CLOSED-LOOP STABILITY CONDITIONS OF A NON-COOPERATIVE DISTRIBUTED MPC CONFIGURATION

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Abstract— This work evaluates the stability conditions of a non-cooperative Distributed Model Predictive Controller configuration, which couple nominally stabilizing model predictive controllers. Two case studies using a four-tank system assess the characteristics of the distributed formulation. In the first case, for a set of system parameters, it is showed that if enough information is shared between local controllers, their cost function are non-increasing. In the second case, it is mapped the combination of system parameters, in which the stability conditions of the distributed MPC formulation are held.

Keywords— DMPC, IHMPC, feasibility, Lyapunov stability.

Resumo— Este trabalho avalia as condições para estabilidade de uma configuração não cooperativa para um controlador preditivo distribuído, através do acoplamento de controladores preditivos nominalmente estáveis. Dois estudos de caso, utilizando um sistema de quatro tanques acoplados, avaliam as características desta formulação. No primeiro caso, dado um certo conjunto de parâmetros da planta, é mostrado que se uma quantidade suficiente de informações é trocada entre os controladores locais, as suas funções custo não crescem. No segundo caso, é mapeada as combinações de parâmetros do sistema nas quais as condições para estabilidade da formulação distribuída de controladores MPC são válidas.

Palavras-chave— DMPC, IHMPC, viabilidade, estabilidade por Lyapunov

1 Introduction

Centralized control schemes for large scale plants face, in general, difficulties in coordination, maintenance and solution of the optimization problem in real time (Pourkargar et al., 2017; Ferramosca et al., 2013). On the other hand, fully decentralized control strategies are simpler to implement, but ignore the interconnections among systems and are not able to drive the global system to an optimum (Venkat et al., 2008). In the meantime, there has been an increase in availability of sensor information, actuators capability and network-based availability of wired and wireless data (Christofides et al., 2013), which allows sharing of information among different agents in a process plant. Therefore, the use of such communication capabilities allows the application of local optimizing controllers that share information, which can bypass some issues of centralized control approaches while improving the performance of decentralized control approaches.

DMPC (Distributed Model Predictive Controller) is developed to achieve such objectives, and is classified in two main approaches (Venkat et al., 2008; Ferramosca et al., 2013): cooperative controllers and non-cooperative (or communication based) controllers. In the former, each local controller minimizes a global cost function, while, in the later, each local controller only optimizes the cost function of its subsystem, making use of available information from other controllers (Maestre et al., 2011). Venkat et al. (2008) indicate that it is necessary to apply cooperation based DMPC in order to achieve system-wide objectives.

However, in some conditions it is possible to design non-cooperative stabilizing DMPC's formulations, as exemplified by Li and Xi (2010). These authors propose an algorithm to couple finite horizon model predictive controllers in a distributed manner, dealing with a scenario in which communication is limited. They indicated the need of limiting communication times necessary for the local controllers reach an agreement, as well as, choosing proper initial estimates for the control actions.

Then, this work aims to address the closed-loop stability requirements for a non-cooperative DMPC configuration, applying nominally stabilizing model predictive controllers as local agents. It is assessed when the controllers cost functions are non-increasing.

The rest of this article is structured as follows. Section 2 presents the proposed DMPC formulation. Section 3 details the closed-loop stability conditions for the distributed control formulation. Section 4 presents the case study in order to exemplify the features of the control system. Finally, Section 5 offers some concluding remarks.

2 DMPC Formulation

2.1 Centralized approach

González and Odloak (2009) proposed a nominally stabilizing model predictive controller, for systems with solely stable poles and zone control:

Problem P0:

$$\min_{\Delta \boldsymbol{u}_k, \boldsymbol{y}_{\mathrm{sp},k}, \boldsymbol{\delta}_{y,k}} V_k = \sum_{j=0}^{\infty} \left\| \boldsymbol{y}(k+j|k) - \boldsymbol{y}_{\mathrm{sp},k} - \boldsymbol{\delta}_{y,k} \right\|_{\boldsymbol{Q}}^2 +$$

$$+ \sum_{j=0}^{m-1} \left\| \Delta \boldsymbol{u}(k+j|k) \right\|_{\boldsymbol{R}}^2 + \left\| \boldsymbol{\delta}_{y,k} \right\|_{\boldsymbol{S}_y}^2,$$

suject to:

$$\boldsymbol{x}^{\mathrm{s}}(k+m|k) - \boldsymbol{y}_{\mathrm{sp},k} - \boldsymbol{\delta}_{y,k} = \boldsymbol{0} \tag{1a}$$

$$-\Delta \boldsymbol{u}_{\max} \leq \Delta \boldsymbol{u}(k+j|k) \leq \Delta \boldsymbol{u}_{\max}, j = 0, \ldots, m-1, \tag{1b}$$

$$\boldsymbol{u}_{\min} \leq \boldsymbol{u}(k+j|k) \leq \boldsymbol{u}_{\max}, \quad j = 0, \ldots, m-1, \tag{1c}$$

$$\boldsymbol{y}_{\min} \leq \boldsymbol{y}_{\mathrm{sp},k} \leq \boldsymbol{y}_{\max}, \tag{1d}$$

where $\boldsymbol{x}^{\mathrm{s}}(k+m|k)$ is a vector of artificial integrating states at the time step $k+m$ given information available at time step k (included by the model formulation), $\Delta \boldsymbol{u}(k+j|k)$ is the movement increment in the input variables, $\boldsymbol{y}(k+j|k)$ is the output vector at time step $k+j$, $\boldsymbol{y}_{\mathrm{sp},k}$ is the reference vector, m is the control horizon, $\boldsymbol{\delta}_{y,k}$ is a vector of slack variables that aims to enlarge the attraction domain of solutions, and $\boldsymbol{Q}$, $\boldsymbol{R}$ and $\boldsymbol{S}_y$ are weighting matrices.

The model of the system applied in such a formulation is a canonical state space based on the analytical form of the step response of the system (González and Odloak, 2009):

$$\begin{bmatrix} \boldsymbol{x}^{\mathrm{s}}(k+1) \\ \boldsymbol{x}^{\mathrm{st}}(k+1) \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_{ny} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{F}^{\mathrm{st}} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{x}^{\mathrm{s}}(k) \\ \boldsymbol{x}^{\mathrm{st}}(k) \end{bmatrix} + \begin{bmatrix} \boldsymbol{B}^{\mathrm{s}} \\ \boldsymbol{B}^{\mathrm{st}} \end{bmatrix} \cdot \Delta \boldsymbol{u}(k), \tag{2}$$

$$\boldsymbol{y}(k) = \begin{bmatrix} \boldsymbol{I}_{ny} & \boldsymbol{\Psi}^{\mathrm{st}} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{x}^{\mathrm{s}}(k) \\ \boldsymbol{x}^{\mathrm{st}}(k) \end{bmatrix}, \tag{3}$$

where $\boldsymbol{x}^{\mathrm{st}}$ is the stable states vector of the system. The other matrices included in the model are detailed in González and Odloak (2009).

Such a controller can be applied in underacted systems due to the degrees of freedom included by the zone control approach.

In order to simplify the application of such a controller, the equality constraint, Equation (1a), can be directly included into the objective function, which gives:

Problem P1:

$$\min_{\Delta \boldsymbol{u}_k, \boldsymbol{y}_{\mathrm{sp},k}} V_k = \sum_{j=0}^{\infty} \left\| \boldsymbol{y}(k+j|k) - \boldsymbol{x}^{\mathrm{s}}(k+m|k) \right\|_{\boldsymbol{Q}}^2 +$$

$$+ \sum_{j=0}^{m-1} \left\| \Delta \boldsymbol{u}(k+j|k) \right\|_{\boldsymbol{R}}^2 + \left\| \boldsymbol{x}^{\mathrm{s}}(k+m|k) - \boldsymbol{y}_{\mathrm{sp},k} \right\|_{\boldsymbol{S}_y}^2,$$

suject to: equations (1b), (1c), (1d).

Remark 1 *It is straightforward to show that Problem P1 preserves the properties of Problem P0, namely: (i) nominal stability, (ii) feasibility (there is always a solution for the optimization problem that fulfill the constraints for all time steps).*

2.2 Distributed approach

In this approach, it is necessary to decompose the system in different subsystems, which are controlled by local agents with some level of communication (Pourkargar et al., 2017). Here it is assumed that the controllers only exchange their input increments.

The model applied in the centralized controller, Equations (2) and (3), is already on a canonical form. This allows the direct decomposition of the subsystems, which can be represented by selecting the states related to each subsystem - $\boldsymbol{x}_i^{\mathrm{s}}$, $\boldsymbol{x}_i^{\mathrm{st}}$ - and including manipulated variables of adjacent subsystems - $\Delta \boldsymbol{u}_j$ - as disturbances:

$$\begin{bmatrix} \boldsymbol{x}_i^{\mathrm{s}}(k+1) \\ \boldsymbol{x}_i^{\mathrm{st}}(k+1) \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_{ny_i} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{F}_i^{\mathrm{st}} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{x}_i^{\mathrm{s}}(k) \\ \boldsymbol{x}_i^{\mathrm{st}}(k) \end{bmatrix} + \begin{bmatrix} \boldsymbol{B}_i^{\mathrm{s}} \\ \boldsymbol{B}_i^{\mathrm{st}} \end{bmatrix} \cdot \Delta \boldsymbol{u}_i(k)$$

$$+ \sum_{j \neq i} \begin{bmatrix} \boldsymbol{B}_j^{\mathrm{s}} \\ \boldsymbol{B}_j^{\mathrm{st}} \end{bmatrix} \cdot \Delta \boldsymbol{u}_j(k), \tag{5}$$

$$\boldsymbol{y}_i(k) = \begin{bmatrix} \boldsymbol{I}_{ny_i} & \boldsymbol{\Psi}_i^{\mathrm{st}} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{x}_i^{\mathrm{s}}(k) \\ \boldsymbol{x}_i^{\mathrm{st}}(k) \end{bmatrix}, \tag{6}$$

where $\Delta \boldsymbol{u}_j$ can be considered as a known disturbance vector for the system i.

Then, each local controller on this distributed approach is an IHMPC (infinite horizon model predictive controller), which follows Problem P1, applying the model represented by Equations (5) and (6). Consequently, they must take account into a known disturbance vector.

Considering a two agents system, the following non-cooperative DMPC algorithm can be formulated, based on an algorithm proposed by Li and Xi (2010):

1. Define the error tolerance and maximum iteration for the algorithm,

2. Obtain the previously evaluated input increment vectors of each agent and apply the receding horizon for them:

$$\Delta \widetilde{\boldsymbol{u}}_{1_k} = \Big[\Delta \boldsymbol{u}_1(k|k-1)^{*\top} \quad \cdots$$

$$\Delta \boldsymbol{u}_1(k+m-2|k-1)^{*\top} \quad \boldsymbol{0} \Big]^\top, \tag{7}$$

$$\Delta \widetilde{\boldsymbol{u}}_{2_k} = \Big[\Delta \boldsymbol{u}_2(k|k-1)^{*\top} \quad \cdots$$

$$\Delta \boldsymbol{u}_2(k+m-2|k-1)^{*\top} \quad \boldsymbol{0} \Big]^\top, \tag{8}$$

3. Evaluate Problem P1 for each subsystem, using $\Delta \widetilde{\boldsymbol{u}}_{1_k}$ and $\Delta \widetilde{\boldsymbol{u}}_{2_k}$ as disturbance vectors for subsystems 2 and 1, respectively,

4. Exchange the optimum input increments evaluated in step 3, $\Delta \boldsymbol{u}_{1_k}^*$ and $\Delta \boldsymbol{u}_{2_k}^*$. Consider them as disturbance vectors for subsystems 2 and 1, respectively, and solve Problem P1 again for each subsystem,

5. Evaluate the stop criteria: the euclidean norm between the difference of optimum solutions found on steps 3 and 4,

6. Repeat steps 4 and 5 until the tolerance is met or the maximum number of iteration is reached.

7. Implement the instant control actions - $\Delta\boldsymbol{u}_1^*(k|k)$, $\Delta\boldsymbol{u}_2^*(k|k)$ - move for the next time step and return to step 2.

According to Li and Xi (2010), the stop criteria imposed, based on the euclidean norm of the difference, approximately guarantees the Nash optimality, if the tolerance is sufficiently small and the maximum number of iterations is not reached. These authors indicate that such a limitation for the iterations should be included to account for the acceptable communication time during one sampling time.

3 Stability Conditions

In other to discuss the stability of the DMPC strategy presented earlier, it is necessary to address the stability conditions of Problem P1 taking account into known disturbances. This issue is detailed in Theorem 1.

Theorem 1. *For the pair ($\boldsymbol{A}$,$\boldsymbol{B}$) stabilizable, consider a subsystem i with distinct stable poles, in which a set of planned increments in the manipulated inputs of adjacent subsystems, $\Delta\boldsymbol{u}_j$, are available after convergence of the proposed algorithm. Since the solution of Problem P1, $\Delta\boldsymbol{u}_i^*$, is always feasible at any time step, then, based on the available information about $\Delta\boldsymbol{u}_j$, the successive solutions of the algorithm applying Problem P1 drive the closed-loop system asymptotically to a steady-state, provided that the increments in the manipulated inputs of adjacent systems realized, $\boldsymbol{\alpha}\cdot\Delta\boldsymbol{u}_j$, are closer to the ones planned.*

Proof: This proof extends the one presented by González and Odloak (2009) for the case where disturbances are included in the model.

Consider that $\left[\Delta\boldsymbol{u}_{1_k}^*, \boldsymbol{y}_{1_{\text{sp}}}^*\right]$ is a feasible solution for the Problem P1 at time step k after convergence of the algorithm, where $\Delta\boldsymbol{u}_{1_k}^*$ is $\left[\Delta\boldsymbol{u}_1(k|k)^* \quad \cdots \quad \Delta\boldsymbol{u}_1(k+m-1|k)^*\right]^\top$, evaluated assuming a certain trend for the increments in the input variables $\Delta\boldsymbol{u}_{2_k}$, $\left[\Delta\boldsymbol{u}_2(k|k) \quad \cdots \quad \Delta\boldsymbol{u}_2(k+m-1|k)\right]^\top$. Then, moving for time step $k+1$, it is straightforward to show that the inherited solution $\left[\Delta\widetilde{\boldsymbol{u}}_{1_{k+1}}, \boldsymbol{y}_{1_{\text{sp}}}^*\right]$, where $\Delta\widetilde{\boldsymbol{u}}_1(k+1)$ is $\left[\Delta\boldsymbol{u}_1(k+1|k)^* \quad \cdots \quad \Delta\boldsymbol{u}_1(k+m-1|k)^* \quad \boldsymbol{0}\right]^\top$, remains feasible for the controller 1 since it attends the constraints, Equations (1b), (1c) and (1d).

Assume that at time step $k+1$ the increments $\Delta\boldsymbol{u}_{2_{k+1}}$ is not equal to the ones planned after convergence of the algorithm, being represented by $\boldsymbol{\alpha}\cdot\Delta\boldsymbol{u}_{2_k}$. Consequently, the comparison of the objective function evaluated at time k and $k+1$ gives:

$$V_{1_k}^* - \widetilde{V}_{1_{k+1}} = \\ \|\boldsymbol{y}_1(k|k) - \boldsymbol{x}_1^{\text{s}}(k+m|k)\|_{\boldsymbol{Q}}^2 + \|\Delta\boldsymbol{u}_1(k|k)^*\|_{\boldsymbol{R}}^2 + \Omega(\boldsymbol{\alpha}), \tag{9}$$

where $\Omega(\boldsymbol{\alpha})$ is an infinite sum of terms, which is function of the input increments of adjacent systems that actually were realized, $\boldsymbol{\alpha}\cdot\Delta\boldsymbol{u}_{2_k}$, the manipulated increments, $\Delta\boldsymbol{u}_{1_k}$, the calculated setpoint, $\boldsymbol{y}_{\text{sp},k}$, the weighting matrices of the controller and the model of the system.

Then, the application of the inherited solution, systematically decreases the objective function if and only if $\Omega(\boldsymbol{\alpha})$ is positive. In this case, as the controller is not obligated to use the inherited solution at time step $k+1$, one can assume that $V_{k+1}^* \leq V_k^*$.

□

Remark 2 *If $\boldsymbol{\alpha}$ is the identity matrix, i.e the disturbance realized is equal to the one planned, $\Omega(\boldsymbol{I})$ is zero.*

Remark 3 *It is necessary to define the neighborhood of $\boldsymbol{\alpha}$ around the identity matrix, in which Theorem 1 holds.*

Remark 4 *In the non-cooperative distributed configuration, for the controller related to subsystem 1, the planned vector of input increments, $\Delta\boldsymbol{u}_{2_k}$, is evaluated by other controller, thus its values could change along the simulation. This also holds for the controller related to subsystem 2. Therefore, assuming that such changes are small ($\boldsymbol{\alpha}$ is in some neighborhood around the identity matrix) and the control horizon is sufficiently large to guarantee an acceptable sharing of information, then the objective function of the controllers does not increase.*

4 Case Studies

This section aims to present the performance of this non-cooperative DMPC strategy and discuss the applicability of Remarks 3 and 4. The case studies are based on a four-tank system, borrowed from Alvarado et al. (2011). In this system, four tanks are fed by two pumps. Pump A feeds tanks 1 and 4, while pump B feeds tanks 2 and 3. Additionally, the discharge flow of tank 4 feeds tank 2 and the discharge flow of tank 3 feeds tank 1. The model of this system is represented by:

$$\frac{\mathrm{d}h_1}{\mathrm{d}t} = -\frac{a_1}{S} \cdot \sqrt{2 \cdot g \cdot h_1} + \frac{a_3}{S} \cdot \sqrt{2 \cdot g \cdot h_3} + \frac{\gamma_\mathrm{a}}{S} \cdot q_\mathrm{a}, \tag{10}$$

$$\frac{\mathrm{d}h_2}{\mathrm{d}t} = -\frac{a_2}{S} \cdot \sqrt{2 \cdot g \cdot h_2} + \frac{a_4}{S} \cdot \sqrt{2 \cdot g \cdot h_4} + \frac{\gamma_\mathrm{b}}{S} \cdot q_\mathrm{b}, \tag{11}$$

$$\frac{\mathrm{d}h_3}{\mathrm{d}t} = -\frac{a_3}{S} \cdot \sqrt{2 \cdot g \cdot h_3} + \frac{1-\gamma_\mathrm{b}}{S} \cdot q_\mathrm{b}, \tag{12}$$

$$\frac{\mathrm{d}h_4}{\mathrm{d}t} = -\frac{a_4}{S} \cdot \sqrt{2 \cdot g \cdot h_4} + \frac{1-\gamma_\mathrm{a}}{S} \cdot q_\mathrm{a}. \tag{13}$$

where, h_i is the level of tank i, q_a is the flow provided by pump A and q_b is the flow provided by pump B. The other parameters and their nominal values are presented in Table 1.

Table 1: Parameters of the model. Source: adapted from Alvarado et al. (2011).

	Value	Description
a_1	$1.31 \cdot 10^{-4}$ m^2	Discharge constant 1
a_2	$1.51 \cdot 10^{-4}$ m^2	Discharge constant 2
a_3	$9.27 \cdot 10^{-5}$ m^2	Discharge constant 3
a_4	$8.82 \cdot 10^{-5}$ m^2	Discharge constant 4
S	0.06 m^2	Cross section
γ_a	0 to 1	Parameter 3-way valve
γ_b	0 to 1	Parameter 3-way valve

This system is decomposed into two subsystems, each one with a local controller, namely:

- Controller 1: controls the levels of tanks 1 and 4, using as manipulated variable q_a.
- Controller 2: controls the levels of tanks 2 and 3, using as manipulated variable q_b.

4.1 Case 1: characteristics of the controller

In order to assess characteristics of the distributed algorithm, using IHMPC as local agents, the case scenario addresses the system when γ_a and γ_b are 0.6. The system is linearized in the following steady state: 0.72 m (h_1), 0.59 m (h_2), 0.29 m (h_3), 0.21 m (h_4), 1.63 m^3/h (q_a), 2.00 m^3/h (q_b). The sampling time applied is 5 s, and the model obtained for each subsystem, following equations (5) and (6), are composed by two artificial integrating states and three stable states. The model of the plant represents the global system linearized in the same steady state.

The simulation scenario addresses changes in the zones of h_1 and h_2, while the zones of h_3 and h_4 are kept constant to give degrees of freedom for the controllers. This scenario is detailed as follows: both subsystems start in the steady state, at 2.4 min h_1 is raised, at 16.6 min h_2 is raised, at 41.6 min h_1 and h_2 are raised, and finally at 62.4 min h_1 and h_2 are decreased. Table 2 presents the tuning parameters and constraints of each controller.

Table 2: Tuning parameters and constraints of controllers.

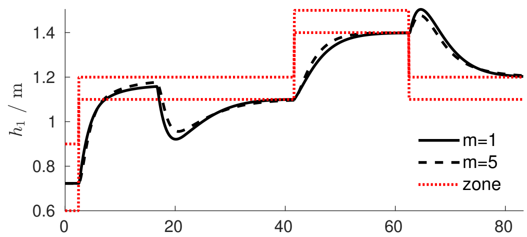
Parameters	Controller 1[1]	Controller 2[1]
m	1 and 5	
$\boldsymbol{Q}$	diag($\boldsymbol{\iota}(2)$)	
$\boldsymbol{R}$	10.5	
$\boldsymbol{S}_y$	$1 \cdot 10^3 \cdot \mathrm{diag}(\boldsymbol{\iota}(3))$	
$u_{\min}$	0 m^3/h	
$u_{\max}$	3.25 m^3/h	4 m^3/h
$\Delta u_{\max}$	0.5 m^3/h	

[1] $\boldsymbol{\iota}^\top(j)$ is a vector composed of j unitary scalar elements.

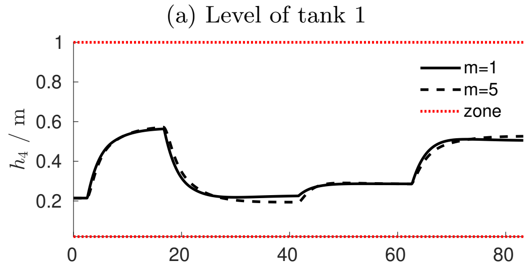
In order to exemplify the features of the distributed control algorithm, specially Remark 4, two control horizons were evaluated: (i) the minimum allowed control horizon, 1, and (ii) a larger control horizon, 5, to comply with Remark 4.

Figures 1 and 2 present the trends for controlled and manipulated variables of the subsystems 1 and 2, respectively. Regarding h_1 and h_2, each local controller guided, in general, such variables towards the inferior or superior bounds of the zone constraints. This can be explained by the interaction between each subsystem, what increases the difficulty of each controller to keep the controlled variables inside the zones.

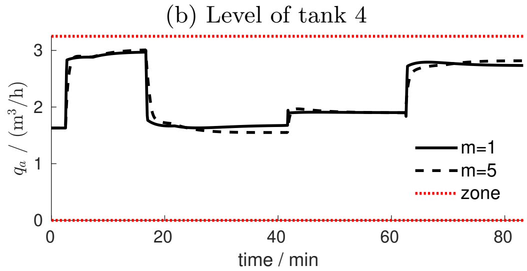
The interaction between these subsystems is



(a) Level of tank 1

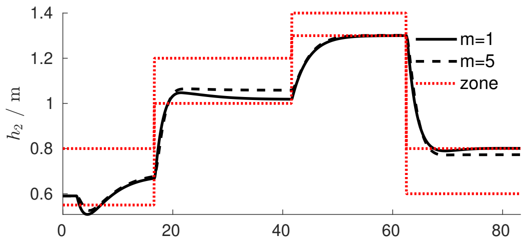


(b) Level of tank 4

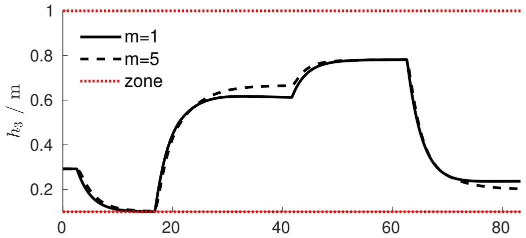


(c) Flow of pump A

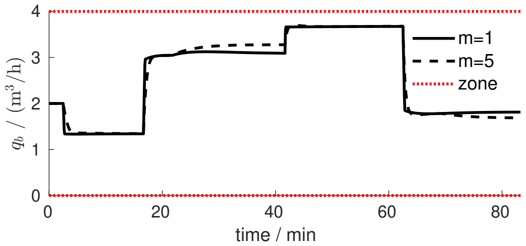
Figure 1: Control structure of subsystem 1



(a) Level of tank 2



(b) Level of tank 3



(c) Flow of pump B

Figure 2: Control structure of subsystem 2

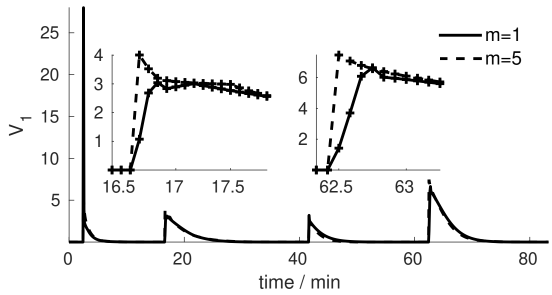
made clear until 41 min of simulation, when the zone changes happen for each subsystem at a time. It is possible to notice the influence of one subsystem in the other and the capability of each local controller to deal with the planned disturbance, $\boldsymbol{\Delta} u_{j_k}$, forcing the controlled variables to their zone, if enough time is given.

From 41 min to the end of the simulation, the zone changes of each controller happen simultaneously, and given enough time, each controller brings its subsystem to the desired zone.

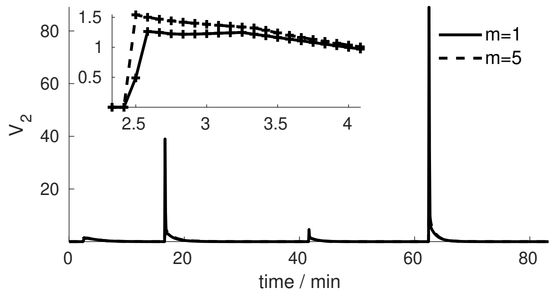
Additionally, despite minor differences in the performance of the controllers with the tuning applied, regarding the control horizons imposed, this case study shows the feasibility of the controllers, since they are able to find a solution even for small control horizons.

In order to evaluate the stability of each local controller, Figure 3 presents the cost function of them, and it includes details for some time periods when the zones have changed.

It is clear that by using the control horizon of 1, the cost function can increase at a series of time steps right after a change of zones. Taking, for example, the controller 1 at times 16.6 min and 62.4 min, it needs from 4 to 7 time steps in order to the cost function continually decrease. Other example is controller 2 at time 2.4 min, when it needs 10 time steps to its cost function continually decrease. However, the use of the control horizon of



(a) Cost function of the controller - subsystem 1.



(b) Cost function of the controller - subsystem 2.

Figure 3: Cost functions of controllers.

5 turns both cost functions non-increasing, what indicates that it gave sufficient information for the controllers, complying with Remark 4.

4.2 Case 2: mapping the closed-loop stability limits

This case study aims to map the region delimited by the system parameters, γ_a and γ_b, in which both local controllers comply with Remarks 3 and 4. This implies that, in this region, the controller cost functions are non-increasing, given an acceptable information sharing.

Then, the same simulation scenario of case study 4.1 is evaluated for different combinations of γ_a and γ_b. The following considerations are taken:

- the zone changes are the same from case study 4.1. However, the alteration of γ_a and γ_b modifies the steady state, then the initial zones of each system are adjusted in such a way to contain these initial states.

- the tuning parameters of each controller and constraints detailed in Table 2 are applied. However, in order to enlarge the domain in which the controllers comply with Remark 4, it is applied a control horizon of 10.

Figure 4 summarizes the results of simulations, which have three possible outcomes: (i) non-increasing control cost of both controllers, complying with remark 4, (ii) increasing control cost of at least one controller, and (iii) saturation of at least one input, not fulfilling the hypothesis of Theorem 1.

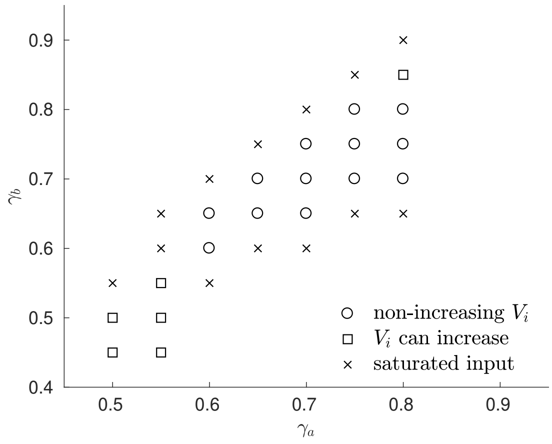


Figure 4: Mapping of the three-way valve parameters.

It can be noticed that, as interaction between the subsystems increase, i.e. γ_a and γ_b decrease, the range in which the controllers present a Lyapunov function reduces. If the difference between these parameters are, in general, greater than 0.1, the controllers tend to saturate their manipulated variables in order to attend the zones imposed.

For the point where γ_a and γ_b are both 0.55, only the objective function of controller 1 is not always decreasing after a change of zones. Then, the control horizon of controller 1 was enlarged to 15, in order to give it more time steps to deal with the disturbance. With this new tuning, both controllers cost functions are non-increasing.

5 Conclusion

This work assesses the stability conditions of a simple algorithm to couple two infinite horizon model predictive controllers, forming a non-cooperative DMPC strategy. The conditions for which the objective functions of the local controllers are non-increasing are summarized as: (i) the trend of successive solutions of each controller can vary inside a limited range, (ii) the control horizon must guarantee a sufficient share of information, and (iii) the system interactions are small. Additionally, the local controller formulations are always feasible, due to the use of slack variables in the terminal constraints.

The case studies exemplified such features using a four-tank system. The first case study addressed the difference of controllers behavior when using two different control horizons, in order to highlight the necessity to have enough information sharing to achieve a non-increasing cost function. However, even with the control horizon of 1, the controllers were able to solve their optimization problem - showing the feasibility of this formulation. The second case study mapped the combination of system parameters, in which this formulation was able to achieve a non-increasing cost function in both local controllers.

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