Piecewise Linear Approximations of the Hydro Power Producer Problem

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***Abstract*—This paper focuses on the solution of the problem faced by a hydro power producer that needs to determine the amount of power to offer to the market for a period of one year. To define the best offers, such producer must solve a bilevel optimization problem that is nonconvex and has local and global minima. The paper analyzes the performance of a piecewise linearization technique when applied to the solution of such problem. The procedure is tested in a small example and in an equivalent of the Brazilian system.**

***Index Terms*—Piecewise linearization methods, energy markets, hydrothermal systems, bilevel optimization.**

I. Introduction

Under energy markets, the agents that participate in the production, transmission, distribution and consumption of electrical energy usually have different economical goals. One of these agents is the owner of generation plants. An important issue for such agent is to define the offers to the energy market that maximize its profits. This is known as the *producer problem*. The optimal decisions of the producer depend not only on the market structure, but also on the generation capacity of such producer and, consequently, its market power. These are key factors to be considered when determining the offers to the energy market.

A large body of work formulates and analyzes the producer problem. The proposed models can be classified in two categories: those that consider a price taker producer [1]–[4], which has no influence on the market price, and those models that take into consideration the influence of the producer on the price of energy [5]–[13]. In the first case, the producer problems can be regarded as particularizations of classical optimal power dispatch problems defined for thermal and hydrothermal systems. Thus, usually the optimal decisions of the producer are obtained by solving a single level optimization problem. In the case that producer decisions affect the price of energy, the optimal offers can be obtained by solving a bilevel optimization problem, with the upper level problem expressing producer goals and constraints, and the lower level problem representing the optimal power dispatch of the system. Depending on the information available to the producer, these optimization problems can be deterministic or stochastic. In both cases, different factors are considered in the analysis of the problem: transmission constraints [5]–[8], [10], [12], the random nature of system loads [8], energy prices and offers made by competing power producers [3], [8], [9], [12], [14] and intermittent renewable generation [8], [13], [14].

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This paper focuses on the solution of the problem faced by a hydro power producer that needs to determine the amount of power to offer to the market in a medium term horizon (one year). Similar problems have been analyzed in the literature, both considering price taker [2] and price maker producers [8], [11], [13], [14]. Here, the problem is formulated for a price maker producer in a predominantly hydro system and considers the stochastic nature water inflows [15]. However, the model described in this last reference has been extended to consider nonlinear characteristics of the hydro generations production functions. Thus, this paper deals with a bilevel nonlinear stochastic optimization problem. It is important to ensure that reliable solutions are obtained for such problem, which is non-convex and have local and global optima [16]. The paper evaluates the use of a piecewise linearization technique in the solution of such problem.

When applied in the solution of the producer problem, mixed integer linear formulations have been adopted to express complementarity constraints [10], [13] and nonlinear constraints that represent some characteristics of the producer [1], [2], [8], [13]. The linearization models used in the previous references are designed to represent those particular constraints. In the present paper, a generic integer programming model, the Disaggregated Convex Combination (DCC) model [17]–[19], is used for piecewise linearization of the nonlinear constraints of the hydro power producer problem [20]. The model can be applied to approximate any nonlinear continuous function that is defined over a compact set. The mixed integer linear formulation is expressed in GAMS platform [21] and solved with CPLEX. Results are obtained for a small example and for an equivalent of the Brazilian system.

The remaining of the paper is organized as follows. Section II presents the bilevel formulation of the hydro power producer problem and the the corresponding single level problem. Section III presents the piecewise linearizatin technique adopted to formulate the mixed integer linear problem. Section IV analyzes the results obtained in tests with hydro systems extracted from the Brazilian system and, finally, section V concludes the paper.

II. The Hydro Power Producer Problem

To supply the system demand, generators make offers to the power market and the system operator is responsible for coordinating all the offers in order to have a feasible dispatch. The objective of each power producer is to maximize its individual profit, which basically depends on the reservoir storage levels, on the river natural water inflows and on the outflows of up-stream reservoirs. This implies that the problem faced by a hydro power producer is to maximize its profit over a given time horizon. To obtain its best offers, the producer must consider that the system operator determines the power dispatch that minimize the amount of thermal generation to be used in the planning horizon considering all the constraints on the operation of the system.

A. Physical and Operational Constraints

In the mathematical framework used throughout the paper, one power plant is associated with each power producer, with one reservoir and one equivalent generation unit. The dispatch procedure is carried out taking into consideration, besides physical and operational limits of the plants, their spatial and temporal interconnections represented by the hydro balance equations. The random nature of natural water inflows is represented through set of N_ω scenarios. For a planning horizon with T periods and a system with H hydro plants, the dispatch obtains the amount of power produced by every plant i in every time period t and for every scenario of water inflow ω. The following hydro plant constraints must be satisfied in the dispatch:

$$
\begin{aligned}
& v_{i,t,\omega} = v_{i,t-1,\omega} + r_{i,t,\omega} - q_{i,t,\omega} - u_{i,t,\omega} \\
& \qquad + \sum_{n\in N_i} [q_{n,t_\omega} + u_{n,t,\omega}] \\
& P_{h_{i,t,\omega}} = k_i\,(h_{v_i} - h_{q_i})\,q_{i,t,\omega} \\
& v_i^{\min} \leq v_{i,t,\omega} \leq v_i^{\max} \\
& q_i^{\min} \leq q_{i,t,\omega} \leq q_i^{\max} \\
& P_{h_i}^{\min} \leq P_{h_{i,t,\omega}} \leq P_{h_i}^{\max} \\
& u_{i,t,\omega} \geq 0
\end{aligned}
\tag{1}
$$

for $i = 1, ..., H$, $t = 1, ..., T$ and $\omega = 1, ..., N_\omega$. In (1), for every scenario ω, $v_{i,t,\omega}$ is the volume of water in reservoir i at the end of time period t, $r_{i,t,\omega}$ is the natural water inflow, $q_{i,t,\omega}$ is the amount of water discharged through the turbines, $P_{h_{i,t,\omega}}$ is the active power generation and $u_{i,t,\omega}$ is the amount of spilled water of reservoir i during period t; N_i is the set of up-stream reservoirs of reservoir i, h_{v_i} and h_{q_i} represent the forebay and afterbay elevations of reservoir i and superscripts $\min$ and $\max$ represent upper and lower limits. The first equation represents the hydro balance in the system, the second represents the input-output characteristics of the plant and the remaining equations set upper and lower limits on reservoir volume, power generation, water discharged through the turbines and water spillage.

The forebay and afterbay elevations of a plant i are expressed by polynomials with degrees that depend on the characteristics of the plant. Thus, we have:

$$
\begin{aligned}
P_{h_{i,t,\omega}} &= \sum_{\gamma=0}^{d_v} \alpha_{\gamma_i} v_{i,t,\omega}^{\gamma} q_{i,t,\omega} \\
&\quad - \sum_{\gamma=0}^{d_q} \beta_{\gamma_i} (q_{i,t,\omega} + u_{t,u,\omega})^{\gamma} q_{i,t,\omega}, \ \forall t, \omega \ ,
\end{aligned}
\tag{2}
$$

where $d_v,\ d_q$ are the degrees of h_{v_i} and h_{q_i}, respectively, and coefficients $\alpha_{\gamma_i},\ \beta_{\gamma_i}$ are empirically obtained.

In addition, the dispatch must respect power balance equations and transmission limits. Let $\mathbf{B}$ be susceptance matrix of the system, $\mathbf{A}_h$ and $\mathbf{A}_y$ zero-one matrices that associate hydro and thermal generators to the system buses, γ_l the susceptance of line l and $\mathbf{A}$ the line-bus incidence matrix of the system. The power balance constraints can be written as:

$$
\begin{aligned}
& \sum_{i=1}^{H} \mathbf{A}_{h_{n,i}}\, ph_{i,t_\omega} + \sum_{i=1}^{N_y} \mathbf{A}_{y_{n,i}} y_{n,t,\omega} - \sum_{j=1}^{N} \mathbf{B}_{n,j}\, \theta_{j,t,\omega} = pd_{n,t}, \\
& -f_l^{\max} \leq \gamma_l \sum_{j=1}^{N} \mathbf{A}_{l,j}\, \theta_{j,t,\omega} \leq f_l^{\max}, \\
& y_{n,t}(\omega) \geq 0,
\end{aligned}
\tag{3}
$$

for $n = 1, ..., N$, $l = 1, ..., L$, and $\forall t, \omega$. In (3), N is the number of buses in the system, N_y is the number of thermal plants, $\theta_{j,t,\omega}$ is the voltage angle of bus j, $y_{n,t,\omega}$ represents the amount of thermal generation necessary to supply the load, $pd_{n,t}$. $f_l^{\max}$ is the transmission limit of line l.

Water inflows in the first planning period can be considered known; in the following planning periods, $r_{i,t,\omega}$ is a random variable represented by N_ω scenarios, each one with probability of occurrence π_ω. Thus, in (1) for every plant i and bus n: $v_{i,1,1} = v_{i,1,2} = ... = v_{i,1,N_\omega},\ q_{i,1,1} = q_{i,1,2} = ... = q_{i,1,N_\omega},\ u_{i,1,1} = u_{i,1,2} = ... = u_{i,1,N_\omega}, P_{h_{i,1,1}} = P_{h_{i,1,2}} = ... = P_{h_{i,1,N_\omega}}, ..., \theta_{n,1,1} = ... = \theta_{n,1,N_\omega}$.

B. The Producer Problem

The objective of each power producer is to maximize its individual profit, which depends on how much water is available to provide power, that is, it depends on the reservoir storage levels, on the river natural water inflows and on the outflows of up-stream reservoirs. When deciding its offer to the next market clearing time, this producer must take into consideration a set of possible future water inflow scenarios and potential future revenues. It is assumed that the producer expresses such criteria by establishing target values for the volumes of water that are stored in its reservoirs at the end of the planning period, v_k^{sp}. Thus, the problem of producer k would be to adjust the amount of offered power in order to minimize the difference between the volume of water stored in its reservoirs at the end of the planning period and v_k^{sp}. The offer for the first planning period, associated with the next market clearing time, does not depend on scenarios; offers for the remaining planning periods depend on the water inflow scenarios. When deciding its offers, the producer must consider that, in every scenario, the system operator will coordinate power production in order to minimize the total amount of thermal generation. For that, it takes into consideration anticipated values for the offers made by the remaining power producers. Thus the producer problem has, in the lower level, N_ω problems that are solved by the system operator to determine feasible dispatches.

To formulate the producer problem, it is supposed that decisions related to the first planning period are made in the first stage, whereas decisions related to the remaining planning periods are made in the second stage considering N_ω streamflow scenarios. Let $O_{k,t,\omega}$ be the amount of power producer k offers to the market at period t and scenario ω and $\mathbf{s}_\omega = \{v_{i,t,\omega}, q_{i,t,\omega}, u_{i,t,\omega}, P_{h_{i,t,\omega}}, y_{t,\omega}, \forall i, t\}$. The problem of producer k can be stated as:

$$
\begin{aligned}
&\min_{O_{k,t,\omega}} \sum_{\omega=1}^{N_\omega} \pi_\omega \left| v_{k,T,\omega} - v_k^{sp} \right| \\
&\text{subject to} \\
&\qquad P_{h_k}^{\min} \le O_{k,t,\omega} \le P_{h_k}^{\max}, \\
&\qquad O_{k,1,1} = O_{k,1,2} = ... = O_{k,1,N_\omega}, \\
&\qquad v_{i,1,1} = v_{i,1,2} = ... = v_{i,1,N_\omega}, \\
&\qquad \vdots \\
&\qquad \theta_{n,1,1} = ... = \theta_{n,1,N_\omega}, \\
&\qquad \mathbf{s}_\omega \text{ solve} \\
&\qquad P(\omega) \begin{cases} \min & \pi_\omega \sum_{t=1}^{T} y_{t,\omega} \\ \text{subject to} & \\ & (1)-(3), \end{cases} \\
&\qquad \omega = 1, ..., N_\omega.
\end{aligned} \tag{4}
$$

It should be noticed that generator offers, $O_{i,t,\omega}, \forall i, t, \omega$, are fixed in the lower level problems.

C. Single Level Problem

The single level problem that is equivalent to (4) is obtained by substituting the conditions that define the optimal solution of every lower level problem into the upper level problem. Lagrangian duality conditions are enforced. The basic supposition is that the lower level problems are "sufficiently convex" to respect such conditions. Thus, an optimal solution to $P(\omega)$, satisfies its constraints, the constraints of its dual problem and the strong duality conditions [22]. To derive the single level problem, the lower level problem $P(\omega)$ is rewritten as

$$
\begin{array}{ll}
\min & f(\mathbf{s}_\omega) \\
\text{s. to} & \mathbf{g}(\mathbf{s}_\omega) = \mathbf{0}, \\
& \mathbf{h}(\mathbf{s}_\omega) \le \mathbf{0},
\end{array} \tag{5}
$$

where $\mathbf{g}$ and $\mathbf{h}$ are composed by the equality and inequality constraints (1)-(3).

The Lagrangian of (5) is defined as

$$
\mathcal{L}(\mathbf{s}_\omega, \lambda, \zeta) = f(\mathbf{s}_\omega) + \lambda^\top \mathbf{g}(\mathbf{s}_\omega) + \zeta^\top \mathbf{h}(\mathbf{s}_\omega), \tag{6}
$$

where λ and ζ are multipliers (dual variables).

Let $\mathbf{s}_\omega^*$ be feasible to (5) and a stationary point of the Lagrangian. Then:

$$
\begin{aligned}
&\frac{\partial \mathcal{L}}{\partial \mathbf{s}_\omega}(\mathbf{s}_\omega^*, \lambda, \zeta) = 0, \\
&\zeta \ge 0.
\end{aligned} \tag{7}
$$

$\mathcal{L}(\mathbf{s}_\omega^*, \lambda, \zeta) \le \mathcal{L}(\mathbf{s}_\omega, \lambda, \zeta)$, for all $\mathbf{s}_\omega$ that is feasible to (5). The dual function is:

$$
h(\lambda, \zeta) = \mathcal{L}(\mathbf{s}_\omega^*, \lambda, \zeta). \tag{8}
$$

It should be noticed that the analytical expression of $h(\lambda, \zeta)$ is obtained by substituting $\mathbf{s}_\omega^*$, obtained from (7), into $\mathcal{L}$.

Let (λ^*, ζ^*) be the maximum of $h(\lambda, \zeta)$ for $\zeta \ge 0$. If the strong duality condition is satisfied, then $h(\lambda^*, \zeta^*) = f(\mathbf{s}_\omega^*)$.

Thus, a point $\mathbf{s}_\omega$ that solves (5) with multipliers (λ, ζ) respects the following conditions:

$$
\begin{aligned}
&\mathbf{g}(\mathbf{s}_\omega) = \mathbf{0}, \\
&\mathbf{h}(\mathbf{s}_\omega) \le \mathbf{0}, \\
&\frac{\partial^\top \mathbf{g}}{\partial \mathbf{s}_\omega}(\mathbf{s}_\omega)\lambda + \frac{\partial^\top \mathbf{h}}{\partial \mathbf{s}_\omega}(\mathbf{s}_\omega)\zeta = \mathbf{0}, \\
&\zeta \ge 0, \\
&h(\lambda, \zeta) = f(\mathbf{s}_\omega).
\end{aligned} \tag{9}
$$

Every lower level problem of (4), $P(\omega)$, is substituted by conditions (9). The result is a single level optimization problem with linear and nonlinear constraints and a non differentiable objective function.

III. Piecewise Integer Linear Model

To solve the producer problem, its objective function and nonlinear constraints are represented by introducing new variables, equalities and inequalities to the problem.

To represent the objective function, let

$$
var_\omega = v_{k,T,\omega} - v_k^{sp}, \ \forall \omega. \tag{10}
$$

$\min \sum_{\omega=1}^{N_\omega} \pi_\omega \left| v_{k,T,\omega} - v_k^{sp} \right|$ is replaced by

$$
\begin{array}{ll}
\min & \sum_{\omega=1}^{N_\omega} \pi_\omega var_\omega, \\
\text{s. to} & v_{k,T,\omega} - v_k^{sp} \le var_\omega, \\
& v_k^{sp} - v_{k,T,\omega} \le var_\omega.
\end{array} \tag{11}
$$

Every nonlinear constraints of the single level problem, f, is approximated by a piecewise linear function, $\tilde{f}$, that satisfies a set of constraints. Let $\tilde{f}(\mathbf{x}) : \mathcal{D} \to \mathbb{R}$ be a continuous piecewise linear function whose domain is a compact set $\mathcal{D} \subseteq \mathbb{R}^d$. Then $\tilde{f}(\mathbf{x}) = \{\mathbf{m}_P^\top \mathbf{x} + c_P, \ \mathbf{x} \in P, \forall P \in \mathcal{P}\}$ where P is a simplex[1], $\mathbf{m}_P$ is a vector, c_P is a scalar and $\mathcal{P}$ is the family of simplexes that represent $\tilde{f}$ in its entire domain.

Fig. 1(a) shows a nonlinear function f and Fig. 1(b) shows the piecewise linear function $\tilde{f}$ that approximates f. The domain of the function is divided into a set of simplexes $\mathcal{P}$ shown in Fig. 1(c). The J1 triangulation [19] is adopted to subdivide $\mathcal{D}$. Each simplex has a set of vertices, $V(P)$. The set of all the vertices in the domain of f is denoted $\mathcal{V}$. The simplexes, and also the vertices of each simplex, are sorted according to pre-defined schemes [20].

In convex combination piecewise linearization models, the representation of a point in $\tilde{f}$, $(\mathbf{x}, \tilde{f}(\mathbf{x}))$, is given by the convex combination of the vertices of the simplexes. To accomplish this, the vertices of every simplex P are numbered and, to each one, ϑ, is associated a weighting variable, $\mu_{P,\vartheta}$ (Figure 1)(c). Then:

$$
\begin{aligned}
&(\mathbf{x}, \tilde{f}(\mathbf{x})) = \sum_{P \in \mathcal{P}} \sum_{\vartheta \in \mathcal{V}} \mu_{P,\vartheta} (\vartheta, \tilde{f}(\vartheta)), \\
&\mu_{P,\vartheta} \ge 0, \forall P \in \mathcal{P}, \forall \vartheta \in \mathcal{V}, \\
&\sum_{P \in \mathcal{P}} \sum_{\vartheta \in \mathcal{V}} \mu_{P,\vartheta} = 1.
\end{aligned}
$$

1 A simplex with dimension d is the polytope which is the convex hull of $d+1$ vertices.

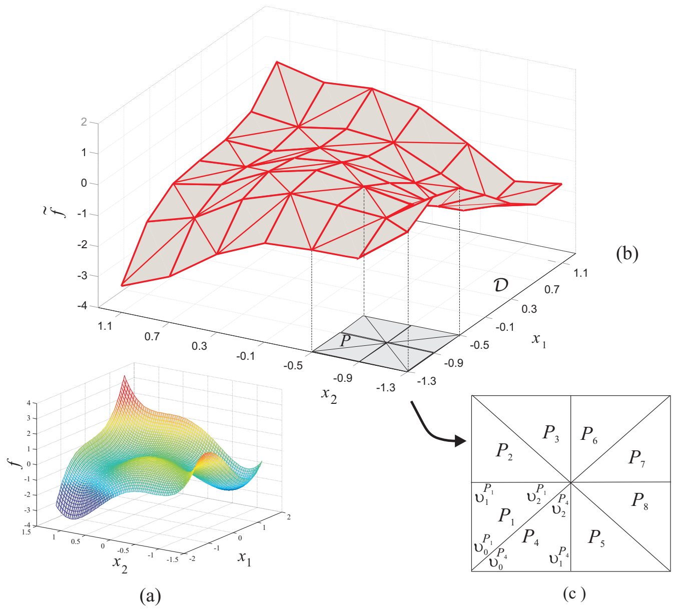

Figure 1. Piecewise Linear Approximation of $f(x)$

A. Disaggregated Convex Combination (DCC) Model

A mixed-integer linear optimization algorithm scans the simplexes that approximate f in the search for the optimal solution to the problem. By choosing a particular piecewise linear model, we define how such search is conducted. In the DCC model, a binary variable, z_P is associated with every simplex P. When the algorithm scans simplex P, $z_P = 1$.

The DCC model is expressed by [18], [19]:

$$\sum_{P\in\mathcal{P}}\sum_{\vartheta\in\mathcal{V}} \mu_{P,\vartheta}\vartheta = \mathbf{x}, \tag{12}$$

$$\sum_{P\in\mathcal{P}}\sum_{\vartheta\in\mathcal{V}} \mu_{P,\vartheta}\tilde{f}(\vartheta) = \tilde{f}(\mathbf{x}), \tag{13}$$

$$\mu_{P,\vartheta} \geq 0, \forall P\in\mathcal{P}, \forall\vartheta\in\mathcal{V}, \tag{14}$$

$$\sum_{\vartheta\in V} \mu_{P,\vartheta} = z_P, \forall P\in\mathcal{P}, \tag{15}$$

$$\sum_{P\in\mathcal{P}} z_P = 1,\; z_P\in\{0,1\}, \forall P\in\mathcal{P}. \tag{16}$$

Constraints (12)-(14) represent a point $(\mathbf{x}, \tilde{f}(\mathbf{x}))$ in the graph as a convex combination of $(\vartheta, \tilde{f}(\vartheta))$. Constraints (15) and (16) limit the convex combination to a single simplex as, from (16), only one variable z_P can be equal to 1. When $z_P = 1$, the weighting variables associated to P may vary within $[0, 1]$, whereas the remaining weighting variables are set to zero.

The single level problem, which is equivalent to the producer problem, has three types of nonlinear constraints: the power production functions (2), the derivatives of $\mathcal{L}$ with respect to $v_{i,t,\omega}, q_{i,t,\omega}$ and $u_{i,t,\omega}$ and the strong duality condition, $h(\lambda, \zeta) = f(\mathbf{s}_\omega)$. Function (2) is expressed in terms of three variables $v_{i,t,\omega}, q_{i,t,\omega}$ and $u_{i,t,\omega}$, therefore the simplexes used its linearization are tetrahedrons whose vertices are defined for given values of these three variables. The piecewise linearization shown in Figure 1 is extended to the three-dimension case shown in Figure 2. The tetrahedrons used in the PWL approximation are derived from the subdivision of small cubes defined by the intervals of variation of the three variables. In this case, the J1 triangulation is carried out using subdivision 1 or 2 of each cube (Figure 2(c)).

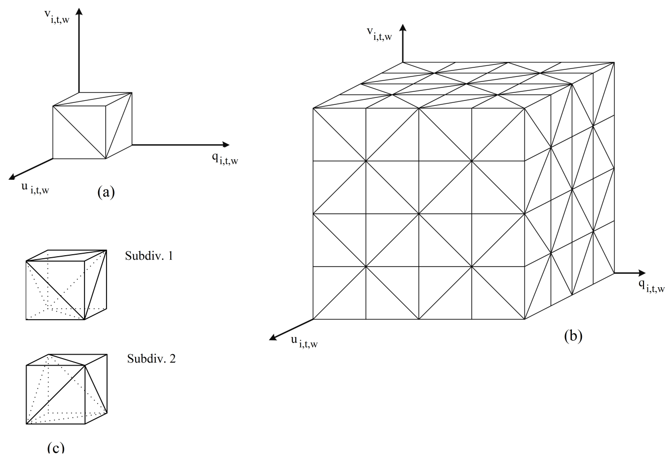

Figure 2. Triangulation in Three Dimensions

The piecewise linearization of (2) is given by (17). It is expressed in terms of weighting variables $\mu_{P,\vartheta,i,t,\omega}$, each one associated with vertex ϑ of simplex P, defined in the domain of a particular variable of the problem that is associated with hydro plant i, period t and scenario ω.

$$\begin{aligned}
&\sum_{P\in\mathcal{P}}\sum_{\vartheta\in\mathcal{V}} \mu_{P,\vartheta,i,t,\omega}\vartheta = v_{i,t,\omega},\\
&\sum_{P\in\mathcal{P}}\sum_{\vartheta\in\mathcal{V}} \mu_{P,\vartheta,i,t,\omega}\vartheta = q_{i,t,\omega},\\
&\sum_{P\in\mathcal{P}}\sum_{\vartheta\in\mathcal{V}} \mu_{P,\vartheta,i,t,\omega}\vartheta = u_{i,t,\omega},\\
&\sum_{P\in\mathcal{P}}\sum_{\vartheta\in\mathcal{V}} \mu_{P,\vartheta,i,t,\omega}\tilde{Ph}(\vartheta) = \tilde{Ph}_{i,t,\omega},\\
&\mu_{P,\vartheta,i,t,\omega} \geq 0, \forall P\in\mathcal{P}, \forall\vartheta\in V(P),\\
&\sum_{\vartheta\in V(P)} \mu_{P,\vartheta,i,t,\omega} = z_{P,i,t,\omega}, \forall P\in\mathcal{P},\\
&\sum_{P\in\mathcal{P}} z_{P,i,t,\omega} = 1,\; z_{P,i,t,\omega}\in\{0,1\}, \forall P\in\mathcal{P}.
\end{aligned} \tag{17}$$

The remaining nonlinear functions of the single level problem are replaced by sets of constraints that are analogous to (17).

As the piecewise linearization requires that the domains of the functions be bounded, upper bounds should be defined for $u_{i,t,\omega}$, $\forall i, t, \omega$, and upper and lower bounds should be defined for the dual variables λ and ζ, as they appear in the nonlinear constraints (9). These bounds define the number of simplexes used in the linearization and affect the computational time to solve the mixed integer linear problem.

IV. Test Results

Tests are carried out with two small systems, composed by Brazilian hydro plants (Table I). The producer problem is solved for a single streamflow scenario, with water inflows set equal to the average value of monthly inflows during year 2014 [23]. The planning period starts in January and the reservoirs are supposed to be initially at the 80% of their maximum storage capacity. Load curves are constructed by normalizing the annual load profiles of the Southeastern Region of Brazil [24] and multiplying the normalized values by the total generation capacity of the system. Additional data can be found in [20].

The solutions to the nonlinear single level problem are calculated by CONOPT, whereas those of the PWL problem are calculated by CPLEX. The solvers run under GAMS platform in a Intel Core i7-4500, 1600MHz computer with 8GB RAM. CONOPT and CPLEX obtain the solutions to the problems of two different producers.

TABLE I. SYSTEM DATA

System	# Buses	# H. Plants	Gen. Cap.(MW)	Storage Cap.(hm^3)
1	3	2	$1,127$	$19,335$
2	15	4	$7,277$	$46,305$

A. The Problem of Chavantes

In the first test, we obtain the optimal offers of Chavantes power plant, located in the system shown in Figure 3. The objective of the producer is to finish the planning period with its reservoir at 50% of the maximum storage capacity ($v^{sp} = 7274.5$ hm^3).

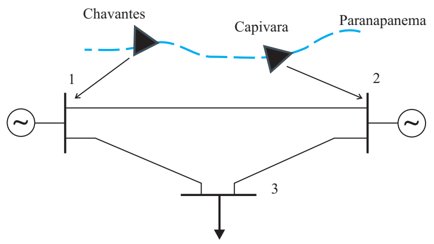

Figure 3. Three Bus System

Figure 4 shows the optimal trajectories of the reservoir volumes at the solution of the nonlinear model (NL) and the PWl model (L). It can be noticed that the solutions of the two models are different. In spite of this, the objective of Chavantes is fulfilled at both solutions. However, the final volume of stored water at Capivara is very different at the two solutions. While, at the solution of the nonlinear model, Capivara has 9222.2hm^3 of stored water at the end of the planning period, at the solution of the PWL model, this amount is 4816hm^3.

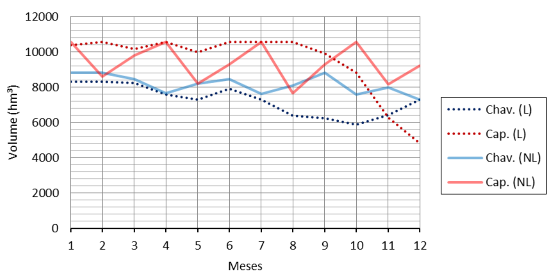

Figure 4. Three Bus System

Form the behavior of the reservoir volumes, it can be inferred that the optimal values of hydro and thermal generation are quite different at the solutions of the nonlinear and PWL models. As nonlinear model is noncovex, depending on the inicialization adopted, the solutions obtained by CONOPT can be local or global optima. The solution of the PWL model, on the other hand, is obtained by exhaustive search, using branch-and-bound algorithm. In this case, the solution to the PWL model is a global optimum. The results shown in Table II confirm this point. Some characteristics of two solutions to the nonlinear model, and the solution of the PWL model are presented: the total energy produced by hydro and power plants, E and Y, the total volume of turbined water, Q, and the total volume of spilled water, U, in the 12 months period. In all these solutions, Chavantes meets its target value for the final reservoir volume. Therefore, the optimal value of the objective function of the upper level problem is the same in all solutions. In spite of this, the total thermal energy is lowest at the solution of the PWL model, and is quite different at the two solutions of the nonlinear model. From the point of view of the system operator (the lower level decision maker), the solution of the PWL model is the best one followed by solution #2 of the nonlinear model. The solution of the PWL model is the global optimum for the problem.

TABLE II. SOLUTIONS - CHAVANTES

Model	Solutions	E (GWh)	Q (hm^3)	U (hm^3)	Y (GWh)
Nonlinear	#1	$71,343.0$	$599,056.0$	$1,077.5$	$36,913.8$
	#2	$35,113.6$	$32,867.4$	$324,339.0$	$73,145.1$
PWL		$76,234.8$	$652,698.8$	280.32	$32,022.2$

B. The Problem of Água Vermelha

This hydro plant is located in the 15-bus system depicted in Fig. 5. When determining its optimal offers, the objective of this producer is to finish the planning period with $10,000$ hm^3 of water in its reservoir.

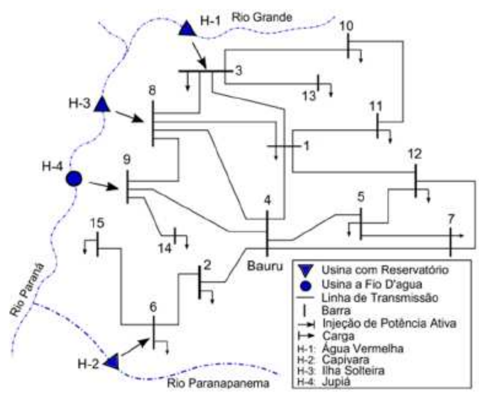

Figure 5. 15-bus System

The solutions obtained for this producer problem using the two different models are given in Table III. In all the solutions, the producer attains its objective, i.e., the optimal value of the objective function of the upper level problem is equal to zero. Comparing solutions #1 and #2 of the nonlinear model, it is possible to notice that the first one is better since it has lower thermal generation. The solution obtained with the PWL model is, once more, the best one.

Figure 6 depicts the hydro power generation of all the plants at solution #1 of the nonlinear model and at the solution of

TABLE III. SOLUTIONS - ÁGUA VERMELHA

Model	Solutions	E (GWh)	Q (hm^3)	U (hm^3)	Y (GWh)
Nonlinear	#1	$400,997.8$	$5,058,505.8$	$542,165.2$	$298,110.7$
	#2	$370,082.8$	$4,466,373.6$	$1,149,075.5$	$329,025.6$
PWL		$406,510.4$	$5,044,393.4$	$529,025.2$	$292,596.3$

the PWL model. The largest differences occur for Ilha Solteira, particularly between the third and the eighth month.

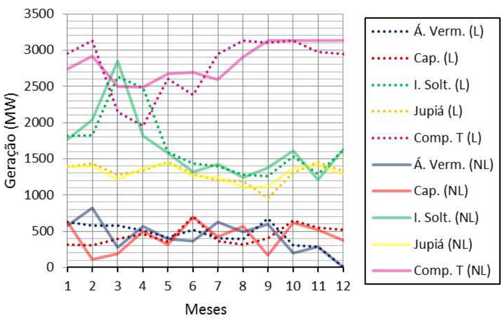

Figure 6. 15-bus System

C. Computational Effort

Some characteristics of the problems and also the computational effort needed to solve the two problems are shown in Table IV. In the PWL process, the interval of variation of each variable is divided in two. It is transparent that the main drawback of the PWL technique is the size of the mixed linear integer problem and the computational effort to obtain its solution.

TABLE IV. COMPUTATIONAL EFFORT

Problem	Solutions	#Cont. Var.	# Bin. Var.	CPU Time (s)	# Iter.
Chavantes	NL #1	580	0	1.75	51
	NL #2	580	0	1.20	43
	PWL	$35,908$	$7,140$	963.9	$2,967,499$
Água Vermelha	NL #1	$2,200$	0	0.33	49
	NL #2	$2,200$	0	0.52	186
	PWL	$72,856$	$14,208$	$35,033.7$	$1,119,455$

V. CONCLUSIONS

The problem faced by a hydro power producer that needs to determine its optimal offers to the power market can be formulated as a bilevel optimization problem. This bilevel problem can be transformed into a single level problem using Lagrangian duality conditions. This single level problem is nonconvex and has local and global optimal solutions. Nonlinear programming algorithms can be trapped in local stationary points, therefore global optimization techniques must be used to properly solve the problem. The disaggregated convex combination model is used in this study to transform the nonlinear single level problem into a mixed integer-linear problem that is solved using branch and bound algorithm. Using this procedure, it is possible to obtain global optimal solutions to the producer problem. The main drawback of the technique is the size of the mixed integer linear problem. To reduce this problem, other PWL techniques found in the literature can be adopted.

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