EVALUATION OF MATRIX PENCIL ALGORITHMS APPLIED IN ELECTRIC POWER SYSTEMS

Renato S. Dorighello*, Gustavo H. C. Oliveira†

*Department of Electrical Engineering, Federal University of Paraná, 81531-980 Curitiba, Brazil

Emails: dorighello@ufpr.br, gustavo@eletrica.ufpr.br

Abstract— Electric power systems sometimes experience transient situations in which their steady state behavior is no longer respected. In these scenarios, the electric power supply can be compromised as well as economic losses may occur. In order to better predict such accidents, system identification uses several methods to define signal parameters. One method is the definition of frequency-dependent network equivalent (FDNE) models to analyse eletromacnetic transitories. Another method is the definition of transient signals' eletromechanic modes based on Wide Area Monitoring (WAM) systems. An important technic of parameter identification is the Matrix Pencil method, which can be applied in both problems described. In this work, it is proposed to study and evaluate the performance of the Matrix Pencil algorithm in the WAM problem and in the estimation of eletromechanic modes. It is also purposed to compare it with the Vector Fitting method, which has been used in similar applications. Three study cases are presented in order to evaluate and compare the problem described: One synthetic test signal and two real cases considering ringdown data extracted from the North American Eastern Interconnection system and from the Brazilian Interconnected Power system.

Keywords— Power Systems Stability, Electromechanical Modes, Ringdown Analysis, System Identification, Matrix Pencil.

1 Introduction

System identification is a very important field in engineering, which enables us to create models of dynamic systems, in order to study and analyse them. When it comes to modelling power systems, the difficulties faced in the process are due to the fact that power systems have become extensive, complicated and ever more interconnected (Pierre et al., 2012). Therefore, its modelling is on the one hand challenging but on the other hand very necessary, given the importance of electricity.

To model these systems and all their complexity, one possibility is to create frequency dependent network equivalents (FDNE) of large power systems in order to study electromagnetic oscillations. In the frequency domain, this model is inferred by using a rational approximation of the network admittance matrix that could be obtained by different methods (Sheshyekani and Tabei, 2014).

Aother application of power systems' modelling, is the estimation of eletromechanic modes from systems that suffer transient interferences. In other words, when the oscillations in the system are originated from a big disturbance, we have a transient response of big intensity, this transient period is called ringdown. However, electric power system suffer constant little perturbations as well, such as load variations and minor topological changes. The measurements during this normal operation time are called ambient data (Pierre et al., 2012). In this paper, the identification will be focused on ringdown signals.

As far as linear methods based on ringdown data are considered, estimating methods were developed working either in the time domain or in the frequency domain (Peng and Nair, 2012), (Hwang and Liu, 2017), (Crow and Singh, 2005), (Schumacher et al., 2018). Among these methods, we have for example, the Prony Method, which was one of the earliest to work with time domain transient signals (Sarkar and Pereira, 1995), the Subspace method, that has applications both in time and frequency response data (McKelvey et al., 1996) (Viberg, 1995), the Vector Fitting (VF), that has been widely used for the fitting of measured or calculated time-domain responses and also applications in ringdown signals (Schumacher et al., 2018).

Another frequently used method is the Matrix Pencil (MP) which was first introduced by Sarkar and Pereira (1995) to identify parameters of time domain transient data. Further on, it was modified by Sheshyekani et al. (2012) to operate also with FDNE models. The MP algorithm shows satisfactory noise filtering results, specially in lower frequencies (Sheshyekani et al., 2012), what, in the field of system identification, could imply in a significant reduction of the computational burden. (Jeremias et al., 2012) The MP method presents a direct process, whereas the VF method, for instance, requires the definition of starting poles. Through the Matrix Pencil, this poles are obtained by a generalized eigenvalue problem, as shown by Sarkar and Pereira (1995). Even though both MP and VF are widely applied in identification problems, studies that compare them in order to evaluate in which case each one has a better performance are still sparse.

This paper presents an application of the MP method in the identification of eletromechanic modes in eletric power systems. Furthermore, the performance of the method in the presence of noise is evaluated. The problem is applied in the time domain with further intention to be transformed to the frequency domain as well. The obtained results will then be compared with the performance of the VF method applied to the same problem as showed by Schumacher et al. (2018), so that a thorough comparison can be presented.

The paper is organizes as follows: In the second section, the problem is stated, in the third section, the MP method is explained. In the forth, the simulations' results are shown. The conclusions are featured in section five.

2 Problem Statement

Due to electric power systems' large scale and small frequency inter-area oscilation, Wide Area Monitoring (WAM) systems are necessary in view of detecting and further counteracting grid instabilities. The measurements are performed by Phasor Measurement Units (PMU), which extract phasors (magnitude and phase angle) of the voltage and current signals in a power system. All phase angles are precisely referenced to a common time frame with the aid of the GPS. The data is collected, time aligned and stored by Phasor Data Concentrators (PDCs). (Annakage et al., 2017) (Ray, 2017).

A WAM system is represented in Figure 1.

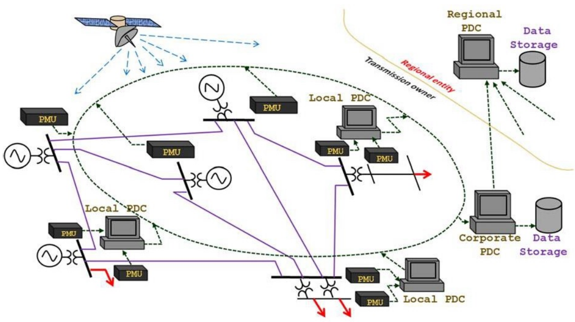

Figure 1: Typical architecture of a wide area synchrophasor network (Annakage et al., 2017).

When an electric power system suffers a sudden disturbance, the representation of this phenomenon is similar to the representation of an impulse applied as input to a linearised system (Kennaugh and Moffatt, 1965), which is much simpler to analyse. In order to best predict the behaviour of the power system in a transient scenario, these signals, which are frequently referred to as ringdown, are well modelled as a sum of damped sinusoids (Pierre et al., 2012).

So, the ringdown data for a SISO system is represented as follows. (Liu, 2010):

$$y(t) = x(t) + n(t) \approx \sum_{1}^{M} R_i e^{(s_i t)} + n(t), \quad (1)$$

$$0 \le t \le T, \qquad i = 1, 2 \cdots, M.$$

where:
$y(t)$: observed time response,
$n(t)$: noise in the system,
$x(t)$: signal,
R_i : residues or complex amplitudes,
$s_i = -\alpha_i + j\omega_i$,
α_i : damping factors,
ω_i : angular frequencies ($\omega_i = 2\pi f_i$),
M : signal's order.

The problem of eletromechanic modes estimation is equivalent to the estimation of angular frequencies and damping factors of a signal modelled by equation 1.

3 Matrix Pencil Method

The Matrix Pencil indentification algorithm uses, in this work, the general representation of a ringdown signal as shown in equation (1) to estimate the system's modes, represented by z_i, as in equation (3) bellow. Hence, for the sampled signal, t will be substituted for $T_s k$, where T_s is the sampling time and k is a natural number:

$$y(T_s k) = x(T_s k) + n(T_s k) \approx \sum_{1}^{M} R_i z_i^k + n(T_s k), \quad (2)$$

$$k = 0, \cdots, N-1.$$

and

$$z_i = e^{(-\alpha_i + j\omega_i)T_s}, \qquad i = 1, 2 \cdots, M. \quad (3)$$

The main goal of the algorithm is to find the best estimates for R_i and z_i. Therefore, once all the N samples are obtained, a Hermitian matrix with size $(N-L)\times(L+1)$ is defined as proposed by Sarkar and Pereira (1995):

$$Y = \begin{bmatrix} y(0) & y(1) & \cdots & y(L) \\ y(1) & y(2) & \cdots & y(L+1) \\ \vdots & \vdots & \ddots & \vdots \\ y(N-L-1) & y(N-L) & \cdots & y(N-1) \end{bmatrix} \quad (4)$$

where L is the Pencil Parameter and it is chosen between $N/2$ and $N/3$.

From Matrix Y, two sub-matrixes, with dimensions $(N-L)\times(L)$, are created as follows:

$$Y_1 = \begin{bmatrix} y(0) & y(1) & \cdots & y(L-1) \\ y(1) & y(2) & \cdots & y(L) \\ \vdots & \vdots & \ddots & \vdots \\ y(N-L-1) & y(N-L) & \cdots & y(N-2) \end{bmatrix} \quad (5)$$

$$Y_2 = \begin{bmatrix} y(1) & y(2) & \cdots & y(L) \\ y(2) & y(3) & \cdots & y(L+1) \\ \vdots & \vdots & \ddots & \vdots \\ y(N-L) & y(N-L+1) & \cdots & y(N-1) \end{bmatrix} \quad (6)$$

As Sarkar and Pereira (1995) proposed, the parameters z_i from equation (2) are equal (in the noiseless case) to the eigenvalues λ_i from the pair of matrixes Y_1 and Y_2. These parameters can be obtained through the solution of a generalized eigenvalue problem represented bellow:

$$Y_1^+ Y_2 - \lambda I \quad (7)$$

where I is an identity matrix, λ is a diagonal matrix containing the eingenvalues of the pair $Y_1^+ Y_2$ and Y_1^+ is the pseudo-inverse of the matrix Y_1 defined by:

$$Y_1^+ = \{Y_1^H Y_1\}^{-1} Y_1^H \quad (8)$$

However, when dealing with noise infected data, the simple eigenvalue problem showed above does not result in the problem's sollution and has to be addapted, in order to find the correct system's modes. The method's next step consists in a singular value decomposition (SVD), which is a factorisation of the matrix Y, in order to obtain three oder matrixes containing eigenvectors and eigenvalues (Nascimento, 2012) as shown bellow:

$$Y = UEV^H \quad (9)$$

in which
$U \rightarrow$ Matrix of the eigenvectors of Y^H,
$V \rightarrow$ Matrix of the eigenvectors of $Y^H Y$,
$E \rightarrow$ Diagonal Matrix containing the singular values of Y,
$[.]^H \rightarrow$ Transposed conjugate of the matrix.

After defining the singular value matrix E for the noise infected data, the sistem's order estimation M is chosen, which is the number of significant singular values of the main matrix, defined by:

$$\frac{\sigma_c}{\sigma_{max}} \geq 10^{-p} \quad (10)$$

in which p is the order of significant decimal digits in the data, chosen by the operator, σ_c represents an evaluated singular value and σ_{max} is the largest singular value of the matrix E. The number of singular values that follow (10) is equal to M.

3.1 Pre-filtering

Before estimating the parameters of the valuated signal, a filtering process is proposed in order to eliminate the noise effect in the signal (Sarkar and Pereira, 1995). The parameter M is extremely important at this moment, because it will help eliminate the elements that are above the signal's order, in other words, eliminate the non-significative values of the sampled data.

New submatrixes are created from matrix V as follows:

$$V' = [V_1, V_2, \cdots, V_M] \quad (11)$$

Values from $M+1$ until L, corresponding to the lower singular values are discarded. Therefore, it is observed that:

$$Y_1 = UE'[V_1']^H \quad (12)$$

$$Y_2 = UE'[V_2']^H \quad (13)$$

in which V_1' is obtained by eliminating the last row of V', V_2' is obtained by eliminating the first row of V' and E' is obtained from the first M columns of E, which correspond to the dominant singular values.

After this reductions, it has been proven in Hua and Sarkar (1990) that the eigenvalue problem shown in equation (7) can be rewritten through the new obtained matrixes V_1' and V_2':

$$\{[V_2']^H - \lambda [V_1']^H\} \rightarrow \{\{[V_2']^H\}^+ [V_1']^H\} - \lambda I\} \quad (14)$$

Through this method, the estimation of the modes z_i ,which are equal to λ_i, in noise presence becomes more precise. Once the modes and M are known, the residues R_i are solved through the following least squares problem:

$$\begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \cdots & z_M \\ \vdots & \vdots & \ddots & \vdots \\ (z_1)^{N-1} & (z_2)^{N-1} & \cdots & (z_M)^{N-1} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_M \end{bmatrix} \quad (15)$$

3.2 Parameters estimation

In order to estimate the model's parameters showed in equations 2 and 3, a set of mathematical properties were used to extrat those parameters from the modes (z_i) and residues (R_i) obtained through the Matrix Pencil Method in equation 15.

The equations used are better described in Nascimento (2012) and Jeremias et al. (2012) but are summarized in Table 1. The operators Re and Im represent the real and the imaginary value of a complex number.

Table 1: Synthetic Signal's Parameters.

Parameter	Equation
Damping (α_i)	$Re(z_i)$
Frequency (ω_i)	$tg^{-1}(\frac{Im(z_i)}{Re(z_i)})$
Amplitude (A_i)	$\|R_i\|$
Phase (ϕ_i)	$angle(R_i)$

4 Results

In this paper, three cases were evaluated in order to validate the Matrix Pencil algorithm accuracy. The cases were taken from Schumacher et al. (2018) where the authors worked with a novel method called Ringdown Time-Domain Vector Fitting (RTD-VF), the results concerning the RTD-VT method's performance can also be found in the article quoted. These three cases were applied then to the MP algorith and a comparison between the two methods was carried out.

In the first study case, a synthetic signal with parameters known beforehand was evaluated by the algorithm. On the other two cases, extracted from the North American Eastern Interconnection (NAEI) system and from the Brazilian Interconnected Power (BIP) system, the parameters were not known. Therefore, the so-called R^2 coefficient of determination is used to compare the performance of RTD-VF and MP methods. The R^2 coefficient of determination is defined by:

$$R^2 = \left(1 - \frac{\sum_{k=0}^{N-1}(y_n[k] - \hat{y}[k])^2}{\sum_{k=0}^{N-1}(y_n[k] - \bar{y}_n)^2}\right) \times 100\% \quad (16)$$

where $y_n[k]$ is the measured ringdown signal with mean value $\bar{y}_n$ and $\hat{y}[k]$ corresponding to the estimated value of $y_n[k]$. This coefficient indicates the fitting rate between measured and estimated data, with $R_2 = 100\%$ representing a perfect fit.

4.1 Synthetic signal identification

The synthetic signal evaluated in the first study case follows the equation 17.

$$\begin{gathered} y(t) = 1 \times e^{-0.1697t}\cos(1.4351t - 2.5122)+ \\ 1.32 \times e^{-0.8150t}\cos(3.9270t - 1.8850)+ \\ 1.13 \times e^{-1.8230t}\cos(6.4654t - 0.3142). \end{gathered} \quad (17)$$

The above described signal was applied to the Matrix Pencil algorithm, considering an estimation order equal to six ($M = 6$) and sampling time $Ts = 0.01s$. However, before the identification process, a white Gaussian noise $n[k]$ with zero mean and 0.05^2 variance was added to the simulated data $y(t)$. Therefore, measurements are actually given by $y_n[k] = y[k] + n[k]$. Figure 2 shows the noise afected signal and the signal estimated by the MP algorithm. It is possible to observe that the noise interference is eliminated by the MP algorithm's performance.

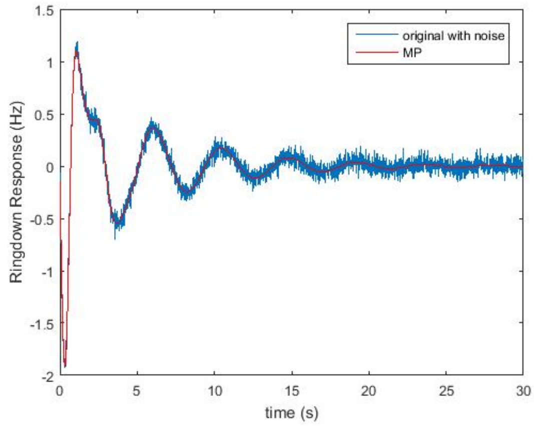

Figure 2: Synthetic signal reconstruction

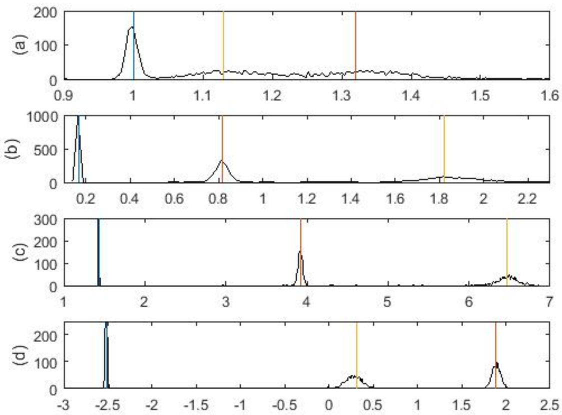

Figure 3: Estimated normalized probability density funtions for: (a) amplitude, (b) damping factor, (c) angular frequency and (d) phase. The real values of the parameters are represented by the highlighted constants

Conjointly, an estimation of the signal's parameters was performed. The parameter estimation procedure was repeated 1000 times for estatistics analysis. The resulting histogram, for the 3 estimated modes, is showed in Figure 3 and resembles a gaussian distribution. Some parameters are more directly identified, whereas the amplitude estimation curve, for example, shows a bigger bias. In Table 2 the estimated parameters' mode is shown and compared with the results given through the RTD-VF method, both estimation values are very close and have estimated the original value of almost all of the parameters.

Table 2: Synthetic Signal's Parameters.

	Method	modes (i)		
		1	2	3
A_i	MP	1	1.14	1.32
	RTD-VF	1	1.13	1.32
α_i	MP	-0.1697	-0.8150	-1,827
	RTD-VF	-0.1697	-0.8150	-1.823
ω_i	MP	1.435	3.927	6.465
	RTD-VF	1.4351	3.927	6.4654
ϕ_i [rad]	MP	-2.513	1.885	0.3142
	RTD-VF	-2.513	1.885	0.3142

4.2 NAEI system

On April 27, 2017, austere weather conditions led to a large generation trip in the Easter Interconnection (EI) system in North America and as a consequence, low frequency ocillations (LFOs) occured between the northern and the southern areas of the system, during the ringdown event (Hwang and Liu, 2017). Two sets of Frequency Disturbance Recorders (FDR) frequency data were chosen, one from Maine and the second from Florida with sampling time of $Ts = 0.1s$. Those two sets of data were gathered as a difference sequence 'Maine-Florida' (signal Maine *minus* signal Florida) to form a ringdown sequence as shown in Figure 4 (a). The signal was then applied (after a specified time of 5.6s) to the MP algorithm and the results of the parameters estimation and signal reconstruction are shown in Table 3 and in Figure 4 (b). In Table 3, the values obtained from the MP algorithm are compared to the ones from the RTD-VF method, which are very close values and present high R^2 rates. Since the original parameters of the system are unknown and the signal reconstruction is well suceeded, the method fullfills it's aim.

4.3 BIP system

As presented by Canizares et al. (2017), after an electrical bushing explosion, on September 02, 2011 at 19h43m3s UTC (Coordinated Universal Time), the Itaipu Hydroeletric was disconnected from the rest of the BIP system and reconnected at 19h49m7s. Both disconnection and reconnection events caused the southern, northern, southeastern and northeastern areas from the system to oscillate against each other with varying dc components. Oscillations were measured by a FDR (located at the Federal University of Santa Catarina), with a sampling time of $Ts = (1/60)s$. The sampled signal is taken (after the time of 1.5s) and the mode indentification MP algorithm is applied. Table 4 shows both parameter estimations from the MP and the RDT-VF. Figure 5 shows the reconstruction of the BIP signal through the modes found by the MP algorithm. In this third case study, the MP method shows once more it's estimation accuracy and when compared to the RTD-VF, presents very satisfying parameter values.

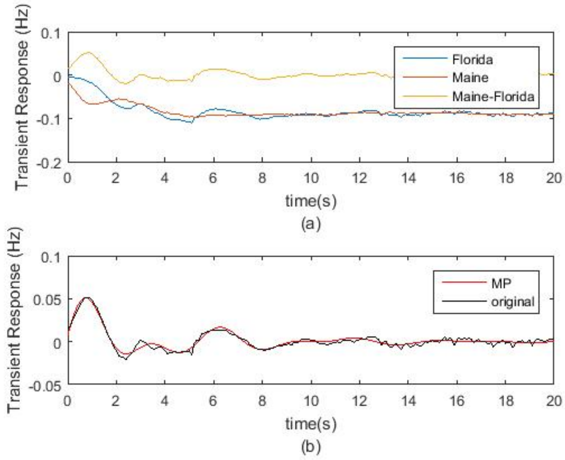

Figure 4: (a) Transient and (b) ringdown response of the NAEI system

Table 3: NAEI Signal's Parameters.

	Method	modes (i)	
		1	2
A_i	MP	0.0341	0.0301
	RTD-VF	0.0345	0.0311
α_i	MP	-0.2024	-0,2207
	RTD-VF	-0.2028	-0,2279
ω_i	MP	1.2475	2.1982
	RTD-VF	1.2522	2.1819
ϕ_i [rad]	MP	-1.4898	-1.4246
	RTD-VF	-1.4612	-1.4448
R^2	MP	94.4422%	
	RTD-VF	94.5728%	

5 Conclusion

In this paper the Matrix Pencil Method was evaluated through an application in eletric power systems. More specific, an estimation of eletromechanic modes based on WAM data captured by PMU equipment was carried out. With the results obtained until now, it can be said that the Matrix Pencil algorithm works well for the identification of time domain signal's parameters even in the presence of noise. Furthermere, when compared to the RTD-VF, which is an identification

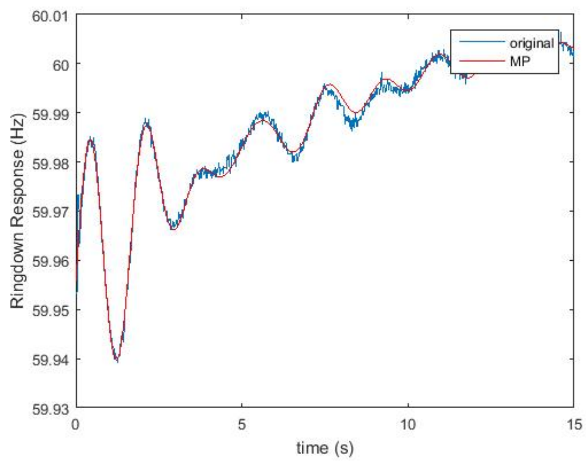

Figure 5: BIP system

Table 4: BIP Signal's Parameters.

	Meth.	modes (i)			
		1	2	3	4
A_i	MP	0.0184	0.0099	0.0091	-0.0415
	VF	0.0145	0.0129	0.0091	-0.0603
α_i	MP	-0.4146	-0.1311	-0.1954	-0.0227
	VF	-0.3837	-0.1729	-0.1862	-0.0993
ω_i	MP	4.1932	3.5908	2.3239	0
	VF	4.3713	3.6016	2.3578	0
ϕ_i	MP	-2.0558	-1.7211	0,6195	0
	VF	-2.2833	-1.6350	0.4951	0
R^2	MP		99.1408%		
	VF		99.2988%		

method with high estimation precision, the results are quite similar and don't present huge variations.

Acknowledgments

This research was partially supported by CAPES Brazil and grant ANEEL/ESBR PD 06631.0006/2017. We would like to thank the authors of Hwang and Liu (2017) and Canizares et al. (2017) for sharing the transient data sets.

References

Annakage, U., Rajapakse, A., Bhargava, B., Chaudhuri, N., Mehrizisani, A., Hauser, C., Wadduwage, D., Ribeiro Campos Andrade, S., Pathirana, V., Katsaros, K. et al. (2017). Application of phasor measurement units for monitoring power system dynamic performance, *Technical report*, Cigré.

Canizares, C., Fernandes, T., Geraldi, E., Gerin-Lajoie, L., Gibbard, M., Hiskens, I., Kersulis, J., Kuiava, R., Lima, L., DeMarco, F. et al. (2017). Benchmark models for the analysis and control of small-signal oscillatory dynamics in power systems, *IEEE Transactions on Power Systems* **32**(1): 715–722.

Crow, M. L. and Singh, A. (2005). The matrix pencil for power system modal extraction, *IEEE Transactions on Power Systems* **20**(1): 501–502.

Hua, Y. and Sarkar, T. K. (1990). Matrix pencil and system poles, *Signal Processing* **21**(2): 195–198.

Hwang, J. K. and Liu, Y. (2017). Idenification of interarea modes from ringdown data by curve-fitting in the frequency domain, *IEEE Transactions on Power Systems* **31**(12): 842–851.

Jeremias, T., Zimmer, V., Decker, I. C., Silva, A. S. and Agostini, M. N. (2012). Estudo de oscilações eletromecânicas no sistema elétrico brasileiro utilizando medidas fasoriais sincronizadas, *XIX Congresso Brasileiro de Automática*, pp. 2364–2371.

Kennaugh, E. and Moffatt, D. (1965). Transient and impulse response approximations, *Proceedings of the IEEE* **53**(8): 893–901.

Liu, G. (2010). *Oscillation Monitoring System Based on Wide Area Phasor Measurements in Power Systems*, PhD thesis, Washington State University.

McKelvey, T., Akçay, H. and Ljung, L. (1996). Subspace-based multivariable system identification from frequency response data, *IEEE Transactions on Automatic Control* **41**(7): 960–979.

Nascimento, M. D. (2012). *Aplicação da técnica da matriz pencil na obtenção do tempo final de sinais*, Mestrado em engenharia elétrica, Universidade de Brasília, Brasília.

Peng, J. C.-H. and Nair, N.-K. C. (2012). Enhancing kalman filter for tracking ringdown electromechanical oscillations, *IEEE Transactions on Power Systems* **27**(2): 1042–1050.

Pierre, J. W., Trudnowski, D., Donnally, M., Zhou, N., Tuffner, F. K. and Dosiek, L. (2012). Overview of system identification for power systems from measured responses, *Symposium on System Identification* **16**(1): 1.

Ray, P. (2017). Power system low frequency oscillation mode estimation using wide area measurement systems, *Engineering Science and Technology, an International Journal* **20**(2): 598 – 615.

Sarkar, T. K. and Pereira, O. (1995). Using the matrix pencil method to estimate the parameters of a sum of complex exponentials, *IEEE Antennas and Propagation Magazine* **37**(1): 48–55.

Schumacher, R., Oliveira, G. H. C. and Kuiava, R. (2018). A novel time-domain linear ringdown method based on vector fitting for estimating electromechanical modes, *Manuscript submitted for publication* .

Sheshyekani, K., Karami, H. R., Dehkhoda, P., Paolone, M. and Rachidi, F. (2012). Application of the matrix pencil method to rational fitting of frequency-domain responses, *IEEE Transactions on Power Delivery* **27**(4): 2399–2408.

Sheshyekani, K. and Tabei, B. (2014). Multiport frequency-dependent network equivalent using a modified matrix pencil method, *IEEE Transactions on Power Delivery* **29**(5): 2340–2348.

Viberg, M. (1995). Subspace-based methods for the identification of linear time-invariant systems, *Automatica* **32**(2): 1835–1851.