_{au}^\top\boldsymbol{M}_{aa}^{-1}\boldsymbol{f}_a$.

The proposed control problem has to ensure that, even in the presence of external disturbances and modeling imprecisions, the vector of generalized coordinates $\boldsymbol{q}$ will follow a desired trajectory $\boldsymbol{q}^d$ in the state space. Hence, defining $\tilde{\boldsymbol{q}} = \boldsymbol{q} - \boldsymbol{q}^d$ as the tracking error vector, both trajectory tracking and set-point regulation could be stated as $\tilde{\boldsymbol{q}} \to \boldsymbol{0}$ as $t \to \infty$.

Consider, as for instance, the sliding mode approach, and let m sliding surfaces be defined in the state space by $\boldsymbol{s}(\tilde{\boldsymbol{q}}) = \boldsymbol{0}$, with $\boldsymbol{s} \in \mathbb{R}^m$ satisfying

$$\begin{aligned} \boldsymbol{s}(\tilde{\boldsymbol{q}}) &= \alpha_a\dot{\tilde{\boldsymbol{q}}}_a + \lambda_a\tilde{\boldsymbol{q}}_a + \alpha_u\dot{\tilde{\boldsymbol{q}}}_u + \lambda_u\tilde{\boldsymbol{q}}_u \\ &= \alpha_a\dot{\boldsymbol{q}}_a + \alpha_u\dot{\boldsymbol{q}}_u + \boldsymbol{s}_r\,, \end{aligned} \qquad (5)$$

where $\boldsymbol{s}_r = -\alpha_a \dot{\boldsymbol{q}}_a^d + \alpha_u \dot{\boldsymbol{q}}_u^d + \lambda_a \tilde{\boldsymbol{q}}_a + \lambda_u \tilde{\boldsymbol{q}}_u$.

Thus, the sliding mode controller must satisfy the following Lyapunov candidate function (Ashrafiuon and Erwin, 2008):

$$V_1(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \frac{1}{2}\boldsymbol{s}^\top \boldsymbol{s} \tag{6}$$

and the so called sliding condition $\dot{V}_1(\boldsymbol{q}, \dot{\boldsymbol{q}}) \leq 0$.

On this basis, the control law is defined as (Ashrafiuon and Erwin, 2008):

$$\boldsymbol{u} = -\hat{\boldsymbol{M}}_s^{-1}\left[\hat{\boldsymbol{f}}_s^{-1} + \dot{\boldsymbol{s}}_r + \boldsymbol{\kappa}\,\mathrm{sgn}(\boldsymbol{s})\right], \tag{7}$$

where $\hat{\boldsymbol{M}}_s$ and $\hat{\boldsymbol{f}}_s$ are, respectively, estimates of $\boldsymbol{M}_s = \alpha_a \boldsymbol{M}_{aa}^{'-1} - \alpha_u \boldsymbol{M}_{uu}^{'-1}\boldsymbol{M}_{au}^\top \boldsymbol{M}_{aa}^{-1}$ and $\boldsymbol{f}_s = \alpha_a \boldsymbol{M}_{aa}^{'-1}\boldsymbol{f}_a^{'} - \alpha_u \boldsymbol{M}_{uu}^{'-1}\boldsymbol{f}_u^{'\top}$. The control gain $\boldsymbol{\kappa}$ should be designed in order to take unmodeled dynamics, parametric uncertainties and external disturbances into account.

Therefore, regarding the development of the control law, the following assumptions should be made:

Assumption 1 *The vector $\boldsymbol{f}_s$ is unknown but bounded, i.e. $|\hat{\boldsymbol{f}}_s - \boldsymbol{f}_s| = |\boldsymbol{d}_s| \leq \boldsymbol{F}_s$.*

Assumption 2 *The vector of generalized coordinates $\boldsymbol{q}$ is available.*

Assumption 3 *The desired trajectory $\boldsymbol{q}^d$ is once differentiable with respect to time. Furthermore, every component of $\boldsymbol{q}^d$ is available and with known bounds.*

However, the presence of a discontinuous term, $\boldsymbol{\kappa}\,\mathrm{sgn}(\boldsymbol{s})$, in the control law leads to the well known chattering effect. To avoid these undesirable high-frequency oscillations of the controlled variable, a thin boundary layer in the neighborhood of the switching surface could be defined by replacing the sign function with a smooth approximation. This substitution can minimize or, when desired, even completely eliminate chattering, but turns *perfect tracking* into a *tracking with guaranteed precision* problem, which actually means that a steady-state error will always remain (Bessa, 2009). In order to improve the tracking performance, some adaptive intelligent algorithm could be used within the boundary layer of smooth sliding mode controllers.

At this point, since fuzzy logic can be considered as an universal approximator (Kosko, 1994), we propose the adoption of a fuzzy based compensation term, $\hat{\boldsymbol{d}}$, within the smoothed version of the control law presented in equation (7):

$$\boldsymbol{u} = -\hat{\boldsymbol{M}}_s^{-1}\left[\hat{\boldsymbol{f}}_s + \hat{\boldsymbol{d}} + \dot{\boldsymbol{s}}_r + \boldsymbol{\kappa}\,\mathrm{sat}(\boldsymbol{\phi}^{-1}\boldsymbol{s})\right], \tag{8}$$

where $\boldsymbol{\phi} \in \mathbb{R}^{m\times m}$ is a diagonal matrix with m positive entries ϕ_i, and $\mathrm{sat}(\cdot)$ is the saturation function:

$$\mathrm{sat}(s_i/\phi_i) = \begin{cases} \mathrm{sgn}(s_i) & \text{if} \quad |s_i/\phi_i| \geq 1, \\ s_i/\phi_i & \text{if} \quad |s_i/\phi_i| < 1\,. \end{cases} \tag{9}$$

Thus, we propose the adoption of an intelligent compensator $\hat{\boldsymbol{d}}$ to cope with the uncertainties and external disturbances related to $\boldsymbol{d}_s$. For each component $\hat{d}_i$, a zero order TSK (Takagi–Sugeno–Kang) inference system is established. The associated fuzzy rules can be stated in a linguistic manner as follows (Jang et al., 1997):

$$\textit{If } s_i \textit{ is } S_{ij}, \textit{ then } \hat{d}_{ij} = \widehat{D}_{ij} \quad ; \quad j = 1, 2, \ldots, N\,,$$

where S_{ij} are fuzzy sets, whose membership functions could be properly chosen.

Considering that the rules define numerical values as output $\widehat{D}_{ij}$, the final output of each $\hat{d}$ can be computed by a weighted average:

$$\hat{d}_i(s_i) = \widehat{\boldsymbol{D}}_i^\top \boldsymbol{\Psi}_i(s_i)\,, \tag{10}$$

where $\widehat{\boldsymbol{D}}_i = [\widehat{D}_{i1}\ \ldots\ \widehat{D}_{iN}]^\top$ are vectors containing the attributed values $\widehat{D}_{ij}$ to the fuzzy rules, $\boldsymbol{\Psi}_i(s_i) = [\psi_{i1}(s_i)\ \ldots\ \psi_{iN}(s_i)]^\top$, with $\psi_{ij}(s_i) = w_{ij}/\sum_{j=1}^N w_{ij}$, and w_{ij} is the firing strength of each rule.

In order to ensure the best possible estimate, let the vector of adjustable parameters be automatically updated by the following adaptation law

$$\dot{\widehat{\boldsymbol{D}}}_i = \varphi_i\, s_i\, \boldsymbol{\Psi}_i(s_i)\,, \tag{11}$$

where φ_i are strictly positive constants related to the adaptation rate.

Remark 1 *Considering that fuzzy logic can perform universal approximation (Kosko, 1994), the output of the TSK inference systems can approximate $\boldsymbol{d}_s$ to an arbitrary degree of accuracy $\boldsymbol{\delta} = \hat{\boldsymbol{d}}^* - \boldsymbol{d}_s$, where $\hat{\boldsymbol{d}}^*$ is the output related to set of optimal parameter vectors $\widehat{\boldsymbol{D}}_i^*$.*

Therefore, by defining the components of $\boldsymbol{\kappa} \in \mathbb{R}^m$ according to $\kappa_i \geq \eta + \delta_i + |\hat{d}_i|$, where η is a strictly positive constant related to the reaching time, Theorem 1 shows that the smooth intelligent controller, equation (8), ensures the convergence of the tracking error to the invariant set defined by the boundary layers.

Theorem 1 *Consider the uncertain underactuated mechanical system (2) subject to Assumptions 1–3. Then, the controller defined by (8), (10), and (11) ensures the convergence of the tracking errors to the manifold $\boldsymbol{\Phi} = \{\tilde{\boldsymbol{q}} \in \mathbb{R}^n \mid |s_i| \leq \phi_i, i = 1, \ldots, m\}$.*

Proof: Let a positive-definite Lyapunov function candidate V_2 be defined as

$$V_2(t) = \frac{1}{2}\boldsymbol{s}_\phi^\top \boldsymbol{s}_\phi\,, \tag{12}$$

where each component of $\boldsymbol{s}_\phi(\tilde{\boldsymbol{q}})$ is a measure of the distance between s_i and its related boundary layer, and is computed as follows:

$$\boldsymbol{s}_\phi(\tilde{\boldsymbol{q}}) = \boldsymbol{s} - \boldsymbol{\phi}\,\mathrm{sat}(\boldsymbol{\phi}^{-1}\boldsymbol{s})\,. \tag{13}$$

Noting that $\dot{\boldsymbol{s}}_\phi = \boldsymbol{s}_\phi = \boldsymbol{0}$ inside $\boldsymbol{\Phi}$, and $\dot{\boldsymbol{s}}_\phi = \dot{\boldsymbol{s}}$ outside of it, then the time derivative of V_2 becomes:

$$\begin{aligned}\dot{V}_2(t) &= \boldsymbol{s}_\phi^\top \dot{\boldsymbol{s}} = \boldsymbol{s}_\phi^\top \left(\boldsymbol{\alpha}_a \ddot{\boldsymbol{q}}_a + \boldsymbol{\alpha}_u \ddot{\boldsymbol{q}}_u + \dot{\boldsymbol{s}}_r\right)\\ &= \boldsymbol{s}_\phi^\top \big[\boldsymbol{\alpha}_a \boldsymbol{M}_{aa}^{'-1}(\boldsymbol{f}_a^{'} + \boldsymbol{u} + \boldsymbol{d}_a^{'})\\ &\quad + \boldsymbol{\alpha}_u \boldsymbol{M}_{uu}^{'-1}(\boldsymbol{f}_u^{'} - \boldsymbol{M}_{au}^\top \boldsymbol{M}_{aa}^{-1}\boldsymbol{u} + \boldsymbol{d}_u^{'}) + \dot{\boldsymbol{s}}_r\big]\\ &= \boldsymbol{s}_\phi^\top \big[\boldsymbol{f}_s + \dot{\boldsymbol{s}}_r + \boldsymbol{M}_s \boldsymbol{u}\big].\end{aligned} \tag{14}$$

Applying the control law (8) to (14), and noting that $\mathrm{sat}(\boldsymbol{\phi}^{-1}\boldsymbol{s}) = \mathrm{sgn}(\boldsymbol{s}_\phi)$ outside $\boldsymbol{\Phi}$ and $|\hat{d}_i - \hat{d}_i^*| \le |\hat{d}_i|$, one obtains

$$\begin{aligned}\dot{V}_2(t) &= -\boldsymbol{s}_\phi^\top \big[\hat{\boldsymbol{d}} - \hat{\boldsymbol{d}}^* + \boldsymbol{\delta} + \boldsymbol{\kappa}\,\mathrm{sgn}(\boldsymbol{s}_\phi)\big]\\ &\le -\eta \,\|\boldsymbol{s}_\phi\|_1 \,,\end{aligned}$$

which implies that $\boldsymbol{s}_\phi \to \boldsymbol{0}$ and $\tilde{\boldsymbol{q}} \to \boldsymbol{\Phi}$ as $t \to \infty$. □

3 Illustrative example: Stabilizing the cart-pole underactuated system

The cart-pole system is composed by a small car with an inverted pendulum on it (see Fig. 1), and the related equations of motion are presented as

$$\begin{bmatrix} m_c + m & ml\cos\theta \\ ml\cos\theta & ml^2 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \\ = \begin{bmatrix} ml\dot{\theta}^2 \sin\theta \\ mgl\sin\theta \end{bmatrix} + \begin{bmatrix} u \\ 0 \end{bmatrix}. \tag{15}$$

Here, x and θ are, respectively, the position of the cart and the angular displacement of the pendulum, m_c is the mass of the cart, m and l represent the concentrated mass and the length of the pendulum, and g is the acceleration due to gravity.

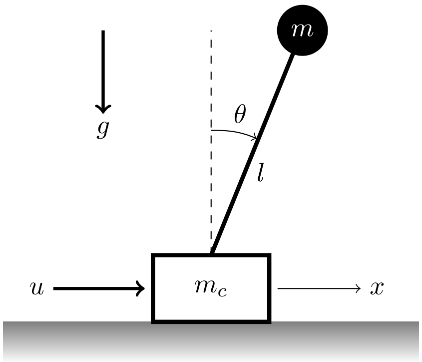


Figure 1: Cart-pole system.

It should be highlighted that, since only the position of the cart x can be directly controlled, the angular displacement θ is considered an unactuated variable. On this basis, Ashrafiuon and Erwin (2008) defined the switching variable as a linear combination of both actuated and unactuated state errors, $s = \alpha_a \dot{\tilde{x}} + \lambda_a \tilde{x} + \alpha_u l \dot{\tilde{\theta}} + \lambda_u l \tilde{\theta}$, with $\tilde{x} = x - x_d$ and $\tilde{\theta} = \theta - \theta_d$ as well as their derivatives representing the state errors. But considering that model uncertainties and external disturbances may occur, we suggest the inclusion of a compensation term $\hat{d}$ in the control law, in order to compensate for modeling inaccuracies and to improve the control performance:

$$u = -\hat{M}_s\big[\hat{f}_s + \hat{d} + \dot{s}_r + \kappa\,\mathrm{sat}(s/\phi)\big], \tag{16}$$

where κ is the control gain, ϕ defines the width of the boundary layer, $\dot{s}_r = -\alpha_a \ddot{x}_d - \alpha_u l \ddot{\theta}_d + \lambda_a \dot{\tilde{x}} + \lambda_u l \dot{\tilde{\theta}}$, $\hat{M}_s = [(\hat{m}_c + \hat{m}\sin^2\theta)l]/[\alpha_a l - \alpha_u \cos\theta]$, and $\hat{f}_s = \big[(\alpha_a l - \alpha_u \cos\theta)\hat{m}l\dot{\theta}^2\sin\theta - [\alpha_a l\hat{m}\cos\theta - \alpha_u(\hat{m}c + \hat{m})]g\sin\theta\big]/(\hat{m}_c + \hat{m}\sin^2\theta)l$. Parameters $\hat{m}_c$ and $\hat{m}$ represent the estimates of the cart and pendulum masses, respectively.

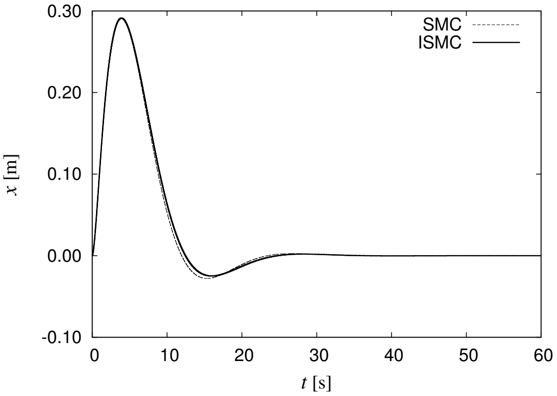
Thus, considering the cart-pole system, equation (15), and the intelligent controller presented in equation (16), numerical simulations were carried out to evaluate the efficacy of the proposed scheme. The simulation studies were performed with sampling rates of 1 kHz for control system and 10 kHz for dynamic model. The differential equations of the dynamic model were numerically solved with the fourth order Runge-Kutta method.

Firstly, in order to demonstrate the robustness of the intelligent controller against structured uncertainties, variations of 15% over the cart and pendulum masses are taken into account. On this basis, the following model parameters are considered: $m_c = 0.4\,\mathrm{kg}$, $m = 0.14\,\mathrm{kg}$, and $l = 0.215\,\mathrm{m}$. Regarding control parameters, the following values were chosen: $\hat{m}_c = 0.34\,\mathrm{kg}$, $\hat{m} = 0.119\,\mathrm{kg}$, $\alpha_a = 0.02$, $\alpha_u = 1$, $\lambda_a = 0.005$, $\lambda_u = 2.5$, $\phi = 0.05$, $\eta = 2$, $\delta = 0.5$, and $\varphi = 100$. Concerning the fuzzy system, triangular (in the middle) and trapezoidal (at the edges) membership functions are adopted, with the central values defined as $C = \{-\phi/4, -\phi/20, -\phi/40, 0, \phi/40, \phi/20, \phi/4\}$. Figure 2 shows the obtained results.

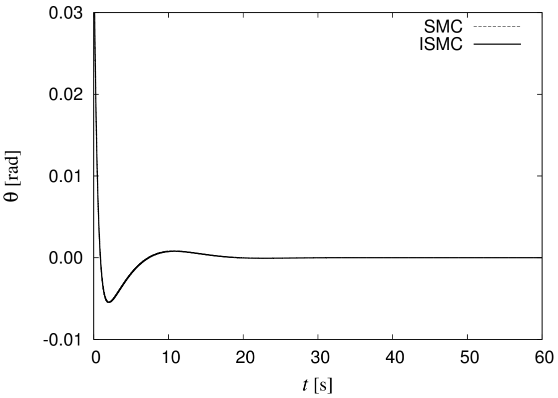
According to Fig. 2, if only small structured uncertainties are taken into consideration, as is the case in (Ashrafiuon and Erwin, 2008), both conventional and the proposed intelligent scheme are able to stabilize the cart-pole system. As observed, the performance of both conventional (SMC) and intelligent (ISMC) controllers is closely similar.

Let us now investigate what happens when the plant is subject to unmodeled dynamics. On this basis, by taking Coulomb friction into account, a dead-zone is also considered in the cart-pole model:

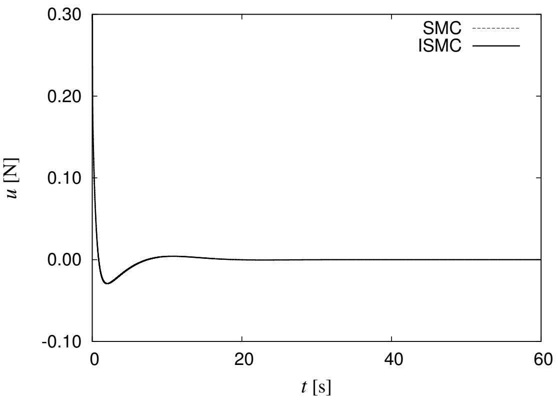
$$\nu = \begin{cases} u + 0.2 & \text{if} \quad u \le -0.2, \\ 0 & \text{if} \quad -0.2 < u < 0.2, \\ u - 0.2 & \text{if} \quad u \ge 0.2\,, \end{cases} \tag{17}$$



(a) Cart position.

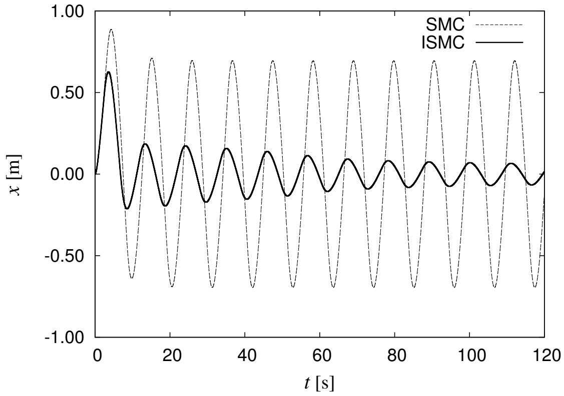


(b) Pendulum angle.

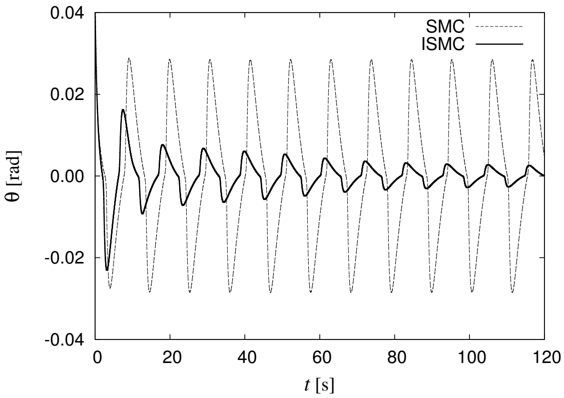


(c) Control action.

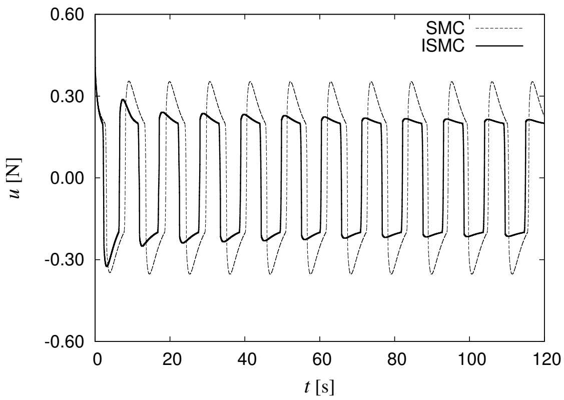
Figure 2: Stabilizing the cart-pole with only parametric uncertainties.



(a) Cart position.



(b) Pendulum angle.



(c) Control action.

Figure 3: Stabilizing the cart-pole with both parametric uncertainties and unmodeled dynamics.

where u is the same control action determined by the (16) and ν represents the new system input:

$$\begin{bmatrix} m_c + m & ml\cos\theta \\ ml\cos\theta & ml^2 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \\ = \begin{bmatrix} ml\dot{\theta}^2 \sin\theta \\ mgl\sin\theta \end{bmatrix} + \begin{bmatrix} \nu \\ 0 \end{bmatrix}. \quad (18)$$

Then, new simulations were carried out by considering the dead-zone input. Besides the unknown dead-zone, all other control and system parameters are kept the same as in the first numerical study. Figure 3 shows the obtained results.

As observed in Fig. 3, when unmodeled dynamics are also considered, the difference between the performance of the two control schemes is remarkable. Figure 3 shows the improved performance of the proposed intelligent controller when compared with the conventional control approach presented in (Ashrafiuon and Erwin, 2008), which confirms that, even in the case of underactuated systems, intelligent control represents the most adequate choice for plants subject to large uncertainties (Bessa and Barrêto, 2010; Bessa et al., 2012). In addition, it should be noted in Fig. 3 that the intelligent scheme is much more efficient than the conventional one. Besides improving the stabilization, Figs. 3a and 3b, it also shows a reduced control effort, Figs. 3c.

4 Concluding remarks

The present work addresses the problem of controlling uncertain nonlinear underactuated systems with a sliding mode control approach, but enhanced by a fuzzy compensation scheme. The convergence properties of the resulting intelligent controller are proved by means of the Lyapunov stability theory. In order to illustrate the controller design method, the proposed scheme is applied to a cart-pole system. The control system performance is confirmed by means of numerical simulations. The adoption of a fuzzy inference system provided an smaller balancing error due to its ability to compensate the performance drop caused by the change of the sign function for the saturation function. Also, the simulation studies show that the intelligent sliding mode controller can more efficiently deal with underactuated systems, even when a high level of uncertainty is involved.

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References

Ashrafiuon, H. and Erwin, R. S. (2008). Sliding mode control of underactuated multibody systems and its application to shape change control, *International Journal of Control* **81**(12): 1849–1858.

Bessa, W. M. (2009). Some remarks on the boundedness and convergence properties of smooth sliding mode controllers, *International Journal of Automation and Computing* **6**(2): 154–158.

Bessa, W. M. and Barrêto, R. S. S. (2010). Adaptive fuzzy sliding mode control of uncertain nonlinear systems, *Controle & Automação* **21**(2): 117–126.

Bessa, W. M., De Paula, A. S. and Savi, M. A. (2012). Sliding mode control with adaptive fuzzy dead-zone compensation for uncertain chaotic systems, *Nonlinear Dynamics* **70**(3): 1989–2001.

Bloch, A., Chang, D. E., Leonard, N. and Marsden, J. (2001). Controlled Lagrangians and the stabilization of mechanical systems II: Potential shaping, *IEEE Transactions on Automatic Control* **46**(10): 1556–1571.

Bloch, A. M., Leonard, N. and Marsden, J. (2000). Controlled Lagrangians and the stabilization of mechanical systems I: The first matching theorem, *IEEE Transactions on Automatic Control* **45**(12): 2253–2270.

Chen, Y.-F. and Huang, A.-C. (2012). Controller design for a class of underactuated mechanical systems, *IET Control Theory & Applications* **6**(1): 103–110.

Consolini, L., Maggiore, M., Nielsen, C. and Tosques, M. (2010). Path following for the PVTOL aircraft, *Automatica* **46**(8): 1284–1296.

Do, K. and Pan, J. (2009). *Control of Ships and Underwater Vehicles: Design for Underactuated and Nonlinear Marine Systems*, Advances in Industrial Control, Springer.

Gómez-Estern, F. and der Schaft, A. V. (2004). Physical damping in IDA-PBC controlled underactuated mechanical systems, *European Journal of Control* **10**(5): 451–468.

Jang, J.-S. R., Sun, C.-T. and Mizutani, E. (1997). *Neuro Fuzzy and Soft Computing: A Computational Approach to Learning and Machine Intelligence*, Prentice Hall, New Jersey.

Kosko, B. (1994). Fuzzy systems as universal approximators, *IEEE Transactions on Computers* **43**(11): 1329–1333.

Lai, X.-Z., She, J.-H., Yang, S. X. and Wu, M. (2009). Comprehensive unified control strategy for underactuated two-link manipulators, *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics* **39**(2): 389–398.

Liu, Y. and Yu, H. (2013). A survey of underactuated mechanical systems, *IET Control Theory & Applications* **7**(7): 921–935.

Muske, K. R., Ashrafiuon, H., Nersesov, S. and Nikkhah, M. (2010). Optimal sliding mode cascade control for stabilization of underactuated nonlinear systems, *Journal of Dynamic Systems, Measurement, and Control* **134**(2): 021020:1–11.

Nguyen, K.-D. and Dankowicz, H. (2015). Adaptive control of underactuated robots with unmodeled dynamics, *Robotics and Autonomous Systems* **64**: 84–99.

Ortega, R., Spong, M., Gómez-Estern, F. and Blankenstein, G. (2002). Stabilization of a class of underactuated mechanical systems via interconnection and damping assignment, *IEEE Transactions on Automatic Control* **47**(8): 1218–1233.

Pucci, D., Romano, F. and Nori, F. (2015). Collocated adaptive control of underactuated mechanical systems, *IEEE Transactions on Robotics* **31**(6): 1527–1536.

Qian, D. W., Liu, X. J. and Yi, J. Q. (2009). Robust sliding mode control for a class of underactuated systems with mismatched uncertainties, *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering* **223**(6): 785–795.

Rapp, C., Kreuzer, E. and Sri Namachchivaya, N. (2012). Reduced normal forms for nonlinear control of underactuated hoisting systems, *Archive of Applied Mechanics* **82**(3): 297–315.

Rudra, S., Barai, R. K. and Maitra, M. (2014). Nonlinear state feedback controller design for underactuated mechanical system: A modified block backstepping approach, *ISA Transactions* **53**(2): 317–326.

Ryalat, M. and Laila, D. S. (2016). A simplified IDA-PBC design for underactuated mechanical systems with applications, *European Journal of Control* **27**: 1–16.

Sankaranarayanan, V. and Mahindrakar, A. D. (2009). Control of a class of underactuated mechanical systems using sliding modes, *IEEE Transactions on Robotics* **25**(2): 459–467.

Seifried, R. (2012a). Integrated mechanical and control design of underactuated multibody systems, *Nonlinear Dynamics* **67**(2): 1539–1557.

Seifried, R. (2012b). Two approaches for feedforward control and optimal design of underactuated multibody systems, *Multibody System Dynamics* **27**(1): 75–93.

Seifried, R. (2013). *Dynamics of Underactuated Multibody Systems: Modeling, Control and Optimal Design*, Solid Mechanics and Its Applications, Springer.

Seifried, R. and Blajer, W. (2013). Analysis of servo-constraint problems for underactuated multibody systems, *Mechanical Sciences* **4**(1): 113–129.

Serrano, M., Scaglia, G., Godoy, S., Mut, V. and Ortiz, O. (2014). Trajectory tracking of underactuated surface vessels: A linear algebra approach, *IEEE Transactions on Control Systems Technology* **22**(3): 1103–1111.

Spong, M. W. (1994). Partial feedback linearization of underactuated mechanical systems, *IROS '94 – Proceedings of the IEEE/RSJ/GI International Conference on Intelligent Robots and Systems*, pp. 314–321.

Sun, N., Fang, Y. and Zhang, X. (2013). Energy coupling output feedback control of 4-DOF underactuated cranes with saturated inputs, *Automatica* **49**(5): 1318–1325.

Tanaka, M. C., de Macedo Fernandes, J. M. and Bessa, W. M. (2013). Feedback linearization with fuzzy compensation for uncertain nonlinear systems, *International Journal of Computers, Communications & Control* **8**(5): 736–743.

Tsiotras, P. and Luo, J. (2000). Control of underactuated spacecraft with bounded inputs, *Automatica* **36**(8): 1153–1169.

Xin, X. and Liu, Y. (2013). Reduced-order stable controllers for two-link underactuated planar robots, *Automatica* **49**(7): 2176–2183.

Xu, L. and Hu, Q. (2013). Output-feedback stabilisation control for a class of under-actuated mechanical systems, *IET Control Theory & Applications* **7**(7): 985–996.

Xu, R. and Özgüner, Ü. (2008). Sliding mode control of a class of underactuated systems, *Automatica* **44**(1): 233–241.