FAULT DETECTION H_2 FILTER FOR MARKOV JUMP LINEAR SYSTEMS

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Abstract— In the paper, we tackle the H_2 Fault Detection Filter (FDF) problem for a Markov Jump Linear System (MJLS) in the discrete-time domain. The main novelty is the synthesis of H_2 FDF for a MJLS in the form of Linear Matrices Inequalities (LMI). We also provide results for the so-called mode-independent case and the design of robust H_2 filter in the sense that the system matrices are uncertain. For the sake of illustrating the suitability of the proposed approaches, a numerical example is presented.

Keywords— Fault Detection, Markovian Jump Linear System, H_2 norm.

1 Introduction

The systems are becoming more complex every day, and this increase in the complexity brings new challenges in engineering. An important field of application is the fault-resilient systems. There is a multitude of strategies that aim to increase the resilience of a system like as reported in (Venkatasubramanian et al., 2003; Kim and Bartlett, 1996; Favre, 1994; Isermann et al., 2002). A possible way to increment the resilience of a system is to implement the Fault Detection and Isolation (FDI) approach. This concept has the main goal of sensing and rearranging the system in order to minimize any kind of loss, see for instance (Hwang et al., 2010).

The FDI framework can be separated into two main stages: i) a residual signal is generated by a filter; ii) the residual signal evaluation. This evaluation process is accomplished by comparing the residue with a pre-determined threshold, so that whenever the residual signal surpasses the threshold it is considered that a fault occurred. The threshold is usually obtained via the observation of the plant in the nominal situation. Fig.1 shows a block diagram representing the FDI framework.

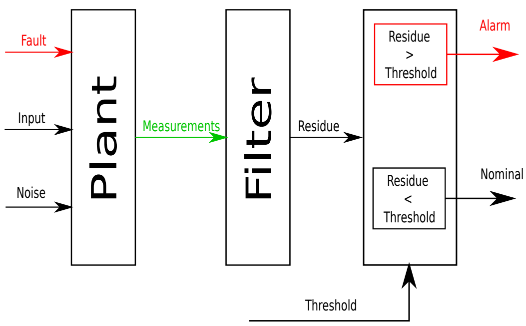

Figure 1: Fault detection framework.

In (Chen and Patton, 2000) two important properties that the filter responsible for generating the residue must have are: i) the high amount of sensibility regarding to the fault and ii) high resilience concerning to the inputs and noise, in order to guarantee a fast detection and low occurrence of false alarms.

The elements that compose the plant and the Fault Detection Filter (FDF) are interconnected via a network (represented by a green arrow in Fig.1), and in some cases, this network is implemented as a wireless network, and it is well known that this kind of networks is more susceptible to communication loss. Aiming to consider this important aspect in the design of a fault detection filter, the usage of Markovian Jump Linear System (MJLS) seems as a suitable choice, as it is possible to model the network behavior in the MJLS framework see, for instance, (Palma and Duran-Faundez, 2016).

Some examples in the literature that tackle this same subject are (Zhong et al., 2003), (Zhong et al., 2005) and (Wang and Yin, 2017), to name a few. In (Zhong et al., 2005), the FDI problem is solved under the MJLS framework and the design of an H_∞ filter working as a residual generator is provided. More recently, in (Wang and Yin, 2017) the design of an H_∞ residual filter in the continuous-time domain was presented. Concerning the works previously cited, as far as the authors are aware of, there is still some research to be made in this field.

In this paper, the main novelty is the synthesis of an H_2 residual mode-dependent filter for the discrete-time domain under the MJLS framework in order to cope with a fault detection problem. Moreover, we also investigate the mode-independent residual filter and the case of uncertain transition probability matrices. In order to illustrate our approach, a numerical example is provided at the end of the paper.

This paper is organized as follows: Section 2 presents the notation used in this work, Section 3 presents a theoretical background, Section 4 describes the FD problem, Section 5 presents the main results which consists of an FDF considering the H_2 norm. Section 5.2 presents the result for the mode-independent case, Section 5.3 presents the case of designing robust H_2 filters, Section 6 presents a numerical example, and Section 7 concludes the paper.

2 Notation

This paper uses a standard notation, with the operator ($'$) denoting the matrix or vector transpose, and ($\bullet$) indicates each symmetric block of a symmetric matrix. The symbol $\mathbb{N}$ denotes a set of natural numbers. We consider the convex set

$$\Upsilon = \left\{ Q; Q = \sum_{l=1}^{V} \iota_l Q^l =, \quad \iota_l \geq 0, \sum_{l=1}^{V} \iota_l = 1 \right\} \quad (1)$$

where V is the number of vertices in the politope. Whenever $Q \in \Upsilon$, we associate the index l with the convex set (1). The set of Markov chain states is represented by $\mathbb{K} = \{1, 2, \ldots, N\}$. The convex combinations of the matrix X_j and the weight ρ_{ij} is given by $\varepsilon_i(X) = \sum_{j=1}^{N} \rho_{ij} X_j$ for $i \in \mathbb{K}$. The symbol $\varepsilon()$ represents mathematical expectations. Considering the stochastic signal $z(k)$, its norm is defined by $\|z\|_2^2 = \sum_{k=0}^{\infty} \varepsilon\{z(k)'z(k)\}$. The set of signals $z(k) \in \mathbb{R}^n$ defined for all $k \in \mathbb{N}$, such that $\|z\|_2 < \infty$ is indicated by $\mathcal{L}^2$.

3 Theoretical Background

In this section, we define the concept of mean square stability (MSS) and H_2 norm, and with that intention, we consider the following general discrete-time MJLS as below

$$\mathcal{G} : \begin{cases} x(k+1) = A_{\theta_k} x(k) + J_{\theta_k} w(k) \\ \quad z(k) = C_{z\theta_k} x(k) + E_{z\theta_k} w(k) \end{cases} \quad (2)$$

where $x(k) \in \mathbb{R}^n$ represents the state, $z(k) \in \mathbb{R}^p$ represents the output, $w(k) \in \mathbb{R}^m$ is an exogenous input and θ_k is a Markov chain taking values in $\mathbb{K}$, with transition probability matrix $\mathbb{P} = [\rho_{ij}]$, where ρ_{ij} satisfies $\rho_{ij} = Pr[\theta_{k+1} = j | \theta_k = i]$ and $\sum_{j=1}^{N} \rho_{ij} = 1$.

3.1 Mean Square Stability

In (Costa and Fragoso, 1993) the MSS for system (2) is defined as:

Definition: System (2) is MSS if for any initial condition $x(0) = x_0 \in \mathbb{R}^n$, initial distribution $\theta(0) = \theta_0 \in \mathbb{K}$ it holds that

$$\lim_{k \to \infty} \varepsilon\{x(k)'x(k) | x_0, \theta_0\} = 0. \quad (3)$$

3.2 Markovian H_2 norm

Considering that the MJLS $\mathcal{G}$ is MSS, the H_2 norm of the system (2) is given by

$$\|G\|_2^2 = \sum_{s=1}^{m} \sum_{i=1}^{N} \mu_i \|z^{s,i}\|_2^2 \quad (4)$$

where μ_i is the initial probability of the Markov chain state θ_0 and $z^{s,i}$ represents the output $z(0)$,$z(1)$, $\ldots$ obtained when

- $x(0) = x_0$ and the input is given by $w(k) = e_s \delta(k)$, where $e_s \in \mathbb{R}^m$ is the n-th column of the identity matrix $m \times m$ and δ_i is the unitary impulse, (Costa et al., 1997).

- $\theta_0 = i \in \mathbb{K}$ with probability $\mu_i = Pr(\theta_0 = i \in \mathbb{K})$

In (Costa et al., 2006) it was shown that if the Markov Chain is ergodic, and $\mu_i = \rho_i$, where ρ_i is the stationary probability distribuition of the Markov chain, then the norm defined in (4) can also be defined as

$$\|G\|_2^2 = \lim_{k \to \infty} \varepsilon[z'(k)z(k)], \quad (5)$$

where $z(k)$ is the system output and $w(k)$ represents a white noise sequence in the wide sense.

4 Problem Formulation

The MJLS we consider in this work is represented by

$$\mathcal{G}_a : \begin{cases} x(k+1) = A_{\theta_k} x(k) + B_{\theta_k} u(k) + B_{d\theta_k} d(k) + B_{f\theta_k} f(k) \\ \quad y(k) = C_{\theta_k} x(k) + D_{d\theta_k} d(k) + D_{f\theta_k} f(k) \\ \quad x(0) = x_0, \end{cases} \quad (6)$$

where $x(k) \in \mathbb{R}^n$ is the state, $y(k) \in \mathbb{R}^q$ is the measured output, $u(k) \in \mathbb{R}^m$ is the known input, $d(k) \in \mathbb{R}^p$ is the exogenous input and $f(k) \in \mathbb{R}^t$ is the fault vector which is considered as an unknown time function. We also consider that $f(k), d(k) \in \mathcal{L}^2$.

The Fault Detection system may be separated into two stages: residual generation and the residual evaluation.

4.1 Residual Generator

The filter responsible for generating the residual signal $r(k)$ is a Markovian observer defined as:

$$\mathcal{F} : \begin{cases} \eta(k+1) = A_{\eta\theta_k} \eta(k) + M_{\eta\theta_k} u(k) + B_{\eta\theta_k} y(k) \\ \quad r(k) = C_{\eta\theta_k} \eta(k) + D_{\eta\theta_k} y(k) \\ \quad \eta(0) = \eta_0 \end{cases} \quad (7)$$

where $\eta(k) \in \mathbb{R}^n$ are the filter states and $r(k) \in \mathbb{R}^l$ denotes the filter residue. We mention that this filter also depends on the Markov mode θ_k, the same index as presented in system (6).

This paper has the main goal of synthesize a residual generator in the form of (7), composed by the matrices $A_{\eta i}$, $B_{\eta i}$, $C_{\eta i}$, $D_{\eta i}$, $M_{\eta i}$, that is mean square stable when $u = 0$, $d = 0$ and $f = 0$ and also minimize the value of γ in

$$\sum_{i=1}^{N} \mu_i trace(W_i) < \gamma \tag{8}$$

In a similar way as in the continuous time case illustrated in (Chen and Patton, 2000) and the discrete time case in (Zhong et al., 2005), a weighting matrix $\mathcal{W}(f)$ is implemented with the purpose of restricting the frequency interval, in which the fault should be identified leading to a performance improvement, see for instance, (Chen and Patton, 2000) and (Niemann and Stoustrup, 2001). A minimal realization of $\hat{f}(z) = \mathcal{W}(z)f(z)$ is

$$\mathcal{W}_f : \begin{cases} x_f(k+1) = A_{wf}x_f(k) + B_{wf}f(k) \\ \hat{f}(k) = C_{wf}x_f(k) + D_{wf}f(k) \\ x_f(0) = 0 \end{cases} \tag{9}$$

where $x_f(k) \in \mathbb{R}^t$ is the weighting filter state vector, and $f(k)$ is the same fault as in (6). The equivalent system represented as a diagram block is shown in Fig.2.

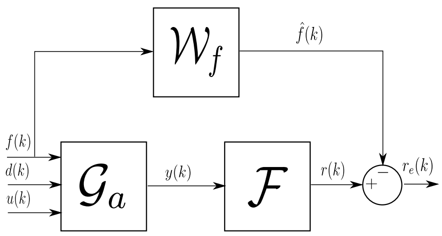

Figure 2: Block diagram.

The augmented system (10) is obtained using the augmented vector $r_e(k) = r(k) - \hat{f}(k)$ yielding to

$$\mathcal{G}_{aug} : \begin{cases} \bar{x}(k+1) = \tilde{A}_{\theta_k}\bar{x}(k) + \tilde{B}_{\theta_k}\bar{w}(k) \\ r_e(k) = \tilde{C}_{\theta_k}\bar{x}(k) + \tilde{D}_{\theta_k}\bar{w}(k) \end{cases} \tag{10}$$

where the augmented state is $\bar{x}(k) = [x'(k)\ \eta'(k)\ x'_f(k)]'$ and $\bar{w}(k) = [u'(k)\ d'(k)\ \hat{f}'(k)]'$ and so

$$\left[\begin{array}{c|c} \tilde{A}_{\theta_k} & \tilde{B}_{\theta_k} \\ \hline \tilde{C}_{\theta_k} & \tilde{D}_{\theta_k} \end{array}\right] =$$

$$\left[\begin{array}{ccc|ccc} A_{\theta_k} & 0 & 0 & B_{\theta_k} & B_{d\theta_k} & B_{f\theta_k} \\ B_{\eta\theta_k}C_{\theta_k} & A_{\eta\theta_k} & 0 & M_{\eta\theta_k} & B_{\eta\theta_k}D_{d\theta_k} & B_{\eta\theta_k}D_{f\theta_k} \\ 0 & 0 & A_{wf} & 0 & 0 & B_{wf} \\ \hline D_{\eta\theta_k}C_{\theta_k} & C_{\eta\theta_k} & -C_{wf} & 0 & D_{\eta\theta_k}D_{d\theta_k} & D_{\eta\theta_k}D_{f\theta_k} - D_{wf} \end{array}\right] \tag{11}$$

A feasible solution for the FDF optimization problem is such that it is possible to obtain matrices that compose the observer (7) in such a way that system (10) is MSS and γ is made as small as possible.

4.2 Residual Evaluation

The evaluation stage uses a evaluation function $J(\bar{r}(k))$ and the threshold $J_{th}(k)$, both introduced in the work (Zhong et al., 2005). The variable L denotes the evaluation time, and with that, it is possible to divide the evaluation into two distinct cases, where the first one is defined by $k - L \geq 0$ and the second one, $k - L < 0$. Thus, we define the auxiliary vectors for each case as

$$\begin{cases} \text{for } k-L \geq 0,\ \bar{r}(k) = [r(k)'\ r(k-1)'\ \dots\ r(k-L)'] \\ \text{for } k-L < 0,\ \bar{r}(k) = [r(k)'\ r(k-1)'\ \dots\ r(0)'] \end{cases} \tag{12}$$

and, given the discrepancy between the intervals, the evaluation functions for each case are set as

$$\begin{cases} \text{for } k-L \geq 0,\ J(\bar{r}(k)) = \left\{ \sum_{\sigma=k}^{k-L} \bar{r}'(\sigma)\bar{r}(\sigma) \right\}^{\frac{1}{2}}, \\ \text{for } k-L < 0,\ J(\bar{r}(k)) = \left\{ \sum_{\sigma=k}^{0} \bar{r}'(\sigma)\bar{r}(\sigma) \right\}^{\frac{1}{2}}. \end{cases} \tag{13}$$

The threshold function is defined as

$$J_{th}(k) = \sup_{d \in \mathcal{L}^2,\ f=0} \varepsilon(J(\bar{r}(k)_{f=0})) \tag{14}$$

where $\bar{r}(k)_{f=0}$ represents the residual signal when the system is operating on the nominal state, meaning that no fault occurs. The occurrence of faults can be detected by comparing the value of $J(\bar{r}(k))$ with $J_{th}(k)$ as follows:

$$\begin{cases} J(\bar{r}(k)) < J_{th}(k), \text{ means that the system is} \\ \qquad \text{in the nominal mode;} \\ J(\bar{r}(k)) \geq J_{th}(k), \text{ means that a fault} \\ \qquad \text{occurred at the instant } k. \end{cases} \tag{15}$$

5 Main Results

In this section, we present the main contribution of this paper, which is a FDF in the MJLS framework. Theorem 1 allows us to design an FDF that depends on the index θ_k. We also present the mode-independent case and the FDF with uncertainties in the matrices of the model.

5.1 Mode-Dependent Filter

Theorem 1 *There exists a mode-dependent FDF in the form of* (7) *satisfying the constraint* (8) *for some* $\gamma > 0$ *if there exist symmetric matrices* Z_i, X_i, W_i, *and the matrices* H_i, Δ_i, O_i, F_i, G_i *with compatible dimensions that satisfy the LMI constraints* (16), (17), (18)

$$\inf \sum_{i=1}^{N} \mu_i trace(W_i) < \gamma \tag{16}$$

$$\begin{bmatrix} W_i^{11} & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ W_i^{21} & W_i^{22} & \bullet & \bullet & \bullet & \bullet & \bullet \\ W_i^{31} & W_i^{32} & W_i^{33} & \bullet & \bullet & \bullet & \bullet \\ \varepsilon_i(Z)B_i & \varepsilon_i(Z)B_{di} & \varepsilon_i(Z)B_{fi} & \varepsilon_i(Z) & \bullet & \bullet & \bullet \\ \varepsilon_i(X)B_i + H_i & \varepsilon_i(X)B_{di} + \Delta_i D_{di} & \varepsilon_i(X)B_{fi} + \Delta_i D_{fi} & \varepsilon_i(Z) & \varepsilon_i(X) & \bullet & \bullet \\ 0 & 0 & \varepsilon_i(T)B_{wf} & 0 & 0 & \varepsilon_i(T) & \bullet \\ 0 & G_i D_{di} & G_i D_{fi} - D_{wf} & 0 & 0 & 0 & I \end{bmatrix} > 0 \tag{17}$$

$$\begin{bmatrix} Z_i & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ Z_i & X_i & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & T_i & \bullet & \bullet & \bullet & \bullet \\ \varepsilon_i(Z)A_i & \varepsilon_i(Z)A_i & 0 & \varepsilon_i(Z) & \bullet & \bullet & \bullet \\ \varepsilon_i(X)A_i + \Delta_i C_i + O_i & \varepsilon_i(X)A_i + \Delta_i C_i & 0 & \varepsilon_i(Z) & \varepsilon_i(X) & \bullet & \bullet \\ 0 & 0 & \varepsilon_i(T)A_{wf} & 0 & 0 & \varepsilon_i(T) & \bullet \\ G_i C_i + F_i & G_i C_i & -C_{wf} & 0 & 0 & 0 & I \end{bmatrix} > 0 \quad (18)$$

$i \in \mathbb{K}$. *If a feasible solution for* (16), (17), (18) *is obtained, then a suitable FDF is given by* $A_{\eta i} = (\varepsilon_i(Z) - \varepsilon_i(X))^{-1}O_i$, $B_{\eta i} = (\varepsilon_i(Z) - \varepsilon_i(X))^{-1}\Delta_i$, $C_{\eta i} = F_i$, $D_{\eta i} = G_i$, $M_{\eta i} = (\varepsilon_i(Z) - \varepsilon_i(X))^{-1}H_i$, *for all* $i \in \mathbb{K}$.

Proof: See the Appendix II.

5.2 Mode-Independent Filtering

In the mode-independent case, filter (7) does not depend on the index θ_k anymore, meaning that a single filter must be designed for all the N modes of the system. Recalling that the solution presented in Theorem 1 is already a sub-optimized solution, we have that, the mode-independent condition adds more conservadorism in the optimization problem.

When we refer to the LMI (16)-(18) in Theorem 2 below we consider that the variables $\Delta_i = \Delta$, $O_i = O$, $F_i = F$, $D_{\eta i} = D_\eta$ and $H_i = H$ no longer vary according to the index i, and moreover we add the following hypothesis

$$\rho_{ij} = \rho_j, \forall (i,j) \in \mathbb{K}. \quad (19)$$

This hypothesis, the so-called Bernoulli case, allow us to derive an LMI constraint capable of designing a mode-independent RFD Filter. This new hypothesis is necessary to remove the dependency on i of the filter matrices, as shown in Theorem 2 below:

Theorem 2 *There exist a mode-independent FDF in the form of* (7) *satisfying the constraint* $\|G\|_2^2 < \gamma$ *if there exist symmetric matrices* Z_i, X_i, W_i, *and matrices* H, Δ, O, F, G *satisfying the LMI* (16)-(18). *If a feasible solution is achieved the matrices that compose the FDF are* $A_\eta = (\varepsilon(Z) - \varepsilon(X))^{-1}O$, $B_\eta = (\varepsilon(Z) - \varepsilon(X))^{-1}\Delta$, $C_\eta = F$, $D_\eta = G$ *and* $M_\eta = (\varepsilon(Z) - \varepsilon(X))^{-1}H$.

Proof: The proof follows the same lines as the proof of Theorem 1 presented in the Appendix II.

5.3 Robust FD filter with uncertainties in the Model

The last special case we tackle is the procedure to add parametric uncertainties in Theorem 1. In order to describe the system (6) with polytopic uncertainties, we consider that for vertex matrices

$$\left[\begin{array}{c|c} A_i^l & B_i^l \\ \hline C_i^l & 0 \end{array}\right], \quad i \in \mathbb{K} \text{ we have that } \left[\begin{array}{c|c} A_{\theta_k} & B_{\theta_k} \\ \hline C_{\theta_k} & 0 \end{array}\right] \in \Upsilon \quad (20)$$

where Υ is the polytope as described in (1), and $l \in \{1, \dots, V\}$ represents the uncertain polytopic vertex. We replace in (17), (18), the matrices A_i, B_i and C_i by respectively A_i^l, B_i^l and C_i^l, so that adding this new index implies in adding V new constraints in (17), (18) for Theorem 1. We have the following result.

Theorem 3 *There exist a mode-dependent FD Filter as in* (7) *satisfying the constraints in Theorem 1, after replacing the matrices* A_i, B_i *and* C_i *by respectively* A_i^l, B_i^l *and* C_i^l, *if there exist symmetric matrices* Z_i, X_i, W_i, *and matrices* H_i, Δ_i, O_i, F_i, G_i *satisfying* (16)-(18). *If a feasible solution is obtained a suitable FD Filter is given by* $A_{\eta i} = (\varepsilon_i(Z) - \varepsilon_i(X))^{-1}O_i$, $B_{\eta i} = (\varepsilon_i(Z) - \varepsilon_i(X))^{-1}\Delta_i$, $C_{\eta i} = F_i$, $D_{\eta i} = G_i$, $M_{\eta i} = (\varepsilon_i(Z) - \varepsilon_i(X))^{-1}H_i$, *for all* $i \in \mathbb{K}$.

Proof: The proof is derived directly from the proof for Theorem 1.

6 Numerical Example

The numerical example used in the present paper was first introduced in (Zhong et al., 2005). The example consists in an MJLS in the discrete-time domain with two modes of operation. The dynamic system is composed of the matrices as in (21), (22), (23), (24)

$$A_1 = \begin{bmatrix} 0.1 & 0 & 1 & 0 \\ 0 & 0.1 & 0 & 0.5 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0.2 \end{bmatrix} A_2 = \begin{bmatrix} 0.3 & 0 & -1 & 0 \\ -0.1 & 0.2 & 0 & -0.5 \\ 0 & 0 & -0.2 & 0 \\ 0 & 0 & 0 & -0.5 \end{bmatrix} \quad (21)$$

$$B_d = \begin{bmatrix} 0.8 \\ -2.4 \\ 1.6 \\ 0.8 \end{bmatrix}, B_f = \begin{bmatrix} 1 \\ 1 \\ 2 \\ -2 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \quad (22)$$

$$D_d = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}, D_f = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \mathbb{P} = \begin{bmatrix} 0.3 & 0.7 \\ 0.6 & 0.4 \end{bmatrix} \quad (23)$$

$$A_{wf} = 0.5, B_{wf} = 0.25, C_{wf} = 1, D_{wf} = 0.5 \quad (24)$$

Using Theorem 1 the obtained RFD filter is presented in (25),(26),(27)

$$A_{\eta 1} = \begin{bmatrix} -1.41 & -0.92 & 0.45 & -1.78 \\ 3.02 & 1.39 & 2.18 & 1.89 \\ -0.86 & -1.56 & -1.43 & -1.85 \\ -0.84 & 0.98 & -1.20 & 1.69 \end{bmatrix}, A_{\eta 2} = \begin{bmatrix} 0.31 & -0.51 & -0.85 & -0.53 \\ 0.72 & 0.37 & 0.74 & -0.19 \\ -0.55 & -1.14 & -0.81 & -1.22 \\ -0.28 & 0.38 & -0.21 & 0.16 \end{bmatrix} \quad (25)$$

$$B_{\eta 1} = \begin{bmatrix} 0.68 & 1.82 \\ -0.53 & -3.65 \\ 1.51 & 0.78 \\ -1.49 & 1.16 \end{bmatrix}, B_{\eta 2} = \begin{bmatrix} 0.52 & -0.02 \\ -0.15 & -0.98 \\ 1.26 & 0.62 \\ -0.63 & 0.39 \end{bmatrix}, C_{\eta 1} = \begin{bmatrix} -0.06 \\ -0.02 \\ -0.02 \\ -0.01 \end{bmatrix}', \quad (26)$$

$$C_{\eta 2} = \begin{bmatrix} -0.06 \\ -0.02 \\ -0.00 \\ -0.00 \end{bmatrix}', D_{\eta 1} = \begin{bmatrix} -0.02 \\ -0.02 \end{bmatrix}' D_{\eta 2} = \begin{bmatrix} -0.05 \\ -0.03 \end{bmatrix}', M_1 = M_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (27)$$

and the obtained H_2 norm value is 4.6556.

In order to show that the theoretical results provided in this paper are suitable solutions to the RFD Filter problem, a simulation is presented. The unknown signal $d_k(k)$ is a white noise sequence with mean equal to 0 and variance equal to 0.7071. The weighted fault signal, denoted by $\hat{f}(k)$, used in the simulation is a unitary step signal starting at $k = 100$ and finishing at $k = 200$. The residual signal, $r(k)$, obtained in the simulation is shown in Fig. 3.

In the graphic shown in Fig. 4 the traced red curve represents the evaluation function (15) in

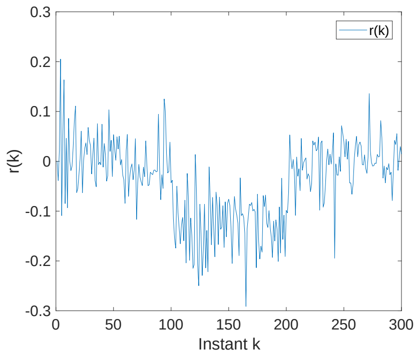

Figure 3: Residual $r(k)$ behavior for the $\|H\|_2$ case.

the case where the fault occurs, the blue curve represents the evaluation function when there is no fault occurrence, and the magenta represents the weighted fault itself.

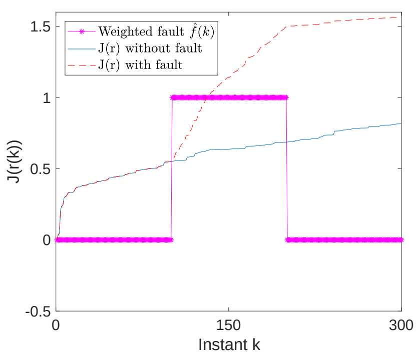

Figure 4: $J(r)$ behavior for the $\|H\|_2$

As explained in Section 4.2 the evaluation process uses the evaluation function $J(r(k))$ and compares its value with the threshold J_{th} obtained via simulation when there is no fault occurrence. It is possible to observe that when the fault, the magenta curve in Fig. 4, starts in this simulation it was needed only $k = 2$ instants to detect the fault, since the traced curve surpasses the blue curve. These results show that our approach can be a convenient solution to the fault detection problem.

We also performed 500 Monte Carlo simulation, and calculated the average number of instants k necessary to detect the fault. The average number obtained was $k = 4.3$, with a standard deviation of 1.7840.

7 Conclusion

The main contribution in this work was to derive a new set of LMI constraints for the design of an H_2 fault detector filter considering a discrete-time MJLS. We aslo analyzed the so-called mode-independent case and the parametric uncertainties case. The numerical example indicates that the presented approach can provide a viable solution to the FDF problem. The following steps in this line of research may be to add the consideration of variable delay in the FDF design.

Appendix I: Auxiliary Results

In (Costa et al., 1997) an LMI constraint for the calculation of the H_2 norm of a MJLS as in (2) was derived. The LMI constraint are presented below:

$$\sum_{i=1}^{N} \mu_i trace(W_i) < \gamma \tag{28}$$

$$\begin{bmatrix} W_i & \bullet & \bullet \\ \varepsilon_i(P)\tilde{J}_i & \varepsilon_i(P) & \bullet \\ \tilde{D}_{zi} & 0 & I \end{bmatrix} > 0 \tag{29}$$

$$\begin{bmatrix} P_i & \bullet & \bullet \\ \varepsilon_i(P)\tilde{A}_i & \varepsilon_i(P) & \bullet \\ \tilde{C}_{zi} & 0 & I \end{bmatrix} > 0 \tag{30}$$

These constraints can be equivalently written as follows:

$$\begin{bmatrix} W_i & \bullet & \bullet \\ \tilde{B}_i & \varepsilon_i(P)^{-1} & \bullet \\ \tilde{D}_{zi} & 0 & I \end{bmatrix} > 0 \tag{31}$$

$$\begin{bmatrix} P_i & \bullet & \bullet \\ \tilde{A}_i & \varepsilon_i(P)^{-1} & \bullet \\ \tilde{C}_{zi} & 0 & I \end{bmatrix} > 0 \tag{32}$$

Appendix II: Proof Theorem 1

The first step to derive the suboptimal condition is to impose the following structure, similar to the structure in (Gonçalves et al., 2011), for the matrices P and P^{-1}

$$P_i = \begin{bmatrix} X_i & U_i & 0 \\ U_i' & \hat{X}_i & 0 \\ 0 & 0 & T_i \end{bmatrix}, \quad P_i^{-1} = \begin{bmatrix} Y_i & V_i & 0 \\ V_i' & \hat{Y}_i & 0 \\ 0 & 0 & T_i^{-1} \end{bmatrix}, \tag{33}$$

and also consider the following structure for the matrices $\varepsilon_i(P)$ and $\varepsilon_i(P)^{-1}$

$$\varepsilon_i(P) = \begin{bmatrix} \varepsilon_i(X) & \varepsilon_i(U) & 0 \\ \varepsilon_i(U)' & \varepsilon_i(X) & 0 \\ 0 & 0 & \varepsilon_i(T) \end{bmatrix}, \varepsilon_i(P)^{-1} = \begin{bmatrix} R_{1i} & R_{2i} & 0 \\ R_{2i}' & R_{3i} & 0 \\ 0 & 0 & T_i \end{bmatrix}. \tag{34}$$

We define the following matrices α_i and δ_i as

$$\alpha_i = \begin{bmatrix} I & I & 0 \\ V_i'Y_i^{-1} & 0 & 0 \\ 0 & 0 & I \end{bmatrix}, \delta_i = \begin{bmatrix} R_{1i} & X_{pi} & 0 \\ 0 & U_{pi}' & 0 \\ 0 & 0 & \varepsilon_i(T_i) \end{bmatrix}. \tag{35}$$

Considering $U_i = Z_i - X_i$ in (33), we get from (33), (35) that $V_i = V_i'$ and $V_i = Z_i^{-1}$. Along side, $U_i = -\hat{X}_i$ we get $R_{1i}^{-1} = \varepsilon_i(X + U) = \varepsilon_i(Z)$, and so we have that

$$\alpha_i' P_i \alpha_i = \begin{bmatrix} Y_i^{-1} & Y_i^{-1} & 0 \\ Y_i^{-1} & X_i & 0 \\ 0 & 0 & T_i \end{bmatrix},$$

$$\delta_i' \tilde{A}_i \alpha_i =$$

$$\begin{bmatrix} R_{1i}^{-1} A_i & R_{1i}^{-1} A_i & 0 \\ \varepsilon_i(X)A_i + \varepsilon_i(U)B_{\eta i}C_i + \varepsilon_i(U)' A_{\eta i} V_i' Z_i & \varepsilon_i(X)A_i + \varepsilon_i(U)B_{\eta i}C_i & 0 \\ 0 & 0 & \varepsilon_i(P^{33})A_{wf} \end{bmatrix},$$

$$\delta_i' \tilde{B}_i =$$

$$\begin{bmatrix} R_{1i}^{-1}B_i & R_{1i}^{-1}B_{di} & R_{1i}^{-1}B_{fi} \\ \varepsilon_i(X)B_i + \varepsilon_i(U)M_{\eta i} & \varepsilon_i(X)B_{di} + \varepsilon_i(U)B_{\eta i}D_{di} & \varepsilon_i(X)B_{fi} + \varepsilon_i(U)B_{\eta i}D_{fi} \\ 0 & 0 & \varepsilon_i(W)B_{wf} \end{bmatrix},$$

$$\delta_i'\varepsilon_i(P)^{-1}\delta_i = \begin{bmatrix} \varepsilon_i(Z) & \varepsilon_i(Z) & 0 \\ \varepsilon_i(Z) & \varepsilon_i(X) & 0 \\ 0 & 0 & \varepsilon_i(T_i) \end{bmatrix},$$

$$\tilde{C}_i\alpha_i = \begin{bmatrix} D_{\eta i}C_i + C_{\eta i}V_i'Z_i & D_{\eta i}C_i & C_{wf} \end{bmatrix}$$

$$\tilde{D}_i = \begin{bmatrix} 0 & D_{\eta i}D_{di} & D_{\eta i}D_i - D_{wf} \end{bmatrix}.$$

Applying the change of variables $\varepsilon_i(U)'B_{\eta i} = \Delta_i$, $\varepsilon_i(U)'A_{\eta i}V_i'Z_i = O_i$, $\varepsilon_i(U)'M_{\eta i} = H_i$, $C_{\eta i}V_i'Z_i = F_i$ and also substituting $\varepsilon_i(Z) = R_{1i}^{-1}$ in (16), allow us to get the following inequalities

$$\sum_{i=1}^{N} \mu_i trace(W_i) < \gamma \tag{36}$$

$$\begin{bmatrix} W_i & \bullet & \bullet \\ \delta_i'\tilde{B}_i & \delta_i'\varepsilon_i(P)^{-1}\delta_i & \bullet \\ \tilde{D}_{zi} & 0 & I \end{bmatrix} > 0 \tag{37}$$

$$\begin{bmatrix} \alpha_i'P_i\alpha_i & \bullet & \bullet \\ \delta_i'\tilde{A}_i\alpha_i & \delta_i'\varepsilon_i(P)^{-1}\delta_i & \bullet \\ \tilde{C}_{zi}\alpha_i & 0 & I \end{bmatrix} > 0 \tag{38}$$

and the inequality (36) is equivalent to the inequality (16). By doing a congruence transformation with the matrix $diag[I, \delta_i^{-1}, I]$ for (37) and $diag[\alpha_i^{-1}, \delta_i^{-1}, I]$ for (38), we get the inequality (31), (32) and with that we can guarantee that $\|\mathcal{G}\|_2^2 < \gamma$.

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