STABILITY ANALYSIS OF A RELAY FEEDBACK STRUCTURE FOR PROCESSES UNDER DISTURBANCES USING POINCARÉ MAP

Moisés Tavares da Silva*, Péricles Rezende Barros*

*Post-Graduate Program in Electrical Engineering - PPgEE - COPELE
Federal University of Campina Grande - Brazil

†Electrical Engineering Department
Federal University of Campina Grande - Brazil

Emails: moises.silva@ee.ufcg.edu.br, prbarros@dee.ufcg.edu.br

Abstract— In this paper, based on Poincaré map, it is analysed the stability of a relay feedback structure that provides a stable and symmetrical oscillation for process under large static disturbances or drift. The relay feedback structure consists of a block which removes static disturbance or drift followed by a relay. The block is composed of a simple high-pass filter followed by a relay plus an integrator. In order to simplify the analysis, an equivalent relay structure is obtained. Thus, for this relay feedback structure, the conditions of existence and local stability of the limit cycle obtained by the relay feedback structure for linear time-invariant (LTI) systems with no delay time are obtained. Simulation studies illustrate the results.

Keywords— Relay Feedback, Poincaré map, Stability Analysis.

1 INTRODUCTION

The relay feedback method proposed by (Åström and Hägglund, 1984) have been used to estimate the ultimate data of the process (phase angle equals to $-180°$). For processes under the effect of static disturbances or drift, the standard relay feedback method results in errors in the ultimate data estimate. Many relay feedback methods have been proposed in order to overcome these errors (Park et al., 1997; Sung and Lee, 2006; Sung et al., 2006; Lee et al., 2011). Among the several methods proposed, of particular interest is the relay feedback structure proposed by (da Silva and Barros, 2017). In this approach, a stable and symmetrical oscillation under static disturbance or drift is obtained, even for large static disturbance values.

In several cases, limit cycle oscillations (a non-trivial periodic orbit that is isolated) occur in relay feedback systems. The limit cycle is symmetric if the periodic solution have equal time intervals between the positive and negative switchings, and unimodal if the relay switches two times per period.

Conditions for limit cycles in linear systems with relay feedback have been obtained in (Åström, 1995) by using a Poincaré map. In addition, conditions for local stability of the limit cycles were obtained and the results were extended to systems with time delays. In (Johansson et al., 1999), the characterization of relay feedback systems that have multiple fast switches is investigated. Alternatively, in (Varigonda and Georgiou, 2001) the conditions for the existence and local stability for a relay with hysteresis are provided. Also for relay feedback systems, global stability results were presented in (Gonçalves et al., 2001) which shows sufficient conditions in terms of a set of linear matrix inequalities. In contrast, in (Varigonda and Georgiou, 2000) a sufficient condition is provided for the global stability of a periodic orbit by applying the contraction mapping theorem.

Limit cycle oscillations are an important phenomenon in control design because it usually imposes undesirable effects on the system. In addition, several controller tuning techniques depend on a correct theoretical prediction of the limit cycles for obtaining their tuning formulas. Thus, the determination of the existence and stability of the limit cycles of relay feedback systems remain an important issues till today (Bazanella and Parraga, 2016; Yoon and Johnson, 2018).

In this paper, the Poincaré map and a state-space representation are used to analyse the relay feedback structure for processes under static disturbances or drift proposed in (da Silva and Barros, 2017). This structure is composed of a block to remove static disturbance or drift followed by a relay. The block consists of a high-pass filter followed by a relay plus an integrator. The integrator is used to compensate the dynamics of the high-pass filter. In order to separate the dynamics of the high-pass filter and the integrator, a relay is used in between.

With the purpose of simplify the analysis of the relay feedback structure under study, an equivalent relay structure is obtained. For the equivalent structure and LTI system with no time delay, the necessary and sufficient conditions for the existence of limit cycle are established. Furthermore, using the Jacobian of the corresponding Poincaré map, the local stability of limit cycle is investigated.

This paper is organized as follows: in Section 2, the relay feedback structure for processes under static disturbance or drift is presented; in Section 3, an equivalent relay structure is obtained in order to simplify the analysis; in Section 4, the analysis of existence and the local stability of limit cycles for the equivalent relay feedback structure are presented; in Section 5, simulation study are shown; in Section 6 conclusions are discussed.

2 The Relay Feedback Structure

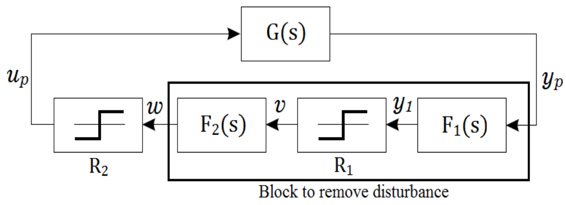

Figure 1: Schematic diagram of the relay feedback structure.

Consider a single-input single-output (SISO) LTI system satisfying the following linear dynamic equations

$$\begin{cases} \dot{x}_p(t) = Ax_p(t) + Bu_p(t) \\ y_p(t) = Cx_p(t) \end{cases} \tag{1}$$

where $x_p \in \mathbb{R}^n$, $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times 1}$, $C \in \mathbb{R}^{1 \times n}$ and A is Hurwitz. The system can also be described by the transfer function

$$G(s) = C(sI - A)^{-1}B. \tag{2}$$

The relay feedback structure of interest is shown in Figure 1. This structure is composed of two blocks. The first block is used to remove static disturbance or drift and consists of a high-pass filter $F_1(s)$, followed by a relay (R_1) and a low-pass filter $F_2(s)$. The second block is a standard relay (R_2).

In this paper, relays with input and output signals $m(t)$ and $z(t)$, respectively, are defined as

$$z(t) = \begin{cases} 1, \text{ if } m(t) > \varepsilon, \text{ or } m(t) \geq -\varepsilon \text{ and } z(t_-) = 1 \\ -1, \text{ if } m(t) < -\varepsilon, \text{ or } m(t) \leq \varepsilon \text{ and } z(t_+) = -1 \end{cases} \tag{3}$$

where $\varepsilon \geq 0$ is the hysteresis parameter.

The high-pass filter $F_1(s)$ is used to remove the static disturbance or drift, and is chosen as

$$F_1(s) = 1 - e^{-s\tau_f}, \tag{4}$$

where τ_d is the filter time constant and can be chosen as the sampling time of the process.

Note that $F_1(s)$ is an approximate derivative, since by using a first-order Taylor series approximation for the term $e^{-s\tau_d}$ of Eq. (4), the following equation is obtained

$$F_1(s) \approx 1 - (1 - s\tau_d) \approx s\tau_d. \tag{5}$$

The frequency response for $F_1(s)$ is given by

$$|F_1(j\omega)| = \left[\left(1 - cos(\omega\tau_f)\right)^2 + \left(sin(\omega\tau_f)\right)^2 \right]^{1/2}$$

$$\angle\theta_1(\omega) = tan^{-1}\left(\frac{sin(\omega\tau_f)}{(1 - cos(\omega\tau_f))} \right).$$

Note that for low frequencies and small values of τ_f, the filter $F_1(s)$ has a phase angle of approximately $+\pi/2$.

The low-pass filter $F_2(s)$ is chosen as an integrator, i.e.

$$F_2(s) = \frac{1}{s}. \tag{6}$$

The integrator is used to compensate the high-pass filter dynamics of $F_1(s)$. In order to separate the dynamics of $F_1(s)$ and $F_2(s)$, the relay R_1 is introduced. The relay R_2 is used as the standard relay to generate a stable oscillation at the process output.

More details on the relay feedback structure for processes under static disturbance or drift can be found in (da Silva and Barros, 2017).

3 Equivalent Relay Feedback Structure

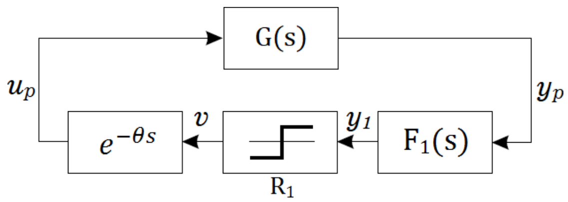

Figure 2: Schematic diagram of the equivalent relay feedback structure.

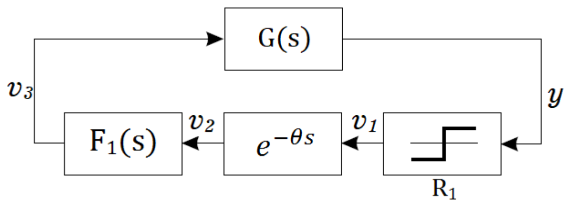

Figure 3: Schematic diagram of the final equivalent relay feedback structure.

In order to simplify the analysis of existence and local stability of limit cycle for the relay feedback structure shown in Figure 1, an equivalent structure is introduced by the following lemma.

Lemma 1 *Consider the relay feedback structure shown in Figure 1. Assume the oscillation period for this relay feedback structure equal to T seconds. The transfer function $F_1(s)$ is given by (4), and $F_2(s)$ is given by (6). Then, except for the initial transient, the relay feedback structure shown in Figure 3 has the same oscillation period T of the structure presented in Figure 1, with θ equal to $T/4$.*

Proof: The proof of the Lemma is divided into two parts.

Part 1: Starting from the relay feedback structure shown in Figure 1, and assume the oscillation period equal to T seconds. The frequency response for $F_2(s) = 1/s$, at the frequency ω_u, is given by

$$F_2(j\omega_u) = \frac{1}{\omega_u} e^{-j\pi/2}.$$

For the input signal $V(s)$, the output of $F_2(s)$ is

$$W(j\omega_u) = \frac{V(j\omega_u)}{\omega_u} e^{-j\pi/2}. \tag{7}$$

Without loss of generality, consider equals amplitudes for both relays. Also consider a square wave ($v(t)$) introduced in the input of the block $F_2(s)$. From Eq. (7), there is a 90° lag between $v(t)$ and $w(t)$ signals. Since the R_2 relay is a symmetric nonlinearity, without hysteresis, there is no lag between $w(t)$ and $u_p(t)$. Thus, there is a 90° lag between $v(t)$ and $u_p(t)$ signals.

Therefore, except for the initial transient, the relay feedback structure shown in Figure 2, with θ equal to $T/4$, has the same oscillation period T of the structure shown in Figure 1.

Part 2: Now from the relay feedback structure in Figure 2, by a straightforward application of the superposition principle, the transfer function $F_1(s)$ can be displaced to the input of the LTI system, since $F_1(s)$ and the system given by (2) are linear.

Thus, the final equivalent relay feedback structure shown in Figure 3 will oscillate with the same period T of the system shown in Figure 1.

□

In order to illustrate the first part of *Lemma 1*, consider the following process

$$G(s) = \frac{1}{(s+1)^4}. \tag{8}$$

This process is simulated for the relay feedback structure, shown in Figure 1, with filter time constant (τ_f) equals to sampling time (0.01s), amplitude of the relay $M = 1$ and hysteresis $\varepsilon = 0$. The oscillation period of the process is $T = 6.2$s. A periodic solution to this system is shown in Figure 4. As it can be seen, there exists a 90° lag between the signals $v(t)$ and $u_p(t)$.

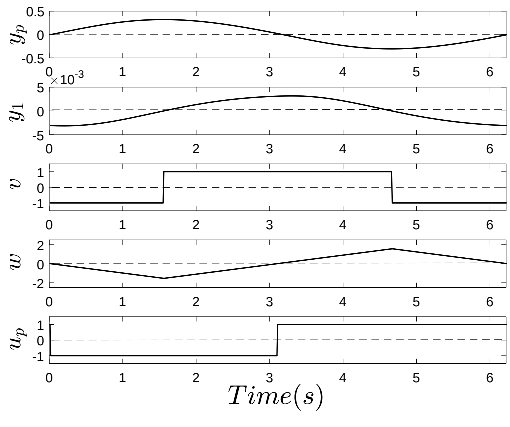

Figure 4: Periodic solution of the relay feedback structure for the system (8).

The second part of *Lemma 1* is also illustrated for the process given by (8). A periodic solution to this system is shown in Figure 5, where $h^* = T/2$ and $\theta = T/4 = 1.55$.

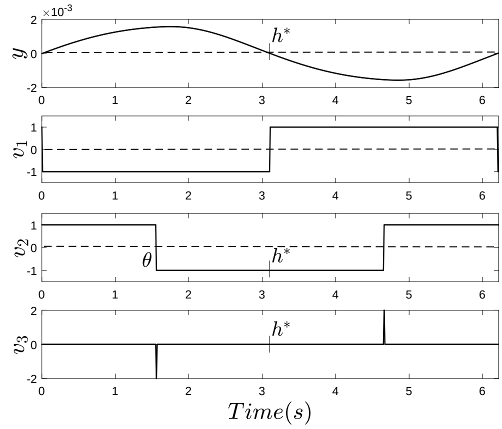

Figure 5: Periodic solution of the final equivalent relay feedback structure for the system (8).

Note that the signal $v_2(t)$ is a delayed version of the relay output signal $v_1(t)$. The signal at the input of the linear system ($v_3(t)$) is similar a derivative of signal $v_2(t)$, and assumes the values of ± 2.

The aim of the equivalent structure is to obtain, in steady state, the same periodic solution of the original structure of the relay (Note that the oscillation period of y_p in Figure 4 is the same as the output signal y in Figure 5). Thus, the difference of the initial transient between the equivalent and original structures does not invalidate the analysis performed in the following sections.

4 Existence and Stability Analysis

In this Section, the conditions for existence of unimodal and symmetrical limit cycles of the relay feedback structure under analysis is presented.

The equivalent system shown in Figure 3 is used in the analysis of existence and local stability of the limit cycle. This system can be represented by the following LTI system

$$\begin{cases} \dot{x}(t) = Ax(t) + Bv_3(t) \\ y(t) = Cx(t) \end{cases} \tag{9}$$

in which A, B and C are the same as the original system represented in Eq. (2).

The system represented in Eq. (9) is the same as

$$\begin{cases} \dot{x}(t) = Ax(t) + B\left[v_2(t-\theta) - v_2(t-\theta-\tau_d)\right] \\ y(t) = Cx(t) \end{cases} \tag{10}$$

For the simplified system the analysis of existence of the limit cycle is equivalent to the analysis presented in Varigonda and Georgiou (2001). Thus, the *switching surface*, composed of an hyperplane of dimension $n-1$, is defined as

$$S = \{x \in \mathbb{R}^n : Cx = \varepsilon\}.$$

Note that the switching surface it is a hyperplane that contains the origin and divides the state space into two distinct regions. In one of them

$$R^- = \{x \in \mathbb{R}^n : Cx > \varepsilon\},$$

in which the system is given by $\dot{x} = Ax - 2B$. In the other region

$$R^+ = \{x \in \mathbb{R}^n : Cx < \varepsilon\},$$

where the system is given by $\dot{x} = Ax + 2B$.

In order to investigate the conditions for existence and local stability of the equivalent relay feedback structure, consider again the process given by Eq. (8) and the periodic solution shown in Figure 5, with $h^* = T/2$ and $\theta = T/4$. Note that the switching occurs at $t = \theta + \tau_d$, and according to Eq. (10), $v_3(t)$ assumes the values of ± 2.

Remarks

1. In order to analyze using the equivalent structure, it is necessary to know in advance the oscillation period of the original relay structure shown in Figure 1. In this paper, the oscillation period is assumed to be known. How to obtain this period directly from the original relay structure is under study.

2. Note that for the LTI system under analysis it is correct to consider only the cases $h^* > \theta$, because the θ value limits the period of oscillation of the system (the oscillation period is never less than the θ delay time). Thus, the periodic solution obtained by the equivalent structure is the same of the original structure.

3. For the simulation of the limit cycle, it is necessary to know only the value of $x(0) = x^*$ at the instant of intersection with the switching surface.

The next theorem makes possible to identify limit cycles of the relay feedback structure under study.

Theorem 1 *Consider the linear system given by (9) connected in feedback with the relay given by (3), as shown in Figure 3. There is a symmetrical and unimodal limit cycle with period $T = 2h^*$ if and only if the following conditions are satisfied:*

$$\text{(i) } g_\theta(h^*) \triangleq 2C(e^{Ah^*}+I)^{-1}\left(\int_\theta^{\theta+\tau_d} e^{A(h^*-\tau)}Bd\tau\right) = \varepsilon, \tag{11}$$

$$\text{(ii) } y(t) = Cx(t) > \varepsilon, \forall t \in (0, h^*), \tag{12}$$

where

$$x^* = 2(e^{Ah^*}+I)^{-1}\left(\int_\theta^{\theta+\tau_d} e^{A(h^*-\tau)}Bd\tau\right), \tag{13}$$

is the initial condition $x(0) = x^*$ which leads to the periodic solution.

Proof: Since the linear systems in relay feedback is symmetric around the origin, the limit cycle only needs to be analyzed over half of its period. The proof is divided into two parts.

Necessity: Assume that there exists a symmetrical limit cycle in the system of Figure 3. Based on Figure 5 of the above example, the solution of the LTI system (Eq. (9)), to $x \in R^-$ is given by

$$\begin{aligned} x(h^*) = \quad & e^{Ah^*}x(0) + \int_0^{h^*} e^{A(h^*-\tau)}Bv_3(\tau)d\tau \\ = \quad & e^{Ah^*}x(0) - 2\int_\theta^{\theta+\tau_d} e^{A(h^*-\tau)}Bd\tau. \end{aligned}$$

Assume $x(0) = x^* \in S$ the point at which the orbit intercepts S. By symmetry, at $t = h^*$ the state of the system is

$$x(h^*) = -x^*.$$

$$\begin{aligned} -x^* &= e^{Ah^*}x^* - 2\int_\theta^{\theta+\tau_d} e^{A(h^*-\tau)}Bd\tau \\ x^* &= 2(e^{Ah^*}+I)^{-1}\left(\int_\theta^{\theta+\tau_d} e^{A(h^*-\tau)}Bd\tau\right). \end{aligned}$$

The initial state is thus given by (13). Hence, since $x^* \in S$, $Cx^* = \varepsilon$, which is equivalent to condition (*i*):

$$g_\theta(h^*) \triangleq 2C(e^{Ah^*}+I)^{-1}\left(\int_\theta^{\theta+\tau_d} e^{A(h^*-\tau)}Bd\tau\right).$$

To ensure that there is no other switch at $0 < t < h^*$, the relay input must satisfy $y(t) = Cx(t) > \varepsilon$, for $0 < t < h^*$, which gives condition (*ii*).

Sufficiency: Assume that $g(h^*) = \varepsilon$, thus $x^* \in S$. Also assume $y(t) > \varepsilon$ for $t \in (0; h^*)$, thus $v_3(t) = -2$, $t \in (0; h^*)$. Therefore, the trajectory from x^* will not reach S again before h^*. It is possible to show that $x(t)$ reaches S after h^* seconds on $-x^*$, which causes $v_3(t)$ to switch to 2. With a similar argument, it is shown that $x(t)$ returns to S after h^* in x^*, so that there exists a periodic orbit through x^*. □

In order to investigate the local stability of the limit cycle, one must calculate the Jacobian of the Poincaré map. This result is presented in the following theorem.

Theorem 2 *Consider the linear system given by (9) connected in feedback with the relay given by (3), as shown in Figure 3. Assume that there is a symmetric periodic solution with $h^* > \theta$. The Jacobian of the Poincaé map is given by*

$$W_\theta = \left(I - \frac{\omega_\theta C}{C\omega_\theta}\right)e^{Ah^*}, \tag{14}$$

where

$$\omega_\theta = e^{Ah^*}Ax^* + 2\left[e^{A(h^*-\theta-\tau_d)} - e^{A(h^*-\theta)}\right]B. \tag{15}$$

The limit cycle is locally stable if and only if W_θ has all its eigenvalues inside the unit disk. It will be unstable if W_θ has at least one eigenvalue outside the unit disk.

Proof: The analysis is performed considering a new origin in $h^*+\delta t$, with $x(0)=x^*+\delta x$. With the new time base, it follows that

$$\begin{aligned}
x(h^*+\delta h) &= e^{A(h^*+\delta h)}x(0) \\
&\quad + \int_0^{h^*+\delta h} e^{A(h^*+\delta h-\tau)}Bv_3(\tau)d\tau \\
&= e^{A(h^*+\delta h)}(x^*+\delta x) \\
&\quad + \int_\theta^{\theta+\tau_d} e^{A(h^*+\delta h-\tau)}B(-2)d\tau \\
&= e^{A(h^*+\delta h)}(x^*+\delta x) \\
&\quad + 2A^{-1}\left[e^{A(h^*+\delta h-\theta-\tau_d)} - e^{A(h^*+\delta h-\theta)}\right]B \\
&= e^{Ah^*}(I+A\delta h)(x^*+\delta x) \\
&\quad + 2A^{-1}\big[e^{A(h^*-\theta-\tau_d)}(I+A\delta h) \\
&\quad - e^{A(h^*-\theta)}(I+A\delta h)\big]B + O(\delta^2) \\
&= -x^* + e^{Ah^*}\delta x + \big[e^{Ah^*}Ax^* \\
&\quad + 2B\big(e^{A(h^*-\theta-\tau_d)} - e^{A(h^*-\theta)}\big)\big]\delta h + O(\delta^2) \\
&= -x^* + e^{Ah^*}\delta x + \omega_\theta \delta h + O(\delta^2), \quad (16)
\end{aligned}$$

where

$$\omega_\theta = e^{Ah^*}Ax^* + 2\left[e^{A(h^*-\theta-\tau_d)} - e^{A(h^*-\theta)}\right]B.$$

Since $x(h^*+\delta h)\in S$ and by symmetry of limit cycle, $x(h^*)=-x^*$:

$$\begin{aligned}
Cx(h^*+\delta h) &= -Cx^* + Ce^{Ah^*}\delta x + C\omega_\theta\delta h + O(\delta^2) \\
&= -\varepsilon.
\end{aligned}$$

Since $Cx^*=\varepsilon$, then

$$Ce^{Ah^*}\delta x + C\omega_\theta\delta h + O(\delta^2) = 0. \quad (17)$$

Neglecting terms of order δ^2 this gives

$$\delta h = -\frac{Ce^{Ah^*}}{C\omega_\theta}\delta x. \quad (18)$$

Substituting (18) in (16), it follows that

$$\begin{aligned}
x(h^*+\delta h) &= -x^* + e^{Ah^*}\delta x - \frac{\omega_\theta Ce^{Ah^*}}{C\omega_\theta}\delta x + O(\delta^2) \\
&= -x^* + \left(I - \frac{\omega_\theta C}{C\omega_\theta}\right)e^{Ah^*}\delta x + O(\delta^2)
\end{aligned}$$

which proves the theorem. □

5 Examples

In this Section, using simulated examples, the existence and local stability of the limit cycle is evaluated for the relay feedback structure for processes under disturbance. In all cases, the amplitude of the relay is $M=1$, the hysteresis is $\varepsilon=0$ and the filter time constant (τ_f) is equal to the sampling time (0.01s).

Example 1: Consider a process given by

$$G(s) = \frac{1}{(s+1)^3}. \quad (19)$$

The oscillation period obtained using the relay feedback structure of Figure 1 is $T=3.58$s. Thus, $\theta = T/4 = 0.895$s.

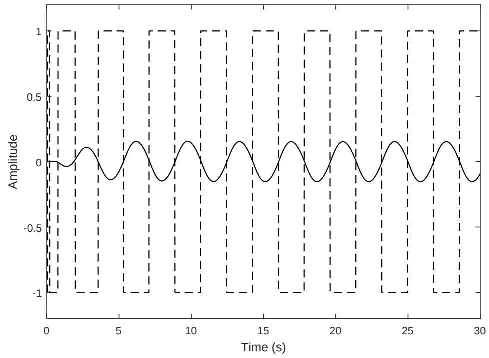

Figure 6: Process input (Dashed line) and output (Solid line) signals of the relay structure of Figure 1.

In Figure 7, the solution of $g_\theta(h^*)$, Eq. (11), is showed. Numerical calculations with Theorem 1 gives two zero for positive h, i.e. $h^*=0.7825$ and $h^*=1.7825$.

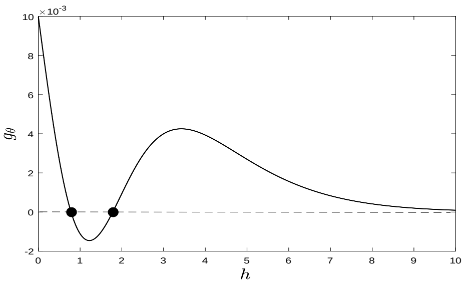

Figure 7: Solution of $g_\theta(h^*)$ for Example 1.

For $h^*=0,7825$ the analysis is not feasible, since that $h^*<\theta$. For $h^*=1.7825$, the initial condition which leads to the periodic solution is

$$x(0) = \begin{pmatrix} 0.0080 \\ 0.0018 \\ -0.0044 \end{pmatrix}.$$

The Jacobian of the Poincaré map can be computed from Theorem 2 as

$$W_\theta = \begin{pmatrix} 0.3152 & 1.1316 & 1.2864 \\ 0.1401 & 0.2690 & 0.0284 \\ 0 & 0 & 0 \end{pmatrix}.$$

The eigenvalues of W_θ are 0, -0.1067 and 0.6909. It can be concluded that the limit cycle is locally stable according to Theorem 2.

Example 2: Consider the non-minimum-phase system

$$G(s) = \frac{-0.5s+1}{(s+1)^3}. \tag{20}$$

The oscillation period obtained from the relay feedback experiment of Figure 1 is $T = 5.08$s. Thus, $\theta = T/4 = 1.27$s.

In Figure 8, the solution of $g_\theta(h^*)$ is showed. From Theorem 1, numerical calculations gives two zero for positive h, i.e. $h^* = 1.2655$ and $h^* = 2.5350$.

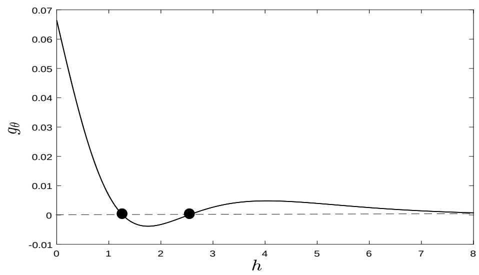

Figure 8: Solution of $g_\theta(h^*)$ for Example 2.

For $h^* = 1.2655$ the analysis is not feasible, since that $h^* < \theta$. For $h^* = 2.5350$, the initial condition to obtain a periodic solution is

$$x(0) = \begin{pmatrix} 0.0007 \\ 0.0042 \\ -0.0038 \end{pmatrix}.$$

From Theorem 2, the Jacobian of the Poincaré map can be computed as

$$W_\theta = \begin{pmatrix} -0.0334 & 0.0078 & 0.1350 \\ 0.1464 & 0.3573 & 0.2161 \\ 0.0732 & 0.1787 & 0.1081 \end{pmatrix}.$$

The eigenvalues of W_θ are -0.0544, 0.4866 and 0. Thus, the limit cycle is locally stable according to Theorem 2.

6 Conclusion

The analysis, based on Poincaré map, of existence and local stability of limit cycle for the relay feedback structure for process under large static disturbances or drift, and LTI system with no time delay was performed. In order to simplify the analysis, an equivalent structure was obtained and the conditions for existence of limit cycles for the relay feedback structure were established. In addition, the local stability of limit cycles was assessed using the Jacobian of Poincaré map. Examples are used to illustrate the analysis.

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