POWER SYSTEM MODELING THROUGH QUADRIPOLES FOR VOLTAGE STABILITY ANALYSIS

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Abstract— The paper presents a methodology of analysis for radial power systems using quadripoles for the components representation. Three-phase elements can be represented this way if they can be considered symmetric and symmetrical components transformations are used in order to work with modal quantities instead of the phase ones. The application of the method is presented using several examples derived from real power systems, allowing interesting studies as voltage stability, reactive compensation, P-Q and P-V curves drawing, voltage variation and reactive power flow, among others. It is shown that the proposed tool demands substantially less computational effort, being, thus, very attractive to evaluate disturbances.

Keywords— Distributed Generation, Power quality, Power systems, Power Flow, Electric System Modeling

Resumo— O artigo apresenta uma metodologia para a análise de sistemas de potência radiais que utiliza quadripolos para representar cada um dos seus equipamentos. Equipamentos trifásicos podem ser representados desta maneira caso se considere que os mesmos são simétricos e utilizando-se componentes simétricas para se operar com as grandezas modais no lugar das de fase. Aplicações deste método são apresentadas utilizando-se exemplos de sistemas elétricos reais e realizando análises interessantes como de estabilidade de tensão, estudo da compensação reativa, obtenção das curvas P-Q e P-V, variações de tensão em função da carga e o fluxo de potência reativa, dentre outros. É mostrado que a ferramenta proposta reduz substancialmente o esforço computacional empregado nas simulações, sendo, portanto, uma opção bastante atrativa para a avaliação de distúrbios.

Palavras-chave— Geração distribuída, Qualidade de energia, Sistemas de potência, Fluxo de Potência, Modelagem de Sistemas Elétricos

1 Introduction

With the substantial increase of the distributed generation in power electrical systems, some problems related to power quality, such as voltage variations and undesirable reactive power flow, may occur if the proper analysis of the system is not performed (Stevenson Jr, 1982; Anderson,1977).

Traditional power flow analysis is based on the resolution of non-linear complex matricial equations (Kundur, 1994), which determine the voltage magnitudes and phases in different bars of the system, so that the power flow between those bars (through each element of the system) be equal to the consumed and/or generated power. Mathematically, the active and reactive power in each bar is given by (Stevenson Jr, 1982):

$$P_i = \sum_k |V_i|\,|V_k|\,[G_{ik}\cos(\theta_i - \theta_k) + B_{ik}\sin(\theta_i - \theta_k)] \quad (1)$$

$$Q_i = \sum_k |V_i|\,|V_k|\,[G_{ik}\sin(\theta_i - \theta_k) - B_{ik}\cos(\theta_i - \theta_k)] \quad (2)$$

where $|V_i|$ and θ_i are, respectively, the magnitude and the phase angle of the voltage in the bar i, P_i and Q_i are the consumed and/or generated active and reactive power in the bar i, the set of k values are the bars connected to i by some element and G_{ik} and B_{ik} are the real and imaginary parts of the admittance of those elements.

When the Equations 1 e 2 are applied for each bar of the system, the result is the matricial equation previous mentioned, and the resolution of the power flow problem consists in determining the values of $|V_i|$ and θ_i for each bar in order to satisfy these equations.

Newton-Raphson or the Gauss-Seidel are the most often chosen methods to solve the matricial equation (Melizi, 2016), and both are iterative methods whose convergence strongly depends on the initial guesses of all unknown variables (voltage magnitudes and angles) (Osano, 1997). For this reason, numerical solvers usually use as initial guesses the solution of the linearized DC power-flow model (Remolino, 2016), as it is more close to the non-linear problem solution than a random guess.

The resolution of the matricial equation may be optimized choosing correctly the indices of the voltages and angles (Athay, 1983) or changing the solving numerical method (Milano, 2009).

This paper proposes an alternative method to solve this problem, which may be applied for radial power systems (i.e., systems where the path between any two elements is unique). It consists in selecting one bar as the infinite bus of the system (which means that voltage is kept constant in this point even when the load changes; a LTC or a ring system may be chosen) and a second bar where a load or generator is located. The path connecting these two points comprises some elements associated in series, which will be represented as quadripoles, to be associated.

As it will be shown in the next section, this method does not use iterations to find the solution, once the resulting equation has a solution, which may be calculated through an explicit expression. This feature will provide computation advantages that will permit to describe correctly the behavior of the system under different conditions.

2 Method Presentation

The most simple case of this method is presented in Fig. 1: an alternate-voltage source (representing the infinite bus bar of the system, sometimes called swing bar, where the voltage modulus and the voltage angle are specified and the active and reactive power supplied depends on the rest of the system), a quadripole (uniquely represented by a 2-by-2 matrix) and a load (by this time it is not necessary to specify the relationship between the voltage and the current in its terminals).

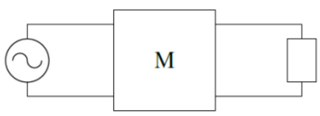


Figure 1: System with a constant voltage source feeding a load

The voltage and the current in these elements are given in 3.

$$\begin{pmatrix} \hat{v}_L \\ \hat{i}_L \end{pmatrix} = M \begin{pmatrix} \hat{V}_S \\ \hat{i}_S \end{pmatrix} \tag{3}$$

where M is a complex matrix. Be H a matrix that associates the modified vectors of input and output:

$$\begin{pmatrix} \hat{v}_L \\ \hat{i}_S \end{pmatrix} = H \begin{pmatrix} \hat{V}_S \\ \hat{i}_L \end{pmatrix} \tag{4}$$

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \longrightarrow H = \frac{1}{D} \begin{pmatrix} det(M) & B \\ -C & 1 \end{pmatrix} \tag{5}$$

The characteristics of the load stipulate a relationship between the current and the voltage in its terminals:

$$\hat{i}_L = f\left(\hat{v}_L\right) \tag{6}$$

The first line of the matrix Equation 4 is written as follows:

$$\hat{v}_L = \frac{1}{D}\left[det(M)\hat{V}_S + Bf\left(\hat{v}_L\right)\right] \tag{7}$$

Solving Equation 7 in the variable $\hat{v}_L$ and then obtaining $\hat{i}_L$ by 6, the second line of the matrix Equation 4 is used to find $\hat{i}_S$, so all the electrical quantities are determined.

If a power system comprises a series association of equipments as those presented in the previous section (as illustrated in Fig. 2), the relation between the current and voltage in its terminals is obtained from the product of the matrices of each equipment, as indicated in Equation 8.

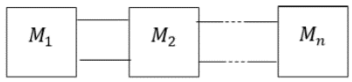


Figure 2: Series associations of multiple system components

$$M_{eq} = M_1 M_2 ... M_n \tag{8}$$

3 Representation of Power System Equipment Through Quadripoles

The method introduced in the previous section consists in representing each one of the equipments of a radial system as a quadripole, which means that all their behavior will be described by the relations between the input and output voltages and currents between its terminal. Once power systems comprise three-phases equipment, representation as a quadripole supposes that all of them are balanced (symmetric) which simplifies the representation.

A quadripole may be entirely described by a 2-by-2 matrix that relates the voltage and current from its terminals. In the sections below, the most common equipment found in power systems and their quadripole matrices will be shown.

3.1 Transformers

The one-phase circuit of the transformer is presented in Fig. 3.

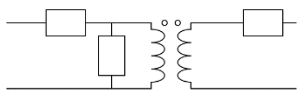


Figure 3: One-phase transformer equivalent circuit

The relation between the primary and secondary voltages and currents is given by the matricial Equation 9.

$$\begin{pmatrix} \hat{v}_2 \\ \hat{i}_2 \end{pmatrix} = \begin{pmatrix} \alpha + \frac{1}{\alpha}\frac{z_2}{z_c} & -\left[\alpha z_1 + \frac{z_2}{\alpha}\left(1 + \frac{z_1}{z_c}\right)\right] \\ -\frac{1}{\alpha}\frac{1}{z_c} & \frac{1}{\alpha}\left(1 + \frac{z_1}{z_c}\right) \end{pmatrix} \begin{pmatrix} \hat{v}_1 \\ \hat{i}_1 \end{pmatrix} \tag{9}$$

If the impedance of the magnetic branch is neglected, then $|z_c| \to \infty$, so Equation 9 may be rewritten as indicated in Equation 10.

$$\begin{pmatrix} \hat{v}_2 \\ \hat{i}_2 \end{pmatrix} = \begin{pmatrix} \alpha & -\left(\alpha z_1 + \frac{z_2}{\alpha}\right) \\ 0 & \frac{1}{\alpha} \end{pmatrix} \begin{pmatrix} \hat{v}_1 \\ \hat{i}_1 \end{pmatrix} \tag{10}$$

3.2 Transmission lines

Assuming that the line is continuously transposed, it can be represented by a single phase circuit, whose representation depends on the length of the line. In the next paragraphs, three circuits and its matrices are presented for three different length ranges (Stevenson Jr, 1982).

- Short-length lines

If the length of the line is less than about 80 km, it can considered a short-length line, so only its resistance and its inductance may be considered. The resulting circuit is a series impedance and the corresponding matrix is indicated in 11.

$$\begin{pmatrix} \hat{v}_2 \\ \hat{i}_2 \end{pmatrix} = \begin{pmatrix} 1 & Z_l \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{v}_1 \\ \hat{i}_1 \end{pmatrix} \tag{11}$$

- B. Medium-length lines

If the transmission line is large enough so its capacitance is no longer neglectable, (which occurs for the length range of 80 km to 240 km), then it is considered a medium-length line and its circuit is indicated in Fig. 4.

$$\begin{pmatrix} \hat{v}_2 \\ \hat{i}_2 \end{pmatrix} = \begin{pmatrix} 1 + \frac{zy}{2} & z \\ y\left(1 + \frac{zy}{4}\right) & 1 + \frac{zy}{2} \end{pmatrix} \begin{pmatrix} \hat{v}_1 \\ \hat{i}_1 \end{pmatrix} \tag{12}$$

The pi-circuit of the Fig. 4 is associated to the matrix of the Equation 12.

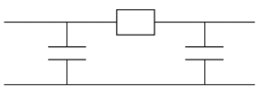


Figure 4: Transmission line equivalent circuit

- C. Long-length lines

For a long-length transmission line, the pi-circuit indicated in 4 is no longer satisfactory, so a more sophisticated model shall be used, as for example the distributed parameter model (Stevenson Jr, 1982), whose matrix is indicated in Equation 13.

$$\begin{pmatrix} \hat{v}_2 \\ \hat{i}_2 \end{pmatrix} = \begin{pmatrix} \cosh(\gamma l) & Z_0 \sinh(\gamma l) \\ \frac{1}{Z_0}\sinh(\gamma l) & \cosh(\gamma l) \end{pmatrix} \begin{pmatrix} \hat{v}_1 \\ \hat{i}_1 \end{pmatrix} \tag{13}$$

3.3 Reactive power compensation equipments

Shunt capacitors and reactors, used in reactive power compensation may be represented by a quadripole whose matrix is indicated in 14.

$$\begin{pmatrix} \hat{v}_2 \\ \hat{i}_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{1}{jZ_Q} & 1 \end{pmatrix} \begin{pmatrix} \hat{v}_1 \\ \hat{i}_1 \end{pmatrix} \tag{14}$$

where $Z_Q = \omega L$ for reactors or $Z_Q = \frac{1}{\omega C}$ for capacitors, where C is the equivalent capacitance of the bank, which may be calculated from the nominal reactive power $C = \frac{Q_0}{V_0^2}$, where V_0 is the nominal voltage of the equipment.

3.4 Loads

The loads are represented in this method as a two-terminal component to be branched to one of the extremities of a quadripole. The relationship between the current and voltage in its terminals is indicated in a column-vector (as indicated in 3) or through the function f of Equation 6. Table I shows some examples of this function for different load types.

TABLE I: Relationship between the current and voltage for different loads.

Quantity constant	Impedance	Power	Current	Voltage
Power dependance	$\propto \|\hat{v}_L\|^2$	cte	$\propto \|\hat{v}_L\|$	–
Example	Resistances	Motors	Induction furnaces	Sync. motors
f	$\frac{\hat{v}_L}{Z_L}$	$\frac{S_L}{\hat{v}_L}$	$\hat{I}_L$	–

Besides the three typical loads considered in the power flow studies, a forth one is included: a constant voltage load, which is a load whose voltage in its terminal may be kept constant regardless the conditions of the system. As an example, we consider a synchronous motor.

3.5 Distributed generation equipment

The generation units are represented in this method in the same way the loads are, but the power load will be inverse (i.e., towards the source). Only a distinction shall be made depending on the type of equipment used to inject power into the grid.

- Synchronous generator-based generation

If the generation is a synchronous generator, it will be represented the same way as a constant-voltage load.

- Inverter-based generation

If the generation unit is based on an inverter, it will be represented the same way as a constant-power load (but injecting power into the grid).

4 Problem Solution and Power System Quantities Determination

The Equation 7 allows to obtain the voltage at the load terminals, which may be used to calculate all the others electrical quantities; however, this equation depends on the function f that describes the behavior of the current in the load as the voltage at its terminals changes.

Replacing f for all the load types described in section 3.4, 7 can be rewritten in the general form:

$$\hat{v}_L = A + B\frac{\hat{v}_L}{|\hat{v}_L|} + C\left(\frac{\hat{v}_L}{|\hat{v}_L|}\right)^2 \tag{15}$$

which is a non-linear complex algebraic equation. The parameters A, B and C are complex number that depends on the system matrix M and the function f.

If the system has a constant-power or a constant-impedance load in its extremity, then $|B| = 0$ and Equation 15 has analytical solutions. Writing the complex variable $\hat{v}_L$ in its Cartesian form $x_0 + jy_0$, the solution is given in 16 and 17

$$x_0 = \frac{-2\Im[A]\Im[C] - \Im[A]^2\Re[A] - \Re[A]^3}{2(\Im[A]^2 + \Re[A]^2)} \pm \frac{1}{2}\sqrt{\frac{-4\Re[A]^2\Im[C]^2 - 4\Im[A]^2\Re[A]^2\Re[C] - 4\Re[A]^4\Re[C] + 2\Im[A]^2\Re[A]^4 + \Im[A]^4\Re[A]^2 + \Re[A]^6}{\left(\Im[A]^2 + \Re[A]^2\right)^2}} \tag{16}$$

$$y_0 = \frac{\Im[A]\,a_0 + \Im[C]}{\Re[A]} \tag{17}$$

If $|B| > 0$ (which occurs for constant-current loads), so the problem assumes its most complex form and no analytical solution has been found. However, the equation can be solved by a numerical iterative method; rewriting 15 in Cartesian form:

$$x_0 = \Re[A] + \frac{\Re[B]x_0 - \Im[B]}{\sqrt{x_0^2 + y_0^2}} + \frac{\Re[C]x_0 - \Im[C]}{\sqrt{x_0^2 + y_0^2}} \tag{18}$$

$$y_0 = \Re[A] + \frac{\Re[B]x_0 - \Im[B]}{\sqrt{x_0^2 + y_0^2}} + \frac{\Re[C]x_0 - \Im[C]}{\sqrt{x_0^2 + y_0^2}} \tag{19}$$

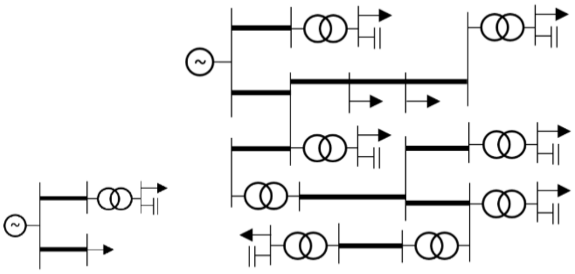
The Jacobian matrix is indicated in 20, which is shown on page 4 and is used to perform Newton-Raphson method for two variables.

$$J = \begin{pmatrix} -1 + \frac{y_0^2\Re[B]+\Im[B]x_0y_0}{\left(\sqrt{x_0^2+y_0^2}\right)^3} + \frac{y_0^2\Re[C]+x_0(2\Im[C]y_0-\Re[C]x_0)}{\left(x_0^2+y_0^2\right)^2} & \frac{-x_0^2\Im[B]-\Re[B]x_0y_0}{\left(\sqrt{x_0^2+y_0^2}\right)^3} + \frac{-x_0^2\Im[C]-y_0(-\Im[C]y_0+2\Re[C]x_0)}{\left(x_0^2+y_0^2\right)^2} \\ \frac{y_0^2\Im[B]-\Re[B]x_0y_0}{\left(\sqrt{x_0^2+y_0^2}\right)^3} + \frac{y_0^2\Im[C]-x_0(\Im[C]x_0+2\Re[C]y_0)}{\left(x_0^2+y_0^2\right)^2} & \frac{x_0^2\Re[B]-\Im[B]x_0y_0}{\left(\sqrt{x_0^2+y_0^2}\right)^3} + \frac{x_0^e\Re[C]-y_0(\Re[C]y_0+2\Im[C]x_0)}{\left(x_0^2+y_0^2\right)^2} \end{pmatrix} \tag{20}$$

5 Comparison with the Traditional Power Flow Method Resolution

The method described in the previous section may be applied to solve power flow problems in radial systems, with an advantage when compared with the classical methods: while the traditional techniques are based upon the resolution of a complex matricial equation, whose matrix size depends on the number of bars of the system and the solving is based on iterative numerical methods (Newton-Raphson or Gauss-Seidel method for n-variables), the method presented is based on simple matrices multiplication and the application of analytical expressions, as those indicated in 16 and 17. For the worst and particular case where a constant-current load is present, a numerical method is employed, but involves a 2-by-2 matrix and, if the initial solution employed is the one obtained from solving of the power-constant problem, the method will probably converge to a solution.

To confirm the advantages of the presented method, it will be tested with a radial system and the results will be compared with those obtained from the commonly used iterative method (Newton-Raphson method for solving the matricial equation obtained from equations 1 and 2, henceforth referred as "traditional method"). Two systems will be analyzed: one is simpler and comprises only 4 bars; a second one, more complex, contains 24 bars and corresponds to a real radial power system located in the southwest region of Brazil. Both systems are represented in Fig. 5.



(a) Four bars system. (b) Twenty-four bars system.

Figure 5: Simulation systems.

5.1 Accuracy

The first and most important comparison to be made is whether the proposed method outputs the same results than the already consolidated, traditional methods. Table II indicates the voltage magnitude and phase at the load terminals for both methods.

As it can be seen, the results are very much closer, which validates the proposed method.

TABLE II: Voltage magnitude and phase at the load terminals.

Quantity	$\lvert V_i \rvert$ (pu)		θ_i (degrees)	
System	4 bars	24 bars	4 bars	24 bars
Proposed method	1.0567	0.9878	-40.6	37.67
Traditional method	1.0567	0.9874	-40.6	37.67

5.2 Simulation time

Besides presenting the same results, the computational effort for the proposed method must be inferior than that of the traditional methods, in order to compensate the restricted topology of the power systems that can be solved.

The first indicator of the computational effort is the time elapsed of the calculations (directly related to the number of calculations performed). Table III shows the simulations times for both methods.

TABLE III: Resolution time (in milliseconds).

System	4 bars	24 bars
Proposed method	7	34
Traditional method	12	142

Once the traditional method are based on iterations (i.e., the same series of calculations are performed more than once in order to approach to the searched solution), it takes longer than the proposed method (applying the equations of section IV returns the right solution in just one step).

It is worth mentioning that simulation times characteristics are highly dependent on how the power flow program was developed, but the result showed in Table III might indicate that the proposed method has potential advantages.

5.3 Allocated memory

The second indicator of the computational effort is total allocated memory (directly related to the number of number stocked during the calculations).

It is not always possible to evaluate how much memory is necessary to perform a simulation because, depending on the platform, there is no direct measurement of this quantity. Here, an analytical evaluation of the necessary memory will be made as follows.

To stock a complex number it is necessary 16 bytes (8 for each double precision floating number). Traditional methods must allocate enough memory to stock the admittance matrix (whose number of non-null components is equal to the number of elements, which in radial system is equal to number of bars plus one). The consumed/generated power in each bar takes another two floating numbers, and another two for the unknown quantities, the magnitude and phase of the voltage. The number of bytes is then given by:

$$NB = (n + 1).16 + 2.8.n + 2.8.n = 16.(3n + 1) \quad (21)$$

In the proposed method, only the current and voltage magnitudes and phases for both load and source (4 complex numbers) need to stock with the quadripole matrix (another 4 complex numbers), totalizing 64 bytes. Thus, table IV shows the memory spent for both proposed and traditional methods.

TABLE IV: Estimated memory allocated during computation (in bytes).

System	4 bars	24 bars
Proposed method	64	64
Traditional method	208	1168

5.4 Divergence

As it was said previously, the proposed method has a major advantage of not using iterative solving methods, so there is no convergence issues. Instead of that, it is possible to know beforehand whether the system admits a valid solution by a check of the implicate variables.

5.5 Limitations

As mentioned in previous sections, this method has its limitations, which are listed below:

- It can only be applied to solve radial power systems

It is an intrinsic property of this method: the system topology cannot contain cycles in order to determine the equivalent tree of the system. Once this tree has a root and a leaf is chosen, the corresponding path will determine the series elements that will compose the system matrix M.

- Only one bar can possess a constant-power or a constant-current load

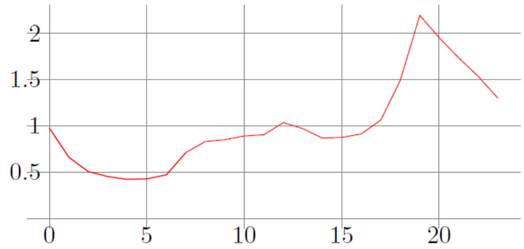
Only one bar in the whole system can possess a constant-power load, and this load must be the one at the extremity of the selected path of the radial system.

The secondary branches of the tree must be represented as an equivalent impedances, and this is only possible with the element at the leaf is a constant-impedance load.

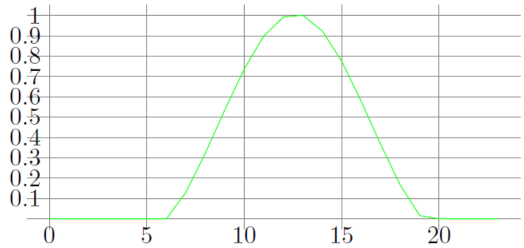
6 Application Example

Once this method can be used to solve many different power flow conditions, it can be particularly interesting to use it to evaluate the voltage variations all over the system for the typical generation and load profiles.

The example to be presented in the next sections corresponds to a real photovoltaic power plant that is been commissioned in Brazil, with peak-power of 3MWp, to be connected in the 13.8kV medium voltage grid circuit. The constant voltage bar of the system to be considered is the secondary of the transformer in the local distribution substations, which feeds the medium voltage circuits. The topology of the system and the line and transformers impedances were provided by a local electric company. Fig. 6 shows the typical generation and load profiles to be used.



(a) Load

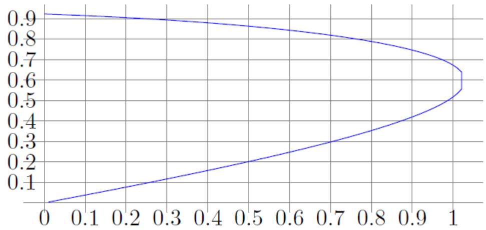


(b) Generation

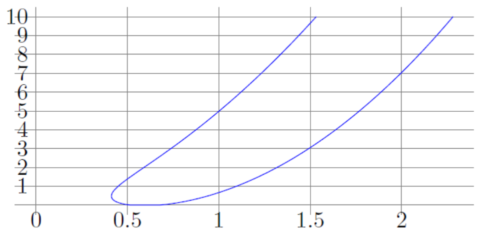
Figure 6: Load and generation profiles.

6.1 Voltage variations during operation under load change

The simulation was performed and the results of the power flow are shown in Fig. 7.



(a) $V = f(P)$



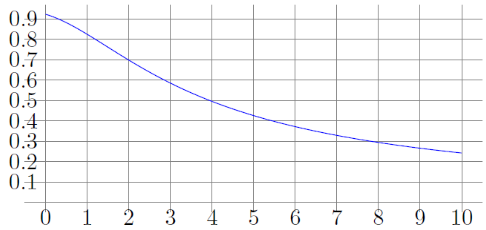
(b) $Q = f(V)$

Figure 7: Voltage stability curves (per unit).

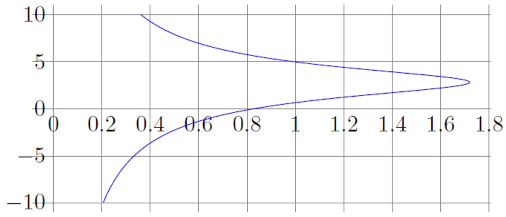
One of the advantages of this method is that is simple to perform different simulations with variation of the power and generation values, providing the necessary data to verify whether there would be power quality violations.

6.2 Voltage stability analysis

Aside from the voltage variations, it is also necessary to know whether the system is close to the voltage stability limit, which is the lowest voltage at a certain point for the system before there is no solution for the power flow problem. In fact, the existence of not-allowed values for the power consumed by a certain load depends on the behavior of this load for voltage variations, as described in section III.D: constant power loads makes the $V \times P$ chart (Fig. 7) to have an 'U' shape, meanwhile constant impedance loads change it to a quasi-linear behavior. Fig. 8 indicates the simulation results when all the loads are considered to have constant impedance behavior.



(a) $V = f(P)$



(b) $Q = f(V)$

Figure 8: Constant impedance loads curves (per unit).

6.3 Reactive compensation and transformers tap determination

Sometimes it is necessary the change the tap of the transformer in order to optimize the voltage range on the consumers or even avoid the transformer to reach the limits of the load tap change.

Another possibility is determine whether the power plant could help with the voltage regulation; usually, non-dispatch power plant generate with unitary power factor, but their machines (inverters, synchronous generator) could be adjusted to work keep maintain the voltage in a certain level.

In addition, in case there is an increase in the reactive power flow, this analysis can provide the necessary to guidance to asset the better place to install reactive compensator, as such capacitive banks.

6.4 Results and Discussion

In the case shown in the previous sections, one can observe that the injection of power in this part of the distribution system will not, in effect, affect or disturb the voltage for the consumer; for the periods of high generation, the power plant will contribute to maintain the voltage in the appropriate levels.

7 Conclusion

The proposed method for the power flow study provides an efficient way to perform the analysis of a radial power system. The reduced computational effort allows making a series of simulations, changing the boundary conditions, in order to estimate the impacts in the system of the insertion of distributed generation. Comparisons of the traditional methods showed the validity of the proposed method and the study case show some possible features in order to evaluate the potential disturbances in the system due to injection of power generation.

References

Stevenson Jr, W. D. (1982). Elements of Power System Analysis. McGraw-Hill, 4th edition edition.

Anderson, P. M. and A.A. (1977). Fouad. Power System Control and Stability. The Iowa State University Press.

Kundur, P. (1944). Power System Stability and Control. Power System Engineering Series.

Melizi, K. et al. (2016). Evaluation of the power flow algorithm for radial distribution system with several penetration of distributed generation. 8th International Conference on Modelling, Identification and Control (ICMIC).

Osano, M. and Capretz, M. A. M. (1977). A distributed method for solving nonlinear equations applying the power load flow calculation. Proceedings of the Thirtieth Hawaii International Conference on System Sciences.

Athay, T. (1983). An Overview of Power Flow Analysis. American Control Conference.

Milano, F. (2009). Continuous Newton's Method for Power Flow Analysis. IEEE Transactions on Power Systems, Volume: 24, Issue: 1, Feb.

Remolino, A. S and Paredes, H. F. R. (2016). An efficient method for power flow calculation applied to the reconfiguration of radial distribution systems. In IEEE PES Transmission & Distribution Conference and Exposition-Latin America (PES T&D-LA).