AN IMPROVED SYMBIOTIC ORGANISMS SEARCH ALGORITHM APPLIED TO NONLINEAR SYSTEM PARAMETER ESTIMATION PROBLEMS

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Abstract— In this paper, we propose a modified version of the Symbiotic Organisms Search (SOS) algorithm. The proposed version of SOS is applied to the nonlinear system parameter estimation problem. Numerical experiments are carried out to evaluate the performance of the proposed algorithm using two nonlinear models: the Output Error Polynomial (OEP) and the Output Error Rational (OER) models. The results show that the proposed algorithm provided good accuracy in both models, outperforming other algorithms. Also, the reduced number of parameters to be chosen simplifies the parameter tuning process of SOS when compared with other metaheuristics. Based on the results, the proposed version of SOS can be considered as a good alternative to solve nonlinear system parameter estimation problems.

Keywords— Symbiotic Organisms Search Algorithm, Parameters Identification, Nonlinear Systems.

Resumo— Neste artigo, é proposta uma versão modificada do Algoritmo de Busca por Organismos Simbióticos (*Symbiotic Organisms Search - SOS*). A versão proposta é aplicada ao problema de estimação de parâmetros de sistemas não lineares. Experimentos numéricos foram realizados com o objetivo de avaliar o desempenho do algoritmo proposto em dois modelos de sistemas não lineares: o modelo OEP (*Output Error Polynomial*) e o modelo OER (*Output Error Rational*). Os resultados mostraram que o algoritmo proposto apresentou boa acurácia nos dois modelos, apresentando melhor desempenho que outros algoritmos. Além disso, o número reduzido de parâmetros a serem definidos simplifica o processo de sintonização de parâmetros do algoritmo SOS, quando comparado a outras metaheurísticas. Com base nos resultados observados, é possível concluir que a versão proposta do SOS pode ser considerada como uma boa alternativa para a solução do problema de estimação de parâmetros de sistemas não lineares.

Palavras-chave— Algoritmo de Busca por Organismos Simbióticos, Identificação de Parâmetros, Sistemas Não Lineares.

1 Introduction

A wide variety of tools is available to solve the parameter identification problem for linear systems such as modal testing (Ewins, 2000) and modal analysis (Maia and Silva, 1997).

However, for nonlinear systems, a mathematical model to describe the system under consideration cannot be obtained by performing the system identification process at a single input level. The model obtained at a single operating point can, at best, provide information about the equivalent linear system at that operating point (Gondhalekar, 2009). Authors divide the nonlinear system identification problem into three different phases: Nonlinearity Detection, Nonlinearity Characterization, and Nonlinear Parameter Estimation (Gondhalekar, 2009).

The first phase (Nonlinearity Detection) consists in detecting the presence of nonlinearities within the system. Different techniques are available in the literature to perform this phase (Vanhoenacker et al., 2002).

The second phase (Nonlinearity Characterization) consists in characterizing the nonlinearities identified in the first phase. In this phase, the type, the location, the form, the intensity, and other characteristics of the nonlinearities in the system are investigated (Kerschen et al., 2006). Some techniques to perform the second phase can be found in the literature (Adams and Allemang, 1999), (Tanrikulu and Ozguven, 1991).

The third phase (Nonlinear Parameter Estimation) consists in estimating the parameter values of the nonlinear model. The techniques used in this phase are commonly divided into spatial methods (Masri and Caughy, 1979) and modal methods (Kerschen et al., 2009).

In this paper, we propose a modified version of the SOS algorithm and apply the proposed version in the third phase of the nonlinear system identification problem. Good results have been achieved in this problem by metaheuristic algorithms such as Artificial Neural Networks (Samad and Mathur, 1992), Genetic Algorithms (Yao and Sethares, 1994) and Particle Swarm Optimization

(Schwaab et al., 2008), among others. Numerical experiments using two nonlinear models are carried out to evaluate the performance of the proposed algorithm. The results obtained using the original version of SOS, as well as the results obtained in previous works using the Cuckoo Search via Lévy Fights algorithm (Souza et al., 2014) and the Teaching-Learning Based Optimization (TLBO) algorithm (Rodrigues et al., 2016) are also computed.

The remaining sections of this paper are organized as follows. Section 2 describes the nonlinear system parameter estimation problem under consideration. Section 3 introduces the SOS algorithm. Section 4 presents the proposed version of SOS. Section 5 presents the results obtained in numerical experiments carried out to evaluate the performance of the proposed version of SOS in two different models. Concluding remarks are presented in section 6.

2 Problem Description

In this section, we describe the nonlinear system parameter estimation problem under consideration. Consider a discrete nonlinear system A. Let $u(k)$ and $y(k)$ be the input and the output of the system at instant k, respectively. In this paper, we assume that the output $y(k)$ is a function of past inputs, past outputs and measurements noise. A mathematical model based on the nonlinear difference equation model is used to describe the behavior of system A according to Equation (1).

$$\begin{aligned} y(k) \;=\; & f(y(k-1),\ldots,y(k-n_y), \\ & u(k-1),\ldots,u(k-n_u)) + e(k) \end{aligned} \tag{1}$$

where $f(\cdot)$ is a nonlinear function, n_y and n_u are the maximum lags in the output and input, respectively, and $e(k)$ is a white noise.

The structure of the model is assumed to be known. Let θ be the parameter vector containing the parameters of the mathematical model of system A. Each element of vector θ is a real number. The nonlinear system parameter estimation problem consists in estimating the values of the elements of vector θ.

Historical data containing an input signal $u(k)$ and the corresponding $y(k)$, with $k \in \{1,\ldots,K\}$ are available. So, it is possible to define an objective function J that quantifies, for a given parameter vector θ, the mean squared error between the estimated output $\bar{y}(k)$ obtained with θ and the real output $y(k)$, when $u(k)$ is used as input. Therefore, estimates for the elements of vector θ can be obtained by minimizing the objective function J presented in Equation 2.

$$J = \frac{1}{M}\sum_{k=1}^{K}\left[y(k)-\bar{y}(k)\right]^2 \tag{2}$$

3 Symbiotic Organisms Search

The Symbiotic Organism Search (SOS) algorithm is a population-based metaheuristic optimization algorithm that simulates the symbiotic interaction strategies adopted by organisms to survive in the ecosystem (Cheng and Prayogo, 2014).

The word symbiosis is used to describe a relationship between two distinct species. The most common symbiotic relationships found in nature are mutualism, commensalism and parasitism (Kanimozhi et al., 2016). Mutualism is a symbiotic relationship between two different species in which both are benefited. One example of mutualism is the relationship between starlings and buffalo illustrated in Figure 1(A). Starlings remove ticks from buffalo skin and the itching on buffalo skin is reduced (Prakash and Rajathy, 2015). Commensalism is a symbiotic relationship between two different species in which one benefits and the other is neutral or unaffected. One example of commensalism is the relationship between remora fish and sharks illustrated in Figure 1(B). A remora fish is attached to a shark and eats the leftover food of a shark without benefiting or harming it (Tejani et al., 2016). Parasitism is a symbiotic relationship between two different species in which one benefits and the other is harmed. One example of parasitism is the deer tick shown in Figure 1(C). The deer tick attaches itself to an animal to suck its blood and in that way it gets benefited. However, the deer tick also carries Lyme disease, causing joint damage and kidney problems to the deer, which also suffers from the loss of blood and may get sick (Cheng and Prayogo, 2014).

The SOS algorithm simulates the different types of symbiotic relationships to find good solutions for the problem at hand. In each phase of the algorithm, one symbiotic relationship is represented by a distinct operator. The SOS algorithm comprises three phases: the mutualism phase, the commensalism phase and the parasitism phase.

The process of generating new candidate solutions in each phase of the SOS algorithm is driven by the characteristics of the symbiotic relationships (Talatahari, 2016). In the Mutualism Phase, new candidate solutions are created for both organisms. In the Commensalism Phase, a new candidate solution is created only for one organism. In the Parasitism Phase, one of the organisms can be replaced by a new solution. The process is repeated until the stop criteria are met. Figure 2 shows the flowchart of the SOS algorithm.

3.1 Mutualism Phase:

In the mutualism phase of SOS, for each organism X_i, an organism X_j, with $j \neq i$, is randomly chosen from the population to interact with organism

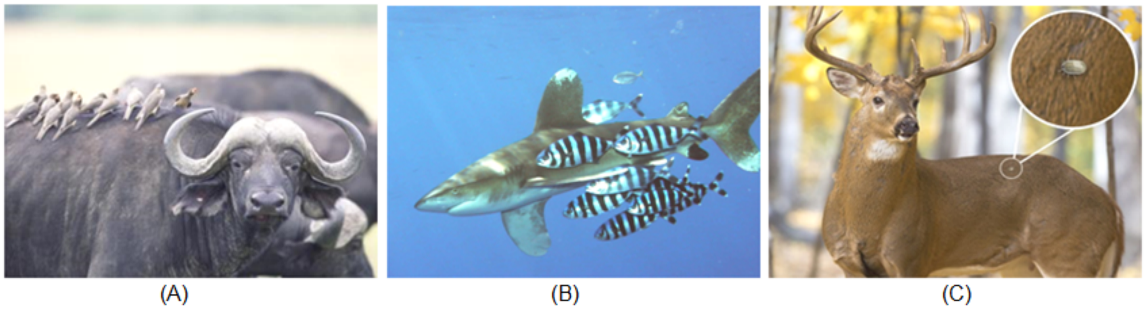

Figure 1: Examples of symbiotic relationships: (A) Mutualism between bufalo and starligs; (B) Commensalism between remora fish and sharks; (C) parasitism between deer and deer tick

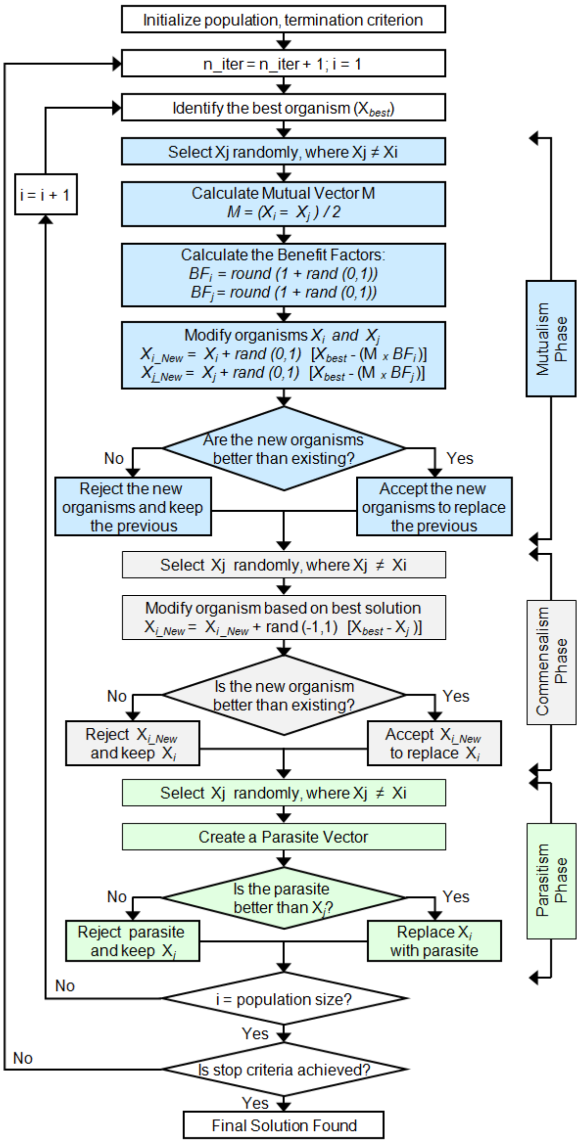

Figure 2: SOS Flowchart

X_i. In a mutualistic symbiosis, both organisms benefit from the relationship. New candidate solutions for both organisms X_i and X_j are calculated according to Equations (3) and (4), respectively.

$$X_{iNew} = X_i + \texttt{rand}(0,1) \times D_i \tag{3}$$
$$X_{jNew} = X_j + \texttt{rand}(0,1) \times D_j \tag{4}$$

where D_i and D_j are given by Equations (5) and (6), respectively.

$$D_i = X_{best} - (M \times BF_i) \tag{5}$$
$$D_j = X_{best} - (M \times BF_j) \tag{6}$$

where BF_i and BF_j are the benefit factors for organisms X_i and X_j, calculated according to Equations (7) and (8), respectively, X_{best} is the organism with the best solution in the ecosystem, and M is the mutual vector that represents the relationship between organisms X_i and X_j, calculated according to Equation (9).

$$BF_i = \texttt{round}\,(1 + \texttt{rand}(0,1)) \tag{7}$$
$$BF_j = \texttt{round}\,(1 + \texttt{rand}(0,1)) \tag{8}$$
$$M = \frac{X_i + X_j}{2} \tag{9}$$

Benefit factors are calculated independently for each organism because some mutualism relationships produce a greater advantage for one of the organisms. In the mutualism phase of SOS, benefit factors BF_i and BF_j are determined randomly as either 1 or 2.

If the new candidate solution X_{iNew} is better than X_i, then solution X_{iNew} is accepted and solution X_i is discarded. Similarly, if the new candidate solution X_{jNew} is better than X_j, then solution X_{jNew} is accepted and solution X_j is discarded.

3.2 Commensalism Phase:

Similar to the mutualism phase, in the commensalism phase of SOS, for each organism X_i, an organism X_j, with $j \neq i$, is randomly chosen from the population to interact with organism X_i. However, in a commensalistic relationship, only organism X_i benefits from the interaction, while organism X_j is not affected. A new candidate solution for organism X_i is calculate according to Equation (10).

$$X_{iNew} = X_i + \texttt{rand}(-1,1) \times (X_{best} - X_j) \quad (10)$$

If the new candidate solution X_{iNew} is better than X_i, then solution X_{iNew} is accepted and solution X_i is discarded.

3.3 Parasitism Phase:

In the parasitism phase of SOS, organism X_i generates a parasite vector. This parasite vector is created by replicating organism X_i and then modifying some randomly selected dimensions using a random number. An organism X_j, with $j \neq i$, is randomly chosen from the population to act as the host of the parasite vector.

The parasite vector is compared with organism X_j using the objective function. If the parasite is better than organism X_j, then parasite kills organism X_j and occupies its position in the ecosystem. Otherwise, X_j is considered immune to the parasite vector and is not affected. The parasite vector is discarded.

4 Proposed Version of SOS

In this section, we present a modified version of the Symbiotic Organism Search algorithm, denoted by MSOS. We add modifications in the commensalism and parasitism phases of SOS.

During the commensalism phase of SOS, an organism X_j is randomly selected to interact with organism X_i. Then, organism X_i is modified to try to improve its solution. However, using this approach, an organism X_j with a worse solution could guide the generation of the candidate solution X_{iNew}, which is not logical considering the commensalism relationships in nature.

In the parasitism phase of SOS, an organism X_j is also randomly selected to interact with organism X_i. Then, organism X_i is modified to create a parasite vector that will try to replace organism X_j in the ecosystem. However, if an organism X_j with a high quality solution is selected, the probability of creating a parasite that will kill organism X_j is low.

In the proposed MSOS, we compare the solutions of organisms X_i and X_j during commensalism and parasitism phases in order to define which organism is going to benefit from the interaction (in the commensalism phase) and which organism will be attacked by the parasite (in the parasitism phase). In the commensalism phase of MSOS, the organism with the worst solution between organisms X_i and X_j will benefit from the interaction. In the parasitism phase of MSOS, the organism with the best solution between organisms X_i and X_j will generate the parasite that will try to kill the organism with the worst solution. The motivation behind the proposed MSOS is to increase the probability of generating new solutions during the execution of the algorithm.

5 Numerical Experiments

In this section, we present the results obtained in numerical experiments carried out to investigate the performance of MSOS in two different models: the Output Error Polynomial (OEP) model and the Output Error Rational (OER) model.

Both the OEP and the OER models were already used to evaluate the performance of metaheuristic algorithms in parameter estimation of nonlinear systems. The Cuckoo Search via Lévy Fights algorithm was used in (Souza et al., 2014), while the Teaching-Learning Based Optimization (TLBO) algorithm was used in (Rodrigues et al., 2016). The results obtained in these works are used in this paper as a comparison reference.

5.1 Parameter Settings

In order to provide a fair comparison among the different algorithms, we will define a fixed number of objective function evaluations. In (Souza et al., 2014), the authors used as stop criterion the maximum number of generations (1,000) without a significant improvement in the objective function, a population size p of 15, a discovery rate r of 25%. In (Ong, 2014), the author concluded that the average number of generations needed for the Cuckoo Search algorithm to converge was about 1,500. Thus, the estimated number of generations, g, needed for the stop criterion adopted in (Souza et al., 2014) to be reached is 2,500. Finally, the number of objective function evaluations in the Cuckoo Search algorithm, n_{CS}, can be obtained based on Equation (11).

$$n_{CS} = p + g \times [1 + (r \times p)] \quad (11)$$

The estimated number of objective function evaluations in the Cuckoo Search algorithm in (Souza et al., 2014) is 11,890. This number of objective function evaluations will be adopted in this paper.

To choose the population size for the SOS algorithm, PS, we used nine possible values: [20, 30, ..., 100]. For each candidate value, a Monte Carlo method with 30 iterations was performed. The best results for the OEP and the OER models were 40 and 30, respectively.

5.2 OEP Model

In this experiment, the Output Error Polynomial (OEP) model originally presented in (Piroddi and

Spinelli, 2003) is considered. The OEP model is described by Equations (12) and (13). Figure 3 shows the Matlab Simulink ® implementation of the OEP model used to run the experiments.

$$\begin{aligned} w(k) &= 0.75w(k-2) + 0.25u(k-1) \\ &\quad -0.2w(k-2)u(k-1) \end{aligned} \quad (12)$$

$$y(k) = w(k) + e(k) \quad (13)$$

Figure 4 shows the signal used as input for the OEP model, $u(k)$, as well as the corresponding output signal, $y(k)$. One characteristic of the OEP model is that it presents an unstable behavior, which brings an additional difficulty to the parameter estimation problem because it may generate multiple local minima in the objective function. In this example, the minimum and the maximum values of $y(k)$ are -3.67x10^{12} and 6.93x10^{3}, respectively.

A series of 300 measurements was used in the experiments. The error $e(k)$ is a white Gaussian noise with mean zero and variance 0.25. A Monte Carlo method with 1,000 iterations was used. Table 1 shows the results obtained with the different algorithms.

The results presented in Table 1 show that, for the OEP model, all the metaheuristics provided an excellent accuracy in the estimates of parameters θ_1 and θ_3. It can also be noticed that θ_2 is the most difficult parameter to be estimated in this model. It was consistently observed in the results of all the metaheuristics. Based on these results, the applicability of SOS to the nonlinear system parameter estimation problem can be validated. Also, the proposed MSOS presented better estimates for all the parameters, with more accurate mean values and lower standard deviations.

5.3 OER Model

In this experiment, the Output Error Rational (OER) model originally presented in (Zhu, 2005) is considered. The OER model is described by Equations (14) and (15). Figure 5 shows the Matlab Simulink ® implementation of the OER model used to run the experiments.

$$w(k) = \frac{N(k)}{D(k)} \quad (14)$$

$$y(k) = w(k) + e(k) \quad (15)$$

where $N(k)$ and $D(k)$ are defined according to Equations (16) and (17), respectively.

$$\begin{aligned} N(k) &= 0.3w(k-1)w(k-2) \\ &\quad +0.7u(k-1) \end{aligned} \quad (16)$$

$$D(k) = 1 + w(k-1)^2 + u(k-1)^2 \quad (17)$$

For the OER model, a random number generation block was used to generate an uniformly distributed input signal with mean zero and variance 0.33. A series of 1,000 measurements were used in the experiments. The error $e(k)$ is a white Gaussian noise with mean zero and variance 0.01. A Monte Carlo method with 1,000 iterations was used. Table 2 shows the results obtained with the different algorithms.

The results presented in Table 2 show that, for the OER model, MSOS outperformed the original SOS algorithm in terms of mean value for all parameters θ_i. MSOS presented lower standard deviations for all parameters, except for θ_4. Again, the results present by MSOS were consistent in comparison with the other algorithms in terms of the relative difficulty to estimate the parameters, with θ_3 presenting the highest standard deviation.

6 Conclusions

In this paper, we proposed a modified version of the Symbiotic Organism Search (SOS) algorithm, denoted by MSOS, and investigated its performance in the solution of the nonlinear system parameter estimation problem. We conducted numerical experiments using the Output Error Polynomial (OEP) and the Output Error Rational (OER) models. The results obtained with MSOS were compared with the results obtained with the original SOS, as well as with the results obtained by (Souza et al., 2014) with the Cuckoo Search via Lévy Fights algorithm and with the results obtained by (Rodrigues et al., 2016) with the Teaching-Learning Based Optimization (TLBO) algorithm.

The comparison with other algorithms validated the applicability of SOS to solve the nonlinear system parameter estimation problem. For the OEP model, MSOS outperformed the SOS algorithm in terms of mean value for all parameters. MSOS also presented a lower standard deviation for all parameters. For the OER model, MSOS outperformed the SOS algorithm in terms of mean value for all parameters. The standard deviation obtained for parameter θ_4 with SOS was lower in comparison with MSOS. For the other parameters, MSOS presented lower standard deviations.

Although many variants for all the metaheuristics considered in this paper can be found in the literature, we used the original version of all the algorithms in this paper. One possible suggestion for extending this paper is to evaluate the performance of different variants of the metaheuristics in the solution of the nonlinear system parameter estimation problem.

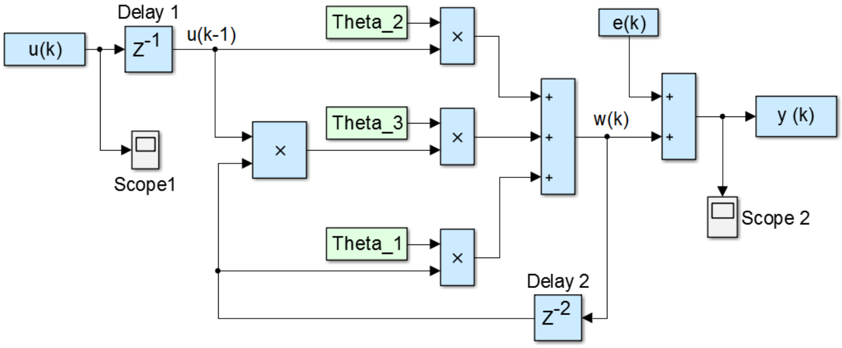

Figure 3: OEP model

Table 1: Simulation results for the OEP model

θ_i	Ref.	CS		TLBO		SOS		MSOS	
		Mean	Std	Mean	Std	Mean	Std	Mean	Std
θ_1	0.75	0.7500	0.0000	0.7499	0.0009	0.7498	0.0008	0.7499	0.0004
θ_2	0.25	0.2336	0.0402	0.2424	0.0508	0.2668	0.0591	0.2597	0.0363
θ_3	−0.20	−0.2000	0.0000	−0.2004	0.0006	−0.2003	0.0008	−0.2001	0.0001

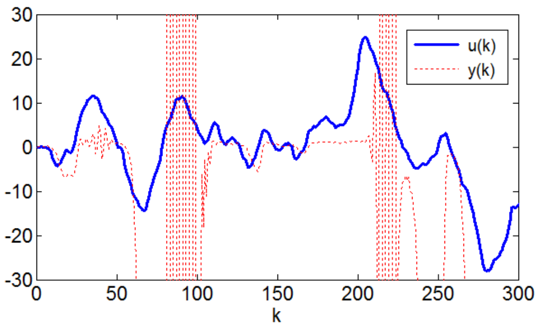

Figure 4: Unstable behavior of the OEP model

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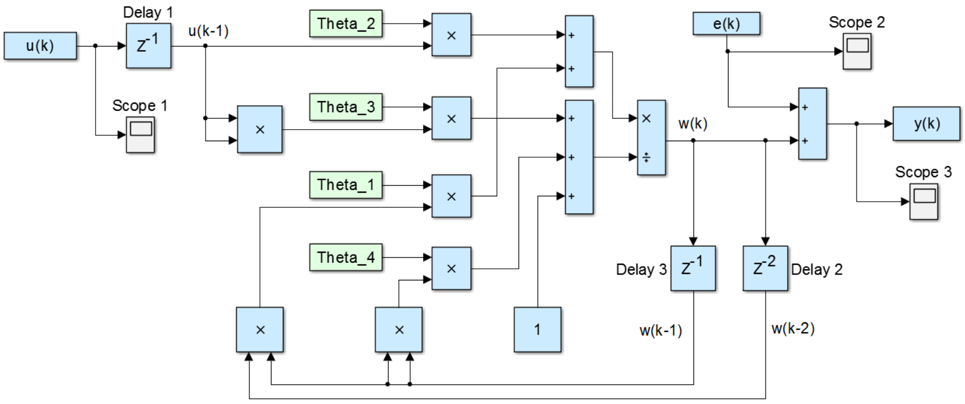

Figure 5: OER simulation model

Table 2: Simulation results for the OER model

θ_i	Ref.	CS		TLBO		SOS		MSOS	
		Mean	Std	Mean	Std	Mean	Std	Mean	Std
θ_1	0.3	0.3008	0.0745	0.2981	0.0184	0.2925	0.0601	0.3027	0.0054
θ_2	0.7	0.6981	0.0266	0.6962	0.0092	0.7151	0.0327	0.7113	0.0139
θ_3	1.0	0.9693	0.5121	0.9795	0.1587	0.9481	0.5905	0.9865	0.4158
θ_4	1.0	0.9971	0.0837	0.9923	0.0440	0.9833	0.0612	1.0049	0.0695

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