MULTIVARIABLE FUZZY IDENTIFICATION OF UNMANNED AERIAL VEHICLES

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Abstract— In this paper, a multivariable fuzzy identification methodology for Unmanned Aerial Vehicles (UAVs) based on Observer/Kalman Filter Identification (OKID) and the Eigensystem Realization Algorithm (ERA) is presented. The UAV is represented by a fuzzy Takagi-Sugeno (TS) model, whose antecedent is constituted by linguistic variables (fuzzy sets) and the consequent is constituted by linear sub-models in state-space discrete representation. The antecedent parameters are obtained using clustering fuzzy algorithms and the consequent parameters (state matrix, input matrix, output matrix and direct transition matrix) are obtained using the OKID/ERA-based algorithm discussed in details in this paper. In order to demonstrate its efficiency, a comparative analysis of the multivariable fuzzy identification methodology presented in this paper and others methodologies accepted in the literature is performed in a traditional multivariable nonlinear benchmark system. In addition, experimental results for identification of a quadrotor UAV are presented, in order to illustrate the applicability of the methodology in a real system.

Keywords— Unmanned Aerial Vehicles, Multivariable Fuzzy Identification, Observer/Kalman Filter, Eigensystem Realization Algorithm, AR.Drone 2.0.

Resumo— No presente artigo é apresentada uma metodologia de identificação fuzzy multivariável para Veículos Aéreos Não Tripuláveis (VANTs) baseada nas teorias de Identificação do Observador/Filtro de Kalman e Algoritmo de Realização de Autossistema. O VANT é representado por um modelo nebuloso Takagi-Sugeno (TS), cujo antecedente é constituído por variáveis linguísticas (conjuntos fuzzy) e o consequente é constituído por submodelos lineares com representação discreta no espaço de estados. Os parâmetros do antecedente são obtidos utilizando-se algoritmos de agrupamento fuzzy e os parâmetros do consequente (matriz de estados, matriz de entrada, matriz de saída e matriz de transição direta) são obtidos utilizando o algoritmo baseado nas técnicas OKID/ERA. De forma a demonstrar a eficiência da metodologia apresentada neste artigo, é feita uma análise comparativa com outras metodologias aceitas na literatura em um problema de identificação de um sistema benchmark não linear multivariável tradicional. Ainda, são apresentados resultados experimentais para identificação de um VANT do tipo quadrirrotor, a fim de ilustrar a aplicabilidade da metodologia em um sistema real.

Palavras-chave— Veículos Aéreos Não Tripuláveis, Identificação Fuzzy Multivariável, Observador/Filtro de Kalman, Algoritmo de Realização de Autossistema, AR.Drone 2.0.

1 Introduction

Learning fuzzy models from data is a recent and powerful tool for applications to Unmanned Aerial Vehicles (UAVs), and nonlinear multivariable systems in general (Angelov et al., 2017), (Vafamand et al., 2018), (Costa and Serra, 2015). On the other hand, most of the proposed techniques have been analyzed considering Multiple-Input and Multiple-Output (MIMO) systems through a set of Single-Input and Single-Output (SISO) systems (Jia et al., 2016), (Münker and Nelles, 2018). Real systems, such as UAVs, that presents interdependence between variables, also known as coupled systems (such as mechanical couplings, magnetic couplings, and so forth), depending on the level of coupling, may have modeling totally or partially compromised using methodologies that represent the systems by a set of SISO subsystems.

In this context, the literature of multivariable fuzzy identification has been reviewed for applications in coupled nonlinear MIMO systems, and methologies recently proposed based on Observer/Kalman Filter Identification (OKID), Eigensystem Realization Algorithm (ERA) and Fuzzy systems, has demonstrated several advantages in this kind of problem. In the Table 1, these methodologies are presented and compared in high level caracteristics. In this paper, an offline approach for quadrotor UAVs modeling is presented. Both, the antecedent and consequent, are estimated in a batch formulation by using Fuzzy C-Means and Batch Fuzzy OKID/ERA algorithms, respectively. Computational results and comparative analysis are performed in a traditional multivariable nonlinear benchmark system. Experimental results for identification of a six degrees of freedom (6-DOF) quadrotor UAV, AR.Drone 2.0, are presented.

2 Multivariable Fuzzy Identification Strategy

In this paper, the multivariable UAV systems are represented by the following fuzzy model structure:

$$R^i = \textbf{IF } z_k^1 \text{ is } F_1^i \text{ and } z_k^2 \text{ is } F_2^i \text{ and } \cdots\ z_k^p \text{ is } F_p^i$$

Table 1: Main works found in the literature based on Observer/Kalman Filter Identification, Eigensystem Realization Algorithm and Fuzzy Systems.

State-of-the-art methodologies based on Fuzzy OKID/ERA				
Paper	(Torres and Serra, 2018)	(Torres and Serra, 2016)	(Pires and Serra, 2018)	(Rodrigues Júnior and Serra, 2017)
Approach	Online	Online	Online	Online
Problem	System modeling	System modeling	System modeling	Forecasting of seasonal time series
Antecedent Estimation	Evolving Fuzzy Clustering	Recursive Fuzzy Clustering	Evolving Fuzzy Clustering	Evolving Neuro-Fuzzy Clustering
Consequent Estimation	Recursive Fuzzy OKID/ERA	Recursive Fuzzy OKID/ERA	Recursive Fuzzy OKID/ERA	Recursive Fuzzy OKID/ERA
Applications	Industrial Evaporator Process and 2DOF Helicopter	Nonlinear Benchmark System	Rocket FTI (or Fogtrein-I)	Seven Benchmark Time Series and Detection of Anomalies Based on ECG Data
Results	Experimental Results	Computational Results	Experimental Results	Experimental Results

$$\textbf{THEN} \begin{cases} x^i_{k+1} = \mathbf{A}^i x^i_k + \mathbf{B}^i u_k \\ y^i_k = \mathbf{C}^i x^i_k + \mathbf{D}^i u_k \end{cases} \tag{1}$$

where R^i denotes the i-th fuzzy inference rule $(i = 1, 2, \cdots, R)$, $z_k = [z^1_k, z^2_k, \cdots, z^p_k]$ are the antecendent variables on k-th instant of time, F^i_j is the i-th fuzzy set of the j-th antecedent parameter $(j = 1, 2, \cdots, p)$. In the consequent part, $\mathbf{A}^i \in \Re^{n\times n}$, $\mathbf{B}^i \in \Re^{n\times r}$, $\mathbf{C}^i \in \Re^{m\times n}$ and $\mathbf{D}^i \in \Re^{m\times r}$ are the parameters of the i-th submodel of order n, r inputs and m outputs, $x^i_k \in \Re^n$ is the state vetor of the i-th submodel, $y^i_k \in \Re^m$ is the output vector of the i-th submodel and $u_k \in \Re^r$ is the input vector of the system.

Let $\mu^i_{F^i_j}(z^j_k) : R \to [0, 1]$ $(j = 1, 2 \cdots, p)$ the activation degree associated with the k-th sample of the linguistic variable, z^j_k, in an universe of discourse U_{z^j} partitioned by fuzzy sets F^i_j, or linguistic terms, then the activation degree of the i-th fuzzy rule is given by:

$$h^i_k = \mu^i_{F^i_1}(z^1_k) \circ \mu^i_{F^i_2}(z^2_k) \circ \cdots \circ \mu^i_{F^i_p}(z^p_k), \tag{2}$$

where $\circ$ denotes a T-norm operator.

The normalized activation degree of the i-th rule is given by:

$$\gamma^i(z_k) = \frac{h^i_k}{\sum\limits_{i=1}^{R}(h^i_k)}. \tag{3}$$

Then, the output of the fuzzy TS model is given by:

$$\begin{cases} \tilde{x}_{k+1} = \sum\limits_{i=1}^{R} \gamma^i(z_k) x^i_{k+1} \\ \tilde{y}_k = \sum\limits_{i=1}^{R} \gamma^i(z_k) y^i_k \end{cases} \tag{4}$$

Replacing Eq. (1) in Eq. (4) gives:

$$\begin{cases} \tilde{x}_{k+1} = \sum\limits_{i=1}^{R} \mathbf{A}^i \gamma^i(z_k) x^i_k + \sum\limits_{i=1}^{R} \mathbf{B}^i \gamma^i(z_k) u_k \\ \tilde{y}_k = \sum\limits_{i=1}^{R} \mathbf{C}^i \gamma^i(z_k) x^i_k + \sum\limits_{i=1}^{R} \mathbf{D}^i \gamma^i(z_k) u_k \end{cases} \tag{5}$$

2.1 Batch Fuzzy Clustering Algorithm

Fuzzy clustering algorithms should be used to estimate the antecedent fuzzy sets F^i_J in Eq. (1) by experimental data sets of the system. Among the most well known algorithms are: Fuzzy C-Means (FCM) (Bezdek et al., 1984); Gustafson-Kessel (GK) (Gustafson and Kessel, 1979); and Fuzzy Maximum Likelihood Estimates (FLME) (Denoeux, 2011). In this paper, the Fuzzy C-Means (FCM) clustering algorithm is chosen.

The objective of FCM is to find a membership matrix $U = [\mu^1; \mu^2; \cdots; \mu^i] \in \Re^{c\times N}$, where c is the number of clusters and N the number of data points, and a centers matrix $V = [v_1; v_2; \cdots; v_c]$, with $v_i \in \Re^p$ and p the dimensionality of a data set z_k such that (Wang, 1997):

$$J_m = \sum_{k=1}^{N}\sum_{i=1}^{c}(\mu^i(z_k))^\eta (d^i_k)^2 \tag{6}$$

where J_m is an objective function to be minimized, $\mu^i(z_k)$ is the membership function of the k-th data point in the i-th cluster, $\eta \in (1, \infty)$ is a weighting constant that control the degree of fuzzy overlap and $d^i_k = \|z_k - v_i\|$ is the Euclidian distance between z_k and cluster center v_k.

Assuming that $\|z_k - v_i\| \neq 0$, $\forall\ 1 \leq k \leq N$ and $\forall\ 1 \leq i \leq c$, then U and V is a local minimum for J_m only if:

$$\mu^i(z_k) = \left(\frac{\sum_{i=1}^{c} \|z_k - v_i\|}{\|z_k - v_i\|} \right)^{\frac{2}{\eta-1}}, \quad (7)$$

where

$$v_i = \frac{\sum_{k=1}^{N} (\mu^i(z_k))^\eta z_k}{\sum_{k=1}^{N} (\mu^i(z_k))^\eta}. \quad (8)$$

The FCM algorithm performs several iterations in order to reduce as much as possible the objective function defined in Eq. (6) until either Eq. (9) or Eq. (10) is satisfied:

$$\|U^{(l+1)} - U^{(l)}\| < \xi \quad (9)$$

$$\|J_m^{(l+1)} - J_m^{(l)}\| < \xi, \quad (10)$$

where l is the current iteration and ξ is a specified minimum threshold or tolerance.

The Fuzzy C-Means algorithm is implemented as follows:

Algorithm 1: Fuzzy C-Means Clustering Algorithm

Receives a data set z_k with $k = 1, 2, \cdots, N$ data points.
Fixes $c \in 2, 3, \cdots, N-1$ and $\eta \in (1, \infty)$.
Initialize $U^{(0)}$, i.e., randomly initialize the cluster membership values, μ^i.
repeat
  Compute the c mean vectors or centers using Eq. (8).
  Update $U^{(l)}$ to $U^{(l+1)}$ according Eq. (7).
  Calculate the objective function J_m with Eq. (6).
until $\|U^{(l+1)} - U^{(l)}\| < \xi$ ***or*** $\|J_m^{(l+1)} - J_m^{(l)}\| < \xi$

2.2 *Fuzzy system Markov parameters estimation*

In order to obtain the submodel parameters $\mathbf{A}^i$, $\mathbf{B}^i$, $\mathbf{C}^i$ and $\mathbf{D}^i$, in Eq. (1), through the input and output data, fuzzy Markov parameters are required for each rule. For this, state sub-observers are used by adding a term $\mathbf{K}^i y_k^i$ on the right side of the of the states of Eq. (1) (Juang et al., 1993):

$$x_{k+1}^i = \bar{\mathbf{A}}^i x_k^i + \bar{\mathbf{B}}^i w_k^i, \quad (11)$$

where

$$\bar{\mathbf{A}}^i = \mathbf{A}^i + \mathbf{K}^i \mathbf{C}^i \quad (12)$$

$$\bar{\mathbf{B}}^i = \mathbf{B}^i + \mathbf{K}^i \mathbf{D}^i \quad (13)$$

$$w_k^i = \begin{bmatrix} u_k \\ y_k^i \end{bmatrix}, \quad (14)$$

and $\mathbf{K}^i \in \Re^{m \times r}$ is the observer gain of the i-th submodel.

Solving Eq. (11) in terms of u_j^i and y_j^i, with $j = 1, 2, \cdots, k$ and $x_0^i = 0$, the following result is obtained:

$$x_k^i = \sum_{j=1}^{k} (\bar{\mathbf{A}}^i)^{j-1} \bar{\mathbf{B}}^i w_{k-j}^i. \quad (15)$$

So, replacing Eq. (15) in Eq. (1), it gives:

$$y_k^i = \sum_{j=1}^{k} \mathbf{C}^i (\bar{\mathbf{A}}^i)^{j-1} \bar{\mathbf{B}}^i w_{k-j}^i + \mathbf{D}^i u_k. \quad (16)$$

Due to the presence of the states observer, it may be considered $(\bar{\mathbf{A}}^i)^s \approx 0$, where s is the number of steps or time instants ahead. Thus, Eq. (16) can be rewritten as:

$$y_k^i = \sum_{j=1}^{s} \bar{\mathbf{M}}_j^i w_{k-1}^i + \mathbf{D}^i u_k, \quad (17)$$

where $\bar{\mathbf{M}}_j^i = \mathbf{C}^i (\bar{\mathbf{A}}^i)^{j-1} \bar{\mathbf{B}}^i$ is the j-th observer Markov parameter of the i-th submodel. This expression can be expressed in matrix form by:

$$y_k^i = \theta_k^i \phi_k^i, \quad (18)$$

where $\theta^i = [\mathbf{D}^i, \bar{\mathbf{M}}_1^i, \cdots, \bar{\mathbf{M}}_s^i]$ is a matrix $\Re^{m \times s(m+r)+r}$ with the observer Markov parameters for each rule and $\phi_k^i = [u_k^T, w_{k-1}^T, \cdots, w_{k-s}^T]^T$ is the regressors matrix for each rule. The subscript k denotes that θ_k^i is estimated using data obtained up to the k-th instant of time.

Replacing Eq. (18) in Eq. (4), it obtains the output of the fuzzy model:

$$\tilde{y}_k = \sum_{i=1}^{R} \gamma^i(z_k) \theta_k^i \phi_k^i. \quad (19)$$

Thus, Eq. (19) can be expanded in matrix form as:

$$\tilde{y}_k = [\theta_k^1, \cdots, \theta_k^R] \begin{bmatrix} \gamma^1(z_k)\phi_k^1 \\ \cdots \\ \gamma^R(z_k)\phi_k^R \end{bmatrix}. \quad (20)$$

As the experimental data remain the same for each fuzzy rule, then $\phi_k^1 = \phi_k^2 = \cdots = \phi_k^R = \phi_k$. Therefore, Eq. (20) can be expressed in batch for $k > s$ as follows:

$$\bar{\mathbf{Y}}_k = \bar{\Theta}_k \bar{\Phi}_k \quad (21)$$

where $\bar{\mathbf{Y}} = [y_{s+1}, y_{s+2}, \cdots, y_k]$ is the output vector, $\bar{\Theta}_k = [\theta_k^1, \theta_k^2, \cdots, \theta_k^R]$ is the vector with the fuzzy observer Markov parameters of all local linear models, $\bar{\Phi} = [\bar{\phi}_{s+1}, \bar{\phi}_{s+2}, \cdots, \bar{\phi}_k]$ is the fuzzy regressors matrix in the k-th instant, the operator $\otimes$ is the Kronecker tensor product, and $\Gamma_k = [\gamma^1(z_k), \gamma^2(z_k), \cdots, \gamma^R(z_k)]^T$ the fuzzy weighting matrix with the normalized membership degrees from Eq. (3).

The least squares solution of Eq. (21) is given as follows:

$$\bar{\Theta}_k = \bar{\mathbf{Y}}_k \bar{\Phi}_k^{\dagger}, \tag{22}$$

where $\bar{\Phi}_k^{\dagger} = \bar{\Phi}_k^T[\bar{\Phi}_k \bar{\Phi}_k^T]^{-1}$ is the pseudoinverse of $\bar{\Phi}_k$.

The fuzzy system Markov parameters of each submodel are obtained by solving the following equations (Juang, 1994):

$$\mathbf{M}_k^i = \bar{\mathbf{M}}_k^{i^{(1)}} + \bar{\mathbf{M}}_k^{i^{(2)}} \mathbf{M}_0^i + \sum_{j=0}^{k-1} \bar{\mathbf{M}}_j^{i^{(2)}} \mathbf{M}_{k-j-1}^i, \quad \text{for } k = 1, 2, \cdots, s \tag{23}$$

$$\mathbf{M}_k^i = -\sum_{j=0}^{k-1} \bar{\mathbf{M}}_j^{i^{(2)}} \mathbf{M}_{k-j-1}^i, \text{for } k > s \tag{24}$$

where $\bar{\mathbf{M}}_k^i = [\bar{\mathbf{M}}_k^{i^{(1)}}, \bar{\mathbf{M}}_k^{i^{(2)}}]$, $\bar{\mathbf{M}}_k^{i^{(1)}} \in \Re^{m \times r}$ and $\bar{\mathbf{M}}_k^{i^{(2)}} \in \Re^{m \times m}$ are partitions of matrix $\bar{\mathbf{M}}_{k_r}^i$ used to find the system Markov parameters through the observer Markov parameters.

2.3 Fuzzy Eigensystem Realization Algorithm

Fuzzy Eigensystem Realization Algorithm is used to obtain the submodel parameters $\mathbf{A}^i$, $\mathbf{B}^i$, $\mathbf{C}^i$ and $\mathbf{D}^i$ from fuzzy Markov parameters which can be defined as:

$$\mathbf{M}_0^i = \mathbf{D}^i \tag{25}$$

$$\mathbf{M}_j^i = \mathbf{C}^i (\mathbf{A}^i)^{j-1} \mathbf{B}^i, \ j = 1, 2, \cdots, s. \tag{26}$$

This algorithm begins with the formation of the generalized Hankel matrix $\mathbf{H}_0^i \in \Re^{\alpha m \times \beta r}$, where α and β are integers such that $\beta r \geq \alpha m$. The Hankel matrix is composed of fuzzy system Markov parameters:

$$\mathbf{H}_0^i = \begin{bmatrix} \mathbf{M}_1^i & \mathbf{M}_2^i & \cdots & \mathbf{M}_\beta^i \\ \mathbf{M}_2^i & \mathbf{M}_3^i & \cdots & \mathbf{M}_{\beta+1}^i \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{M}_\alpha^i & \mathbf{M}_{\alpha+1}^i & \cdots & \mathbf{M}_{\alpha+\beta-1}^i \end{bmatrix}. \tag{27}$$

Replacing Eq. (26) in Eq. (27), it gives:

$$\mathbf{H}_0^i = \begin{bmatrix} \mathbf{C}^i \mathbf{B}^i & \cdots & \mathbf{C}^i (\mathbf{A}^i)^{\beta-1} \mathbf{B}^i \\ \mathbf{C}^i \mathbf{A}^i \mathbf{B}^i & \cdots & \mathbf{C}^i (\mathbf{A}^i)^{\beta} \mathbf{B}^i \\ \vdots & \ddots & \vdots \\ \mathbf{C}^i (\mathbf{A}^i)^{\alpha-1} \mathbf{B}^i & \cdots & \mathbf{C}^i (\mathbf{A}^i)^{\alpha+\beta-2} \mathbf{B}^i \end{bmatrix}. \tag{28}$$

The generalized Hankel matrix can be rewritten in terms of the controllability matrix $\mathbf{P}_\alpha^i$ and the observability matrix $\mathbf{Q}_\beta^i$, as follows:

$$\mathbf{H}_0^i = \begin{bmatrix} \mathbf{C}^i \\ \mathbf{C}^i \mathbf{A}^i \\ \vdots \\ \mathbf{C}^i (\mathbf{A}^i)^{\alpha-1} \end{bmatrix} [\mathbf{B}^i, \mathbf{A}^i \mathbf{B}^i, \cdots, (\mathbf{A}^i)^{\beta-1} \mathbf{B}^i] \tag{29}$$

$$\mathbf{H}_0^i = \mathbf{P}_\alpha^i \mathbf{Q}_\beta^i. \tag{30}$$

The maximum order n of $\mathbf{H}_0^i$, i.e., the number of nonzero singular values is equal to the order of $\mathbf{P}_\alpha$ and $\mathbf{Q}_\beta^i$ in an observable and controlled system. Decomposing $\mathbf{H}_0^i$ in singular values, it gives:

$$\mathbf{H}_0^i = \mathbf{U}^i \mathbf{\Sigma}^i (\mathbf{V}^i)^T, \tag{31}$$

where the columns of the matrices $\mathbf{U}^i$ and $\mathbf{V}^i$ are orthonormal and $\mathbf{\Sigma}^i$ a rectangular matrix:

$$\mathbf{\Sigma}^i = \begin{bmatrix} \mathbf{\Sigma}_n^i & 0 \\ 0 & 0 \end{bmatrix} \tag{32}$$

with

$$\mathbf{\Sigma}_n^i = \begin{bmatrix} \sigma_1^i & 0 & \cdots & 0 \\ 0 & \sigma_2^i & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_n^i \end{bmatrix}. \tag{33}$$

In Eq. (33), $\sigma_1^i > \sigma_2^i > \cdots > \sigma_n^i > 0$ are the n most significant values of $\mathbf{H}_0^i$, considering that above the order n there are no significant singular values. Defining $\mathbf{U}_n^i$ and $\mathbf{V}_n^i$ the first n columns of $\mathbf{U}^i$ and $\mathbf{V}^i$, respectively, the matrix $\mathbf{H}_0^i$ becomes:

$$\mathbf{H}_0^i = \mathbf{U}_n^i \mathbf{\Sigma}_n^i (\mathbf{V}_n^i)^T. \tag{34}$$

Examining Eq. (29), Eq. (30) and Eq. (34), it gives:

$$\mathbf{H}_0^i = [\mathbf{U}_n^i (\mathbf{\Sigma}_n^i)^{1/2}][(\mathbf{\Sigma}_n^i)^{1/2} (\mathbf{V}_n^i)^T] \approx \mathbf{P}_\alpha^i \mathbf{Q}_\beta^i. \tag{35}$$

The approximation of Eq. (35) is useful in cases where there is noise and very small singular values. To compute the matrix $\mathbf{A}^i$, it shifts $\mathbf{H}_0^i$ as below:

$$\mathbf{H}_1^i = \begin{bmatrix} \mathbf{C}^i \mathbf{A}^i \mathbf{B}^i & \cdots & \mathbf{C}^i (\mathbf{A}^i)^{\beta} \mathbf{B}^i \\ \mathbf{C}^i (\mathbf{A}^i)^2 \mathbf{B}^i & \cdots & \mathbf{C}^i (\mathbf{A}^i)^{\beta+1} \mathbf{B}^i \\ \vdots & \ddots & \vdots \\ \mathbf{C}^i (\mathbf{A}^i)^{\alpha} \mathbf{B}^i & \cdots & \mathbf{C}^i (\mathbf{A}^i)^{\alpha+\beta-1} \mathbf{B}^i \end{bmatrix} \tag{36}$$

$$\mathbf{H}_1^i = \mathbf{P}_\alpha^i \mathbf{A}^i \mathbf{Q}_\beta^i = \mathbf{U}_n^i (\mathbf{\Sigma}_n^i)^{1/2} \mathbf{A}^i (\mathbf{\Sigma}_n^i)^{1/2} (\mathbf{V}_n^i)^T. \tag{37}$$

Solving Eq. (37) for $\mathbf{A}^i$, it gives:

$$\mathbf{A}^i = (\mathbf{\Sigma}_n^i)^{-1/2} (\mathbf{U}_n^i)^T \mathbf{H}_1^i \mathbf{V}_n^i (\mathbf{\Sigma}_n^i)^{-1/2}. \tag{38}$$

The matrices $\mathbf{B}^i$ and $\mathbf{C}^i$ are obtained through Eq. (29) and Eq. (35):

$$\mathbf{B}^i = \text{first } \mathbf{r} \text{ columns of } \mathbf{Q}_\beta^i = (\mathbf{\Sigma}_n^i)^{1/2} (\mathbf{V}_n^i)^T \tag{39}$$

$$\mathbf{C}^i = \text{first } \mathbf{m} \text{ rows of } \mathbf{P}_\alpha^i = \mathbf{U}_n^i (\mathbf{\Sigma}_n^i)^{1/2}. \tag{40}$$

3 Results and Analysis

In order to evaluate the performance of the proposed methodology, two case studies are presented. The first case study consists in the identification of a multivariate nonlinear benchmark system, in which a comparative analysis will be used with another methodology already accepted in the state of the art of systems identification. The second case study consists in the identification of an unmanned aerial vehicle (UAV) of the quadrotor type, demonstrating the efficiency and applicability of the proposed methodology in relation to nonlinear coupled systems with fast dynamics.

3.1 Identification of a Multivariable Nonlinear Benchmark System

A traditional multivariable nonlinear benchmark system can be described by the following equations (Narendra and Parthasarathy, 1990):

$$\begin{bmatrix} y_{1,k+1} \\ y_{2,k+1} \end{bmatrix} = \begin{bmatrix} \dfrac{y_{1,k}}{1+(y_{2,k})^2} \\ \dfrac{y_{1,k}\cdot y_{2,k}}{1+(y_{2,k})^2} \end{bmatrix} + \begin{bmatrix} u_{1,k} \\ u_{2,k} \end{bmatrix} \tag{41}$$

whose input signals are given by:

$$\begin{bmatrix} u_{1,k} \\ u_{2,k} \end{bmatrix} = \begin{bmatrix} \sin(2\pi k/25) \\ \cos(2\pi k/25) \end{bmatrix}. \tag{42}$$

For the estimation of this benchmark system, a set of $N = 100$ data samples is used. To perform the estimation of the rule antecedent parameters, the Fuzzy C-means algorithm is implemented using the number of clusters $c = 2$, weighting degree $\eta = 1.25$ and tolerance $\xi = 0.001$. The fuzzy sets obtained for the two outputs are shown in Fig. 1. The normalized activation degree of each rule is shown in Fig. 2.

A generalized fuzzy rule for this model, considering Eq. (2), is given by:

$$R^i = \textbf{IF } z_k^1 \text{ is } F_1^i \text{ and } z_k^2 \text{ is } F_2^i$$

$$\textbf{THEN} \begin{cases} x_{k+1}^i = \mathbf{A}^i x_k^i + \mathbf{B}^i u_k \\ y_k^i = \mathbf{C}^i x_k^i + \mathbf{D}^i u_k \end{cases} \tag{43}$$

where $i = 1, 2$ ($R = 2$), $z_k^1 = [y_{1,k} \;\; y_{2,k} \;\; u_{1,k}]^T$ and $z_k^2 = [y_{2,k} \;\; y_{1,k} \;\; u_{2,k}]^T$. The linguistic variables were chosen in this way because the relation between input and output became stronger, following a sinusoidal pattern due to the input signals, as can be verified in Eq. (41).

Once the degree of normalized activation was obtained, successive tests were carried out by varying the main parameters of the OKID/ERA algorithm, that is, the s, α and β variables. Thus,

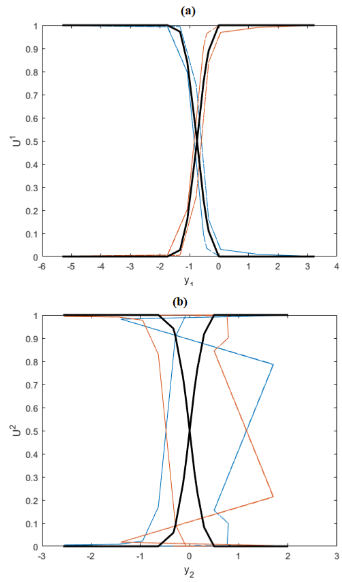

Figure 1: Membership Functions during the Multivariable Nonlinear Benchmark System identification: (a) output y_1, (b) output y_2.

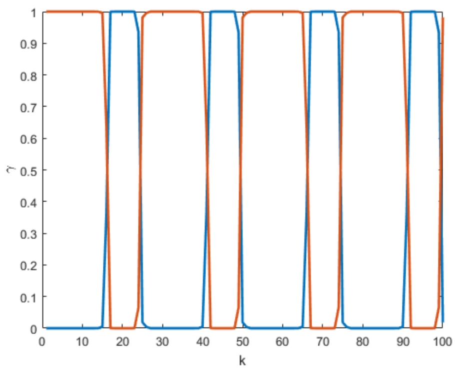

Figure 2: Normalized degree of activation during the Multivariable Nonlinear Benchmark System identification.

initial s input and output data, which may negatively affect the dynamics of the proposed methodology, were disregarded. In addition, after observing the values of the fuzzy Markov parameters of the system obtained, it is noticed that the quantity of parameters with more significant values must be around $\alpha + \beta - 1$, as shown in Eq. (36).

Therefore, through this manual tuning method, the following values for fuzzy identification were considered: $s = 1$, $\alpha = 5$, $\beta = 5$. Checking the singular values of the Hankel matrix for each fuzzy rule shown in Fig. 3, the significant values may not be visible in Σ described in Eq. (32). But, by analyzing the Σ values internally, the chosen order for the identified model is $n = 3$. A comparison between the output of the identified model and the actual response of the system is shown in Fig. 4.

In order to quantify the quality of the pro-

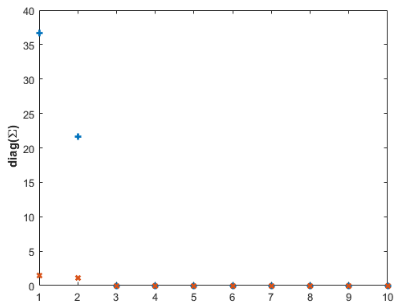

Figure 3: Singular Values of Hankel matrix for each local model of the benchmark system in state-space.

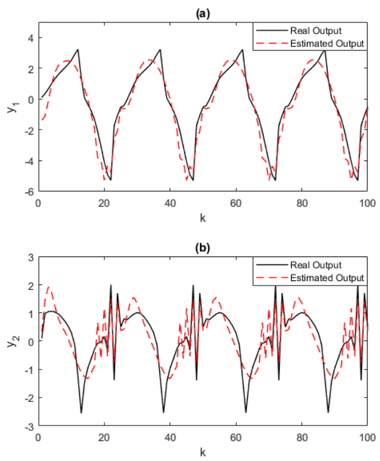

Figure 4: Validation and comparison between (a) real output y_1 and estimated $\bar{y}_1$ (b) and real output y_2 and estimated $\bar{y}_2$.

posed methodology, a function VAF (Variance Accounted For) is defined between two signals, given as follows:

$$VAF = \left(1 - \frac{var(y - \bar{y})}{var(y)}\right) \tag{44}$$

where y is the real output from system and $\bar{y}$ is the estimated output. The VAF measures the average squared correlation between two signals to verify the performance of a model.

In (A. Trabelsi and Enea, 2004), a methodology for identification of nonlinear multivariable system by adaptative fuzzy Takagi-Sugeno model is proposed. This paper presents the results for identification of the multivariable benchmark system, in Eq. (41) and Eq. (42), in two ways: without adaptation (Rec. Id. 1) and with adaptation (Rec. Id. 2). These results are compared with the results of the present paper for the same problem in Table 2.

As can be seen, the proposed methodology presented better approximation of the nonlinear multivariable system, greater VAF value, even with a lower number of rules and without an optimization procedure. In order to further improve the results presented by the proposed methodology, without using an optimization algorithm, one of the possibilities is to use a clustering algorithm with better performance than Fuzzy C-Means, as for example using their variations as PFCM (Possibilistic Fuzzy C-Means), T2FCM (Type-2 Fuzzy C-means) and DOFCM (Density Oriented Fuzzy C-Means) (Gosaina and Dahiya, 2016).

Table 2: Comparison between the results obtained by the proposed methodology and the recursive identification methodology.

	VAF (%)		
Method	***Output 1***	***Output 2***	***Rules***
Proposed	91.02	71.28	2
Rec. Id. 1	49.79	38.61	3
Rec. Id. 2	88.81	69.62	3

3.2 *Identification of a Quadrotor UAV*

The Quadrotor UAV used in this case study is the Parrot AR.Drone 2.0, a nonlinear multivariable system of fast dynamics as shown in Fig. 5. This quadrotor has six degrees of freedom (6-DOF), i.e., it has four inputs and six outputs (Hernandez et al., 2013). Your inputs are: $\{\varphi_{ref}, \vartheta_{ref}, \psi_{ref}, \zeta_{ref}\}$ - roll, pitch and yaw angle references, in radians, and vertical speed reference, in m/s. Your outputs are: $\{\varphi, \vartheta, \psi, \zeta, \dot{x}, \dot{y}\}$ - roll, pitch and yaw angles in radians, altitude in meters and linear velocities in the longitudinal and transversal axes in m/s .

Figure 5: The Quadrotor UAV AR.Drone 2.0.

For this case study, only three inputs and three outputs will be considered, that is, $u = \{\varphi_{ref}, \vartheta_{ref}, \psi_{ref}\}$ and $y = \{\varphi, \vartheta, \psi\}$. To estimate the proposed model, a set of $N = 101$ sample data was used[1]. The Fuzzy C-Means parameters are: $c = 4$, $\eta = 1.25$ and $\xi = 0.001$. The fuzzy sets obtained for the three outputs are shown in Fig. 6, with the same type of fit of the membership functions as shown in the first case study. The

[1]http://bit.ly/2JmTyOt

normalized activation degree of each rule is shown in Fig. 7.

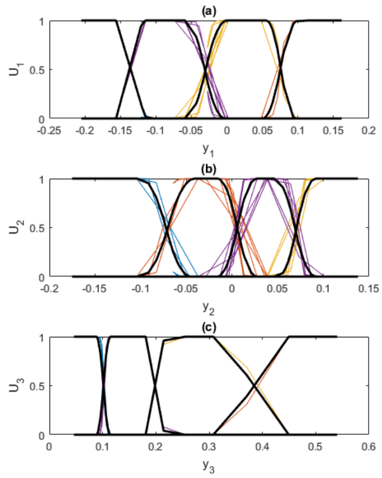

Figure 6: Membership Functions for: (a) output y_1, (b) output y_2, (c) output y_3.

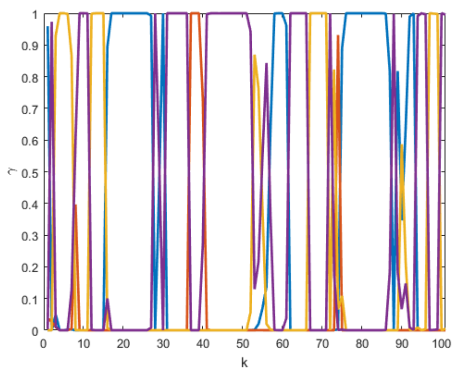

Figure 7: Normalized degree of activation during the Quadrotor UAV identification.

For this case, a generalized fuzzy rule is given by:

$$R^i = \textbf{IF } z_k^1 \text{ is } F_1^i \text{ and } z_k^2 \text{ is } F_2^i \text{ and } z_k^3 \text{ is } F_3^i$$
$$\textbf{THEN} \begin{cases} x_{k+1}^i = \mathbf{A}^i x_k^i + \mathbf{B}^i u_k \\ y_k^i = \mathbf{C}^i x_k^i + \mathbf{D}^i u_k \end{cases} \quad (45)$$

where $i = 1, 2, 3, 4$ $(R = 4)$, $z_k^1 = [y_{1,k}]^T$, $z_k^2 = [y_{2,k}]^T$ and $z_k^3 = [y_{3,k}]^T$.

Considering the same method of manual tuning used in the first case study, the parameters used in the consequent of fuzzy rules in this case are: $s = 1$, $\alpha = 5$, $\beta = 5$. Checking the singular values of the Hankel matrix for each fuzzy rule shown in Fig. 8, it is seen that for each fuzzy rule there are two significant values of Σ. Therefore, the chosen order is $n = 2$. A comparison between the output of the identified model and the actual response of the system is shown in Fig. 9.

The proposed methodology obtained VAF $=$ 86.26% for the roll angle φ, VAF $=$ 82.81% for the pitch angle ϑ and VAF $=$ 99.98% for the yaw angle ψ. These results validate the efficiency of the proposed model in real systems, which presented a good approximation of the Quadrotor UAV.

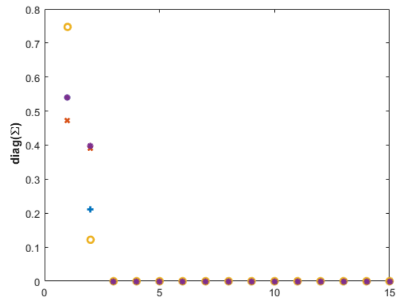

Figure 8: Singular Values of Hankel matrix for each local model of the UAV in state-space.

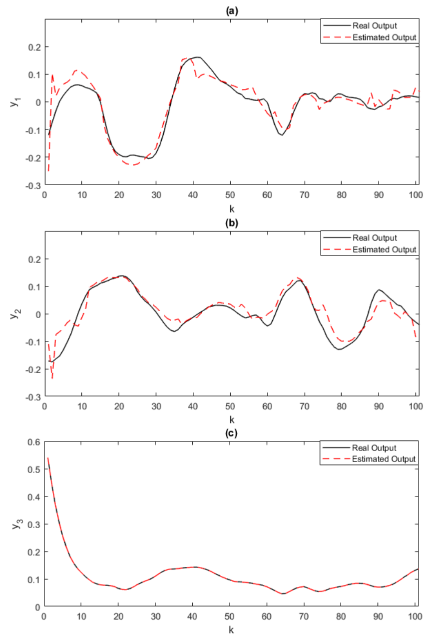

Figure 9: Validation and comparison between (a) real and estimated roll angle, (b) real and estimated pitch angle, (c) and real and estimated yaw angle.

4 Conclusions

In this article, a multivariable fuzzy identification methodology for Unmanned Aerial Vehicles (UAVs) based on Observer/Kalman Filter Identification (OKID) and the Eigensystem Realization Algorithm (ERA) was presented. The comparative analysis for identification of a traditional multivariable nonlinear benchmark demonstrate the efficiency of the methodology presented in this paper in relation with others accepted methodologies of the literature. The experimental results shown the applicability of this methodology for identification of an UAV mechanical coupled systems, without having to decorrelate the multiple-input multiple-output data. In addition, the fuzzy model obtained is of minimum-order (smallest order possible) and can be seen as the decomposition of a nonlinear system into a collection of local linear state-space submodels. So, it has a wide range of aplications and future works, such as gain-schedulling, optimal, robust and intelligent control of UAVs.

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