CLASSIFYING TEXTURE IMAGES USING ARTIFICIAL AGENTS, FRACTAL DIMENSION AND EXTREME LEARNING MACHINES

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Abstract— This work proposes a new method of representing textures on digital images through their maximum (or minimum if the negative of the image is used) and different intensity borders by means of artificial beings called artificial hikers that search for the maximum of a texture image and in doing so, represent the different characteristics of the image. The technique has two main parameters that can be adjusted in order to emphasize the greatest maximum of an image and different frequency borders. For the classification of textures, the technique of artificial hikers was combined with fractal dimension analysis and extreme learning machine and it presented superior results compared to previous works dealing with texture classification with artificial agents.

Keywords— Texture classification, Artificial agents, Artificial hikers, Fractal dimension, Extreme learning machine.

Resumo— Este trabalho propõe um novo método para representar texturas em imagens digitais por meio de seus máximos (ou mínimos se o negativo da imagem é utilizado) e suas bordas de diferentes intensidades através de agentes artificias chamados de *hikers* artificiais que buscam os máximos de uma imagem de textura e, neste processo, representam diferentes características da imagem. A técnica possui dois parâmetros principais que podem ser ajustados para enfatizar as maiores intensidades da imagem e as bordas de diferentes frequências. Para a classificação de texturas, a técnica de *hikers* artificiais foi combinada com a análise da dimensão fractal e máquinas de aprendizado extremo e apresentou resultados superiores se comparada com trabalhos anteriores que lidam com a classificação de texturas usando agentes artificiais.

Palavras-chave— Classificação de texturas, Agentes artificiais, *Hikers* artificiais, Dimensão fractal, Máquinas de aprendizado extremo.

1 Introduction

Digital image processing has many applications on diverse areas as automation, industry, medicine, agriculture, robotics, and many others (Gonzalez and Woods, 2006; Oliva and Cuevas, 2017). Even though humans can easily read and understand the different elements on an image, it is not that simple with digital computers. In this context, textures can help on the representation of a digital image by segmenting different regions of the image according to the texture information contained on it.

While textures do not have a precise definition on the literature, it can be regarded as a visual characteristic with repeated basic primitives that describes the appearance of the surface of some object (Neiva et al., 2017; Nair and Jacob, 2017; Zhang et al., 2017). Many techniques were proposed in order to represent textures on digital images, based on structural properties (Chen and Dougherty, 1994), statistical properties (Haralick et al., 1973; Haralick, 1979; Chetverikov, 1999; Ojala et al., 2002; Tan and Triggs, 2010), frequency spectrum (Azencott et al., 1997; Bianconi and Fernández, 2007; Florindo and Bruno, 2016b), mathematical models (Cross and Jain, 1983; Chellappa and Chatterjee, 1985; Chaudhuri and Sarkar, 1995) and artificial agents based models (Zhang and Chen, 2004; Zhang and Qiu Chen, 2005; Gonçalves et al., 2014; Machado et al., 2016).

In this paper a new technique for texture image representation and classification is proposed based on artificial agents searching for the maximum and minimum of a digital image while, at the same time, detecting different borders on the image. This technique is called artifical hikers, since it is based on the energy expenditure of hikers crossing natural ledges while trying to reach for the highest place in a region.

While the artificial hikers represents the texture image, for the classification process, features must be extracted from the artificial agents. In this case, the artificial hikers method is combined with the Bouligand-Minkowski fractal dimension in order to generate features that discrimination of textures from the hikers information. The features are then applied on an extreme learning machine in order to be classified.

In this sense, this paper aims to present a technique based on artificial agents that represent and classify textures with a smaller computational cost and greater classification score than other existent methods on the literature.

The text is divided in six sections. On section I, an introduction about digital image representation, segmentation and computational evolution techniques was presented. Section II explains the proposed artificial hikers method, section III explains the concepts of the fractal descriptors used to extract features, section IV presents the extreme learning machine algorithm used to classify the textures, section V presents the results of the proposed technique on comparison with the technique from which the proposed method was based, and section VI concludes the paper.

2 Artificial Hikers

This proposal is based on the method of artificial crawlers proposed by Zhang and Qiu Chen (2005) and refined by Gonçalves et al. (2014) and Machado et al. (2016). In the paper of Zhang and Qiu Chen (2005), an artificial life technique was proposed where artificial crawlers would try to reach the greatest intensities of a digital image, losing energy as they walked and absorbing energy from the medium proportionally to the gray level intensity of the pixel according to the following movement rules:

- If the maximum intensity (the gray intensity of the pixel) of the neighbors of an individual is lower than the intensity of the individual itself, it will settle down.
- If the maximum intensity of the eight neighbors of an individual is higher than the intensity of the individual and is unique, then it moves to the pixel of the highest intensity.
- If the maximum intensity of the eight neighbors of an individual is hiher than the intensity of the invidual and is not unique, it will move to a pixel that has already been occupied by another artificial crawler. Otherwise, it moves to either of the pixels of the highest intensity.

At each movement, an artificial hiker loses some energy, but also absorbs some energy that is proportional to the gray intensity of the pixel to which the agent has moved.

Gonçalves et al. (2014) and Machado et al. (2016) extended its concept so that the crawlers would also move, on a second analysis, to the lowest intensities of the image.

The original and improved methods were applied on texture image classification on the Brodatz image database (Brodatz, 1966), shown in Figure 1 using four different curves from the artificial crawlers evolution: the crawler evolution curve, the habitant settlement curve, the colony formation curve, and the distribution of colonies with a certain number of individuals, which, respectively, counts the number of alive crawlers on each iteration, the number of settled on each iteration, the number of colonies within a certain radius, and the number of colonies with a certain quantity of individuals (Zhang and Chen, 2004; Zhang and Qiu Chen, 2005).

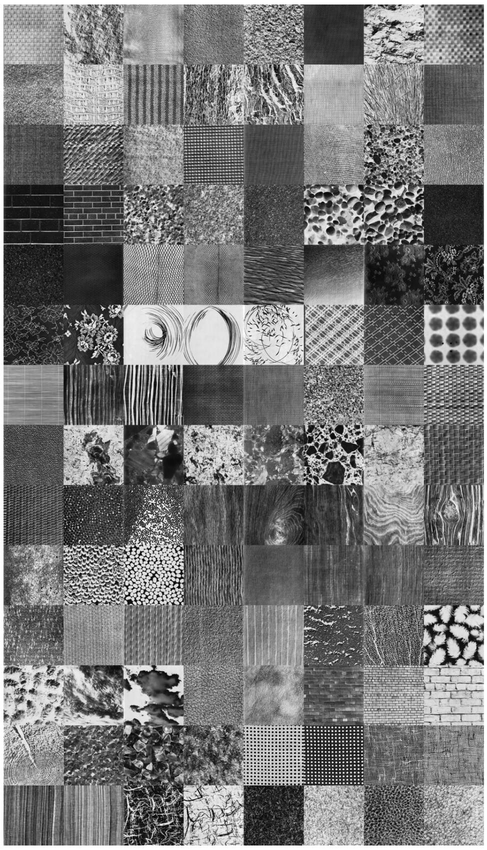

Figure 1: Images on Brodatz database.

In this proposal, a digital image can be imagined as a three-dimensional surface, with positions x and y given by the pixel line and column, respectively, and the height of the surface given by its pixel intensity $f(x, y)$.

Firstly, an initial population with N individuals is randomly placed on a digital image with an uniform probability distribution, meaning that each pixel has the same probability of being initialy occupied by an artificial hiker with an initial energy of $\epsilon_{initial}$.

Each n-th individual can be described by its position on the image x_n and y_n and by its energy e_n. The neighborhood of a given hiker is defined as the pixels with a distance smaller than $\sqrt{2}r$ pixels, where r is the radius of a neighborhood.

For example, if $r = 1$, the neighborhood of a pixel are the 8-connected pixels and if $r = 2$, the neighborhood of a pixel are the 24-connected pixels, and son on.

For each iteration k, the hiker searches in its neighborhood for the greatest pixel intensity and moves to it. When it moves, it loses part of its energy according to the difference of the pixel intensity where the hiker is and the pixel intensity of the pixel with the greatest intensity:

$$e_n[k] = e_n[k-1] - \eta \left[f(x_n[k], y_n[k]) - f(x_n[k-1], y_n[k-1]) \right], \quad (1)$$

where η is an energy dispersion parameter and $e_n[k]$ is the energy of the n-th hiker on the k-th iteration.

Movement rules for other situations are described as follows:

- If more than one pixel share the same greatest intensity, then the artificial hiker randomly chooses any one of them with an equal probability.
- If there are not any pixels with an intensity greater than the intensity of the pixel where the artificial hiker is located, then it settles down and does not move in the next iterations.
- If the energy of the hiker decreases below a minimum energy value, ϵ_{min} then it dies and does not move in the next iterations.

Notice that, opposed to the artificial crawlers technique, the artificial hikers do not absorb any energy, but lose it when they try to reach a pixel with a higher intensity.

The expected result is that in the final iteration, some pixels will have reached the local maxima of the image (the peaks of the 3D surface) and some will have died in the borders of the image. In this way, the method describes the peaks of the image and its borders.

The same technique can be applied with the negative of the image, as done with artificial crawlers in the work of Gonçalves et al. (2014) and Machado et al. (2016), so that a representation of the local minima and the borders of the image is obtained. The negative of the image is calculated as

$$f_{negative}(x, y) = 1 - f(x, y), \quad (2)$$

for each pixel (x, y) of the image.

The represented features to be described with the technique can be enhanced by adjusting two main parameters: the initial energy of the artificial hikers, $\epsilon_{initial}$, and the neighborhood radius, r.

As the initial energy increases, more hikers are able to reach the local maximum of the image, and if the neighborhood radius also increases, then the greatest local maximum are found by the hikers.

If the initial energy decreases, the number of dead hikers increases and the borders of the image are emphasized. In this case, the radius is an important parameter to guide the hikers towards the maximum (or minimum, if the negative is used) of the image.

To illustrate the process, the artificial hikers was applied on Brodatz D26 image shown on Figure 2. The dead hikers are presented in black on Figure 3, the alive hikers that reached local maxima of the image are shown in black on Figure 4, and the alive hikers that reached the local minima of the image are shown in black on Figure 5. It can be seen that the artificial hikers not only represent the maxima and minima of the the texture image, but also its borders, which represent an important discriminative characteristic of the texture.

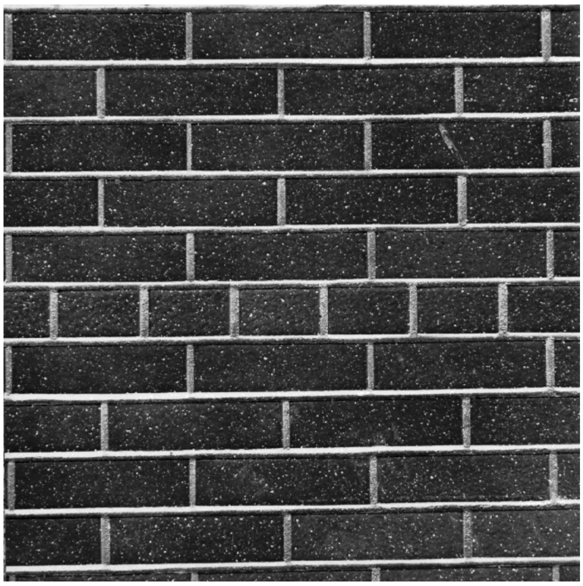

Figure 2: Brodatz D26 image.

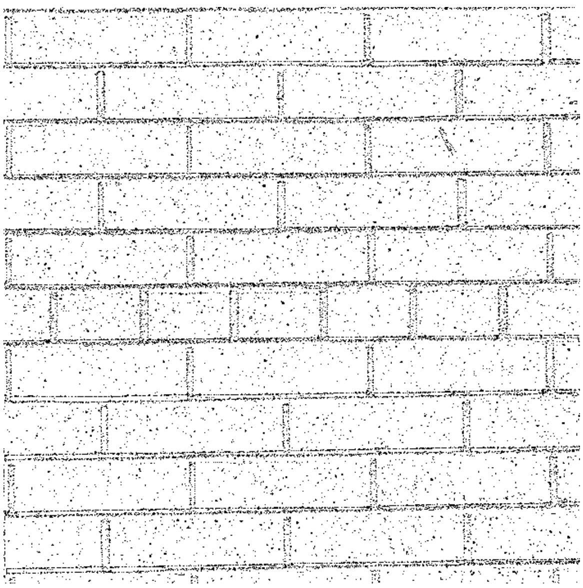

Figure 3: Dead hikers representing the borders of Brodatz D26 image.

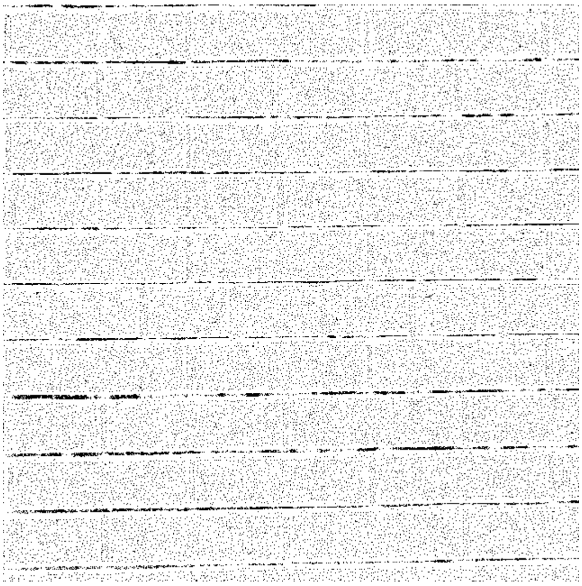

Figure 4: Alive hikers representing the local maxima of Brodatz D26 image.

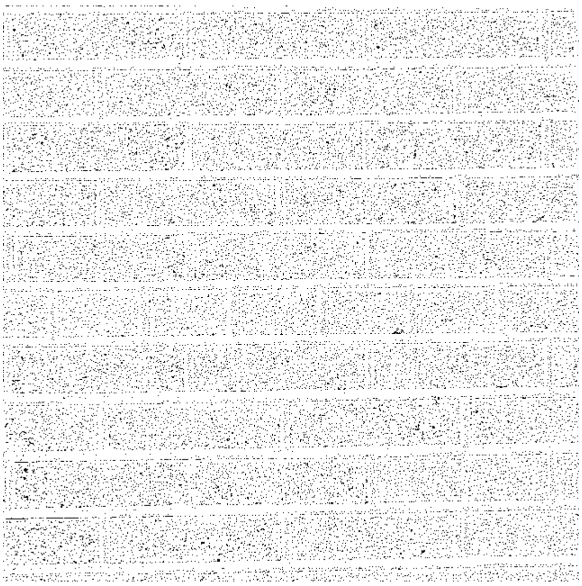

Figure 5: Alive hikers representing the local minima of Brodatz D26 image.

3 Fractal Dimension

In order to classify the textures from the artificial hikers, features from the alive and dead hikers must be extracted in order to discriminate the differente textures. In this case, the Bouligand-Minkowski fractal dimension is calculated from those hikers and a feature vector is generated for the classification stage.

A fractal can be regarded as a structure that presents infinite complexity and infinite self-similarity (Florindo and Bruno, 2016a), that is, it shows the same characteristics on any scale. They were first described by Mandelbrot (Mandelbrot, 1983) when trying to describe mathematically the coastline and observing that Euclidian geometry could not describe it properly.

Mathematically, the fractal dimension can be defined as the Hausdorff-Besicovitch dimension (Mandelbrot, 1983) which is not feasible to be applied on digital images. Due to this difficulty, many alternative techniques are presented on the literature to estimate the fractal dimension like the Bouligand-Minkowski (Tricot, 1994) and the box-counting method (Sarkar and Chaudhuri, 1994) and is normalized based the equation

$$D(X) = \lim_{\epsilon \to 0} \frac{\log N(\epsilon)}{\log \frac{1}{\epsilon}}, \tag{3}$$

where $N(\epsilon)$ is the number of objects of size ϵ needed to cover the whole object X.

The Bouligand-Minkowski fractal dimension was used by (Gonçalves et al., 2014) and (Machado et al., 2016) to increase the discrimination obtained from the artificial crawlers method. The Bouligand-Minkowski fractal dimension can be described by

$$D_{BM}(X) = \lim_{\epsilon \to 0} \left(D_T - \frac{\log V(X \oplus Y_\epsilon)}{\log \epsilon} \right), \tag{4}$$

where D_T is the topological dimension, X is the object which has its fractal dimension being calculated, Y_ϵ is a disk (if $D_T = 2$) or a sphere (if $D_T = 3$) of diameter ϵ and V is the volume of the dilation between elements X and Y_ϵ (Gonçalves et al., 2014).

Gonçalves et al. (2014) and Machado et al. (2016) proposed to calculate the Bouligand-Minkowski fractal dimension on the energy crawlers information, treating it as a three-dimensional surface where two axes are the position of the alive crawler on the last iteration and the other axes is its energy, thus obtaining a volumetric binary image.

A three-dimensional dilation process using spheres with different radius is then applied on this volumetric binary image and the influence volume for each radius is calculated and stored. Figure 6, extracted from Gonçalves et al. (2014), is useful to the understanding of this process.

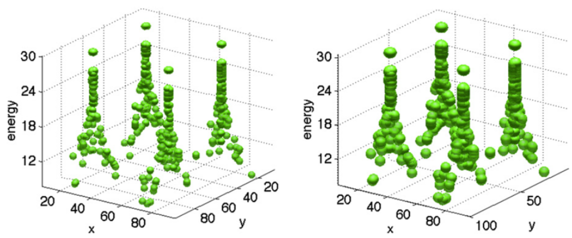

Figure 6: Dilation of the energy position of the hikers considering radius of 2 on the left image and radius of 3 on the right image.

Instead of calculating the fractal dimension by using the curve of volume and radius, Gonçalves et al. (2014) and Machado et al. (2016) used all the influence volumes for each radius as a feature vector to classify the binary images.

Since the artificial hikers technique also represents the borders of the texture image by the position of its dead hikers, this paper proposes to calculate the Bouligand-Minkowski fractal dimension on the two-dimensional binary image that represents the position of the dead hikers.

In this case, a two-dimensional dilation process using disks with different radius is applied on the dead hikers image and the influence surfaces for different radius are also used as the components of the features vector.

In this case the feature vector has the influence volumes obtained from the position and the energy alive artificial hikers through the dilation process explained and the influence surfaces obtained from the position of the dead artificial hikers through the dilation process.

4 Extreme Learning Machines

With the features vector of each digital image calculated as described on the previous section, a classifier must be employed to discriminate between the different textures. In this sense, an extreme learning machine was chosen in order to combine the nonlinearities capabilities of neural networks with a faster training algorithm other than backpropagation.

Extreme learning machine (ELM) is a training algorithm for single-layer feedforward neural networks (SLFN) that not only aims to reach the smallest training error but also the smallest norm of weights, which improves the generalization performance of the network (Huang et al., 2006).

For a set of N arbitrary distinct samples $(\mathbf{x}_i, \mathbf{t}_i)$ where $\mathbf{x}_i = [x_{i1}, x_{i2}, \ldots, x_{in}]^T \in \mathbb{R}^n$ and $\mathbf{t}_i = [t_{i1}, t_{i2}, \ldots, t_{im}]^T$, the outputs of a single-layer feedforward neural network with M hidden nodes and activation function $g(x)$ are

$$\sum_{i=1}^{M} \beta_i g(\mathbf{w}_i.\mathbf{x}_j + b_i) = \mathbf{o}_j, \quad j = 1, \ldots, N, \tag{5}$$

where $\mathbf{w}_i = [w_{i1}, w_{i2}, \ldots, w_{in}]^T$ is the weight vector connect the i-th hidden node and the input nodes, $\beta_i = [\beta_{i1}, \beta_{i2}, \ldots, \beta_{im}]^T$ is the weight vector connect the i-th idden node and the output nodes, and b_i is the threshold of the i-th hidden node.

Huang et al. (2006) proposed to randomly choose the weights and the bias of the hidden layer and represent the problem as a linear equation written as

$$\mathbf{H}\boldsymbol{\beta} = \mathbf{T}, \tag{6}$$

where

$$\mathbf{H} = \begin{bmatrix} g(\mathbf{w}_1.\mathbf{x}_1 + b_1) & \ldots & g(\mathbf{w}_M.\mathbf{x}_1 + b_M) \\ \vdots & \ldots & \vdots \\ g(\mathbf{w}_1.\mathbf{x}_N + b_1) & \ldots & g(\mathbf{w}_M.\mathbf{x}_N + b_M) \end{bmatrix}_{N \times M}$$

is the hidden layer output matrix of the neural networks, and

$$\boldsymbol{\beta} = \begin{bmatrix} \beta_1^T \\ \vdots \\ \beta_M^T \end{bmatrix}_{M \times m} \quad \text{and} \quad \mathbf{T} = \begin{bmatrix} \mathbf{t}_1^T \\ \vdots \\ \mathbf{t}_N^T \end{bmatrix}_{N \times m}.$$

The problem can then be solved using the generalized Moore-Penrose inverse of $\mathbf{H}$, $\mathbf{H}^+$ as

$$\hat{\boldsymbol{\beta}} = \mathbf{H}^+\mathbf{T} \tag{7}$$

This is the minimum norm least-squares solution of the linear system described by equation 6 and tends to give a good generalization performance.

5 Results and Discussion

A comparison of the techniques of artificial hikers, proposed on this paper, and artificial crawlers, proposed on the paper of Zhang and Qiu Chen (2005) and improved by Gonçalves et al. (2014) and Machado et al. (2016), was made on the context of texture classification.

The Brodatz texture database (Brodatz, 1966) was employed and each image was divided in 25 images with a size of 100 × 100 pixels, so that each of the 112 classes had 25 samples. A 5-fold validation method was employed, with each fold containing 20 images for training and 5 images for testing. The tests were performed on a 8GB RAM and i7-4770K processor digital computer using the software MATLAB.

For the artificial crawlers, the parameters were the same used on the previous works with artificial crawlers (Zhang and Chen, 2004; Zhang and Qiu Chen, 2005; Gonçalves et al., 2014; Machado et al., 2016): a initial energy of 10 units, a minimum energy of 1 unit, a maximum energy of 12 units, and an energy absorption rate of 0.01. Since the size of the images used on this paper is different from the ones used by Zhang and Qiu Chen (2005), Gonçalves et al. (2014), and Machado et al. (2016), the coverage rate was kept, that is, 1,000 artificial crawlers were initially placed randomly on the image following an uniform random distribution.

For the artificial hikers, a initial energy of 10 units, a minimum energy of 1 unit, and an energy dispersion parameter of 0.1 were considered with an initial number of 1,000 artificial hikers placed randomly on the image.

For both techniques, a total number of 40 iterations was considered for the individuals evolution.

The time elapsed for the application of each technique on the 2,800 images are presented on Table 1. The time elapsed by the artificial hikers technique is almost 60% of the time elapsed by the artificial crawlers technique. This is due to the more simple processing on the first technique, since the hikers do not interact with each other and they do not search for a path where another hiker has already been.

Technique	Time elapsed (seconds)
Artificial crawlers	711,30
Artificial hikers	439,83

Table 1: Time elapsed on each technique.

As for the classification process, a radial basis function was first used as the activation function, and different numbers of neurons were tested on the hidden layer. The best result was obtained with 990 hidden neurons.

The percentage of correct classification for each fold are shown on Table 2.

Fold	Artificial crawlers	Artificial hikers
1	76,07%	80,00%
2	79,11%	84,82%
3	82,32%	87,50%
4	80,54%	83,75%
5	79,11%	82,68%
Mean	**79,43%**	**83,75%**

Table 2: Correct rate of texture classification on each fold using a radial basis activation function with 990 hidden neurons.

A sigmoid activation function was then considered and the best result was obtained with 950 hidden neurons. The percentage of correct classification for each fold are shwon on Table 3.

Fold	Artificial crawlers	Artificial hikers
1	80,71%	83,93%
2	83,04%	87,68%
3	84,64%	89,29%
4	85,89%	88,21%
5	82,32%	87,14%
Mean	**83,32%**	**87,25%**

Table 3: Correct rate of texture classification on each fold using a sigmoid activation function with 950 hidden neurons.

Finally, a extreme learning machine with a kernel (Huang et al., 2012) was employed on the training of the neural network, with a regularization coefficient of 2^{24} and a kernel parameter of 2^7, and the results are presented on Table 4.

As it can be seen on Tables 2, 3, and 4, the results employing artificial hikers were superior

Fold	Artificial crawlers	Artificial hikers
1	85,00%	89,29%
2	84,82%	88,39%
3	86,25%	90,18%
4	87,14%	90,54%
5	85,89%	86,96%
Mean	**85,82%**	**89,07%**

Table 4: Correct rate of texture classification on each fold considering a radial basis kernel function.

than the ones employing artificial crawlers in every case. This is due to the fact that artificial hikers also represent the borders of the images as a complimentary description of a texture along with maximum and minimum intensities.

6 Conclusion

The experimental results show that the proposed technique is able to identify both the maximum (or the minimum, if the negative of the image is used) and the borders of the image.

Although is has not been done in the context of texture classification, by controlling two main parameters: the initial energy and the neighborhood radius; it is possible to enhance the greatest maximum of the image, or the different intensity borders, which corresponds to different frequencies.

In comparison to the artificial crawlers technique, by making the artificial individuals lose energy while going to higher intensity pixels instead of absorbing, it is possible to detect the borders of the image. On the application of texture classification the artificial hikers technique demonstrates to be faster and to achieve better results due to its representation of image borders.

The use of a robust classifier as a single hidden layer neural network trained using Extreme Learning Machine proved to provide a good classification rate for the Brodatz image database.

Future works will include consider different and more challenging texture databases with different illumination and rotation angles on its images.

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