IMPROVING THE CHOICE OF DISTURBANCE REFERENCE MODEL IN DATA-DRIVEN CONTROL METHODS

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Abstract— This work aims to study and address the problem of choosing the reference model within a model reference control design problem for disturbance rejection. Existing theory on the design of the reference model for disturbance is revisited, and adaptations for data-based applications, in which the process' mathematical model is unknown, are proposed. Finally, this paper presents an alternative way of accounting for this choice directly in a data-based control tuning method. Simulation results show that a sensible choice of reference model improve controller tuning and that the proposed strategy is able to manage this choice in a systematic manner without using the process' model.

Keywords— Data-driven control, model reference control, Disturbance rejection, PID control.

Resumo— Este trabalho tem como objetivo estudar e resolver o problema da escolha de modelo de referência em um problema de projeto de controle por modelo de referência para perturbação. A teoria existente sobre a escolha do modelo de referência para perturbação é revisitada, e adaptações para aplicações baseadas em dados, nas quais o modelo matemático do processo é desconhecido, são propostas. Finalmente, este artigo apresenta uma forma alternativa de considerar esta escolha diretamente em um método de controle baseado em dados. Resultados em simulação mostram que uma escolha sensata de modelo de referência melhora a sintonia do controlador e que a estratégia proposta é capaz de lidar com esta escolha de forma sistemática sem utilizar o modelo do processo.

Palavras-chave— Controle baseado em dados, controle por modelo de referência, rejeição à perturbação, controle PID.

1 Introduction

Model reference control design has become an usual approach in control tuning, for its ability to incorporate performance specifications in a direct manner. In this technique, the desired closed-loop behavior is specified as a transfer function between output and, in most cases, the reference signal, which is called reference model. The controller is therefore tuned aiming to achieve in closed-loop the desired dynamics, e.g. by minimizing a certain criterion.

This strategy has been explored for model-based scenarios, and successfully developed in data-driven and adaptive control (Goodwin and Sin, 2014; Hjalmarsson et al., 1998; Campi et al., 2002), where usually the reference tracking problem is tackled. Being the most important design variable, the choice of the reference model has a major impact on the resulting controller. This choice is addressed partially in Aguirre (1993), Bazanella et al. (2012) and Bazanella et al. (2008) and is further detailed in Gonçalves da Silva, Bazanella and Campestrini (2018) for reference tracking. It is shown in the literature how critical it is to specify the reference model in a sensible manner, i.e. so that the desired behavior is (approximately) achievable by the available control structure, given the process' characteristics.

The literature on model reference control strategies concerning the disturbance rejection problem is limited compared to reference tracking. As in the analogous problem, the choice of the reference model for disturbance is also important to improve closed-loop stability. This choice has been studied for model-based scenarios, in which the process' parametric model is known (Szita and Sanathanan, 1996; Szita and Sanathanan, 2000). However in data-driven control methods for disturbance rejection, where such model is unavailable, this choice remains challenging.

In this work, the design of the reference model for disturbance will be explored, particularly for data-based approaches. The method Virtual Disturbance Feedback Tuning (VDFT) (Eckhard et al., 2018), which consists in a data-based model reference control method for disturbance, is used so as to illustrate the effect of reference model choice on the resulting closed-loop performance. An adaptation to the VDFT method is proposed so as to handle the choice of the reference model for disturbance systematically.

This work is divided in the following manner: section 2 introduces the model reference control problem for disturbance with a motivational example. Section 3 explores the existing guidelines for the choice of reference model for disturbance and when the process model is known. Later, in section 4, a systematic procedure in defining a reference model, with some free parameters, is presented and the data-driven control method VDFT is adapted in a flexible formulation to handle the reference model choice issue; in section 5, simulation results are presented to support the flexible VDFT method. Finally, section 6 presents some conclusions from the work.

Notation: In this work, the operator $\Gamma[\cdot]$ represents the relative degree of a transfer function and Deg$[\cdot]$, the degree of a polynomial, e.g. for a given transfer function $X(q) \triangleq \frac{y(q)}{z(q)}$, $\Gamma[X(q)] = \text{Deg}[z(q)] - \text{Deg}[y(q)]$.

2 Problem description

Consider the linear time-invariant discrete single-input single-output (SISO) system described as

$$y(t) = G(q)u(t) + \nu(t) \tag{1}$$

where q is the forward shift operator, such that $qx(t) = x(t+1)$ for a signal $x(t)$. Moreover, $G(q)$ represents the process' transfer function, while $y(t)$, $u(t)$ and $\nu(t)$ are the output, input and noise signals respectively.

It is assumed that, in closed-loop, the system's input is subject to a disturbance signal $d(t)$, such that $u(t) = d(t) + u_c(t)$, where $u_c(t)$ is the manipulated variable. In this context, the controller design problem aims to attenuate the disturbance effect over the process' output, by tuning a set of controller parameters $\rho \in \mathbb{R}^m$, characterizing a linear time-invariant controller $C(q,\rho)$, so as to achieve a desired closed-loop response regarding the disturbance behavior.

Considering the feedback control framework, $u_c(t) = C(q,\rho)(r(t) - y(t))$, the input signal $u(t)$ can be thus expressed as $u(t) = d(t) + C(q,\rho)(r(t) - y(t))$, with $r(t)$, the reference signal. The closed-loop response is then represented according to

$$y(t) = T(q,\rho)r(t) + Q(q,\rho)d(t) + S(q,\rho)\nu(t) \tag{2}$$

with

$$T(q,\rho) \triangleq \frac{G(q)C(q,\rho)}{1 + G(q)C(q,\rho)} \tag{3}$$

$$Q(q,\rho) \triangleq \frac{G(q)}{1 + G(q)C(q,\rho)} \tag{4}$$

$$S(q,\rho) \triangleq \frac{1}{1 + G(q)C(q,\rho)} \tag{5}$$

In the model reference approach, the desired closed-loop response is specified through a desired closed-loop transfer function, i.e. a reference model: $T_d(q)$ for reference tracking and $Q_d(q)$, for disturbance. Then, an H_2 performance criterion is minimized in order to find a controller that minimizes the difference from the desired behavior and the obtained behavior. In the disturbance rejection problem, this criterion is defined as

$$\begin{aligned} &\min_{\rho} J^{DM}(\rho) \\ &J^{DM}(\rho) \triangleq \sum_t \left\{[Q(q,\rho) - Q_d(q)]d(t)\right\}^2 . \end{aligned} \tag{6}$$

which represents an adapted formulation from model reference control for reference tracking (where the controller is tuned to approximate $T(q,\rho)$ and the reference model $T_d(q)$ (Bazanella et al., 2012)).

The ideal controller $C_d(q)$ which provides the exact desired behavior in closed-loop $Q_d(q)$ is given by

$$C_d(q) = Q_d^{-1}(q) - G^{-1}(q) \tag{7}$$

whose expression was derived from (4), where $C(q,\rho)$ was substituted by $C_d(q)$ and $Q(q,\rho)$ by $Q_d(q)$. In order to obtain a causal ideal controller and an internally stable closed loop, $Q_d(q)$ cannot be chosen freely, as we will show in next section.

Data-driven control design can be seen as the identification of the ideal controller, which is performed as: collect input-output data from the process, define a desired closed-loop behavior ($Q_d(q)$ in the case dealt in this work) and choose a fixed controller class, i.e. $\mathcal{C} = \{C(q,\rho) : \rho \in \Omega \subseteq \mathbb{R}^p\}$; minimize a performance criterion in order to find the controller $C(q,\rho)$ without deriving a process model.

In most cases the ideal controller is not achievable, i.e. $C_d(q) \notin \mathcal{C}$, however a good design practice is to choose sensibly design parameters so that $C_d(q)$ is drawn closer to the controller class $\mathcal{C}$. This can be done by choosing a more flexible controller structure, or a reference model $Q_d(q)$ considering some information on the process, since the mathematical process model is unknown.

It is well known for the reference tracking problem that the choice of $T_d(q)$ affects the achieved optimization results of (6): if the desired performance is unrealistic for the given controller class and process order, the optimization can lead far from the best controller within $\mathcal{C}$ (Bazanella et al., 2012).

Throughout the examples here presented, the data-driven control method Virtual Disturbance Feedback Tuning (VDFT) (Eckhard et al., 2018) is used in order to tune PID controllers based on closed-loop data collected from the system. As most data-driven control methods, this method aims to tune a linearly parametrized controller, with fixed poles, by solving a quadratic optimization problem. This framework can be restrictive since the ideal controller $C_d(q)$ will hardly belong to $\mathcal{C}$. It is nonetheless important to choose $Q_d(q)$ wisely so that $C_d(q)$ lies close to $\mathcal{C}$ and the obtained closed-loop behavior is not excessively far from the desired one.

The following example illustrates how a poor choice of reference model for disturbance $Q_d(q)$ leads to an also poor performance in a data-driven control approach.

Example 1 *Consider the discrete-time SISO system given by*

$$G_1(q) = \frac{0.1}{(q-0.8)(q-0.4)} \tag{8}$$

unknown by the user, with sampling time of $1\,\mathrm{s}$*, and operating in closed-loop with the PI controller* $C_0(q) = \frac{0.7(q-0.6)}{(q-1)}$.

Using the VDFT method, we seek to improve the controller tuning, using only data collected from the process in order to achieve the following reference model for disturbance:

$$Q_d(q) = \frac{0.09(q-1)}{(q-0.7)^2} \tag{9}$$

which rejects step disturbances and imposes a shorter settling time ($20\,\mathrm{s}$*) and smaller peak. For that, we choose a PID structure as controller class* $\mathcal{C}$.

The careless choice of $Q_d(q)$ *in* (9) *is equivalent to trying to identify the ideal controller* $C_d(q) = \frac{-10(q-1.976)(q-0.7575)(q-0.5775)}{q-1}$*, which does not belong to* $\mathcal{C}$ *and is not realizable. The controller tuned using VDFT provides a disastrous closed-loop behavior, which can be seen in Figure 1.*

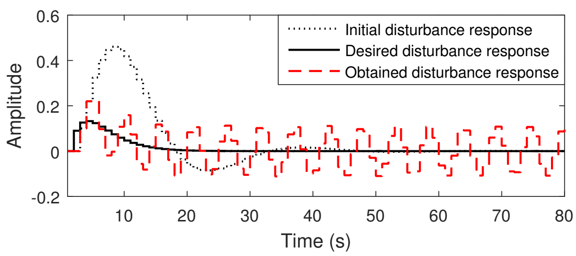

Figure 1: Comparison among desired, initial and obtained closed-loop behavior for disturbance.

Motivated by the example above, the following sections of this work aim to determine practical guidelines for choosing the reference model for disturbance. We first present some model-based choices from the literature and then we adapt these choices for the case where the process model is unknown.

3 Model-based choice of $Q_d(q)$

Considering the model reference problem for disturbance, some results on the choice of the reference model exist in the literature based on a more comprehensive knowledge of the plant.

A first result comes directly from the analysis of the closed-loop expression in (4). Assuming that $G(q)C_d(q)$ is strictly proper, the relative degree of $Q_d(q)$ is given as

$$\begin{aligned} \Gamma[Q_d(q)] &= \Gamma[G(q)] - \Gamma[1 + C_d(q)G(q)], \\ &= \Gamma[G(q)] - \min\{0, \Gamma[C_d(q)G(q)], \} \\ &= \Gamma[G(q)], \end{aligned} \tag{10}$$

where tropical algebra is applied to derive this result (Gonçalves da Silva, Bazanella and Campestrini, 2018). Therefore $Q_d(q)$ should have necessarily the same relative degree as $G(q)$.

In order to derive the restriction for controller causality and stability, let the transfer functions $G(q)$ and $Q_d(q)$ be decomposed into $G(q) \triangleq \frac{nG(q)}{dG(q)}$ and $Q_d(q) \triangleq \frac{nQ_d(q)}{dQ_d(q)}$, where $nG(q)$ and $dG(q)$ are coprime, and $nQ_d(q)$ and $dQ_d(q)$ are likewise coprime.

3.1 Causality

The ideal controller will be causal when

$$\Gamma[C_d(q)] \geq 0. \tag{11}$$

Substituting (7) in (11), the following theorem can be derived:

Theorem 1 (Szita and Sanathanan (1996)) *The controller* $C_d(q)$ *will be proper if and only if*

$$\mathrm{Deg}[dQ_d(q)nG(q) - dG(q)nQ_d(q)] \leq \mathrm{Deg}[nQ_d(q)nG(q)] \tag{12}$$

with $\Gamma[G(q)] = \Gamma[Q_d(q)]$. $\square$

Indeed (12) holds true if at least λ terms are canceled in the expression $dQ_d(q)nG(q) - dG(q)nQ_d(q)$, where $\lambda \triangleq \Gamma[G(q)] = \Gamma[Q_d(q)]$. It can be noted that in order for such cancellations to occur, some significant knowledge on the process' model is necessary. To deal with this issue, consider that

$$nQ_d(q) = (\alpha_0 q^a + \alpha_1 q^{a-1} + \cdots + \alpha_a)\times (q^l + \gamma_1 q^{l-1} + \cdots + \gamma_l) \tag{13}$$

$$dQ_d(q) = q^b + \beta_1 q^{b-1} + \cdots + \beta_b \tag{14}$$

with $\lambda = b - a - l$, where the polynomial $\gamma(q)$ contains zeros which are fixed by the user.

Cancellations in $dQ_d(q)nG(q) - dG(q)nQ_d(q)$ will occur by satisfying the following expression

$$\begin{bmatrix} 1 & 0 & \dots & 0 \\ \beta_1 & 1 & & 0 \\ \vdots & & \ddots & \vdots \\ \beta_{\lambda-1} & \beta_{\lambda-2} & \dots & 1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_\lambda \end{bmatrix} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ \gamma_1 & 1 & & 0 \\ \vdots & & \ddots & \vdots \\ \gamma_{\lambda-1} & \gamma_{\lambda-2} & \dots & 1 \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_{\lambda-1} \end{bmatrix} \tag{15}$$

where h_i corresponds to the i-th non-zero coefficient of the process' impulse response. For a sampled system, $h_i = g_i - g_{i-1}$, where g_i, corresponds to the step response at instant i.

In a data-driven control design, the user could thus choose both the denominator of $Q_d(q)$ according to the desired closed-loop performance and the fixed part of its numerator, considering the type of disturbance that should be rejected (step, ramp, etc.); thus determine the remaining part of the numerator of $Q_d(q)$ (polynomial $\alpha(q)$) using (15) and measured open-loop data from the system (step response, for instance).

Remark: In the reference tracking problem, a causal controller was obtained by simply choosing $\Gamma[T_d(q)] \geq \Gamma[G(q)]$ (Bazanella et al., 2012).

Example 2 *Consider again the system from Example 1. The goal is to tune a PID controller based on data collected from the process $G_1(q)$ so as to reject in closed-loop step disturbances with the behavior described by $Q_d(q)$. In order to employ* (15)*, we also consider that the impulse (or step) response of $G_1(q)$ was measured from the system.*

Since $\Gamma[G(q)] = 2$, the relative degree of $Q_d(q)$ should also be 2, and, according to Theorem 1, two parameters α_0, α_1 should be identified to assure that $C_d(q)$ is causal. Also, only the first two elements of the impulse response must be known ($h_1 = 0.1$, $h_2 = 0.12$). Therefore, the structure of $Q_d(q)$ is chosen to be

$$nQ_d(q) = (\alpha_0 q + \alpha_1)(q-1) \tag{16}$$
$$dQ_d(q) = (q-0.6)^4 \tag{17}$$

Solving system (15) *results in $\alpha_0 = 0.1$ and $\alpha_1 = -0.02$ and*

$$Q_d(q) = \frac{0.1(q-1)(q-0.2)}{(q-0.6)^4} \tag{18}$$

with the causal ideal controller

$$C_d(q) = \frac{2(q-0.4211)(q-0.7789)}{(q-0.2)(q-1)}.$$

Now employing the VDFT method, with the reference model in (18)*, the following PID controller is tuned:*

$$C(q, \hat{\rho}_1) = \frac{2.0909(q-0.2955)(q-0.7829)}{q(q-1)}, \tag{19}$$

which provides in closed-loop the behavior observed in Figure 2.

It should be noted that, since $C_d(q) \notin \mathcal{C}$ for the VDFT method in general, the ideal controller is not achieved. However Figure 2 shows that the obtained behavior is quite close to $Q_d(q)$ and significantly improved compared to the response obtained in Example 1.

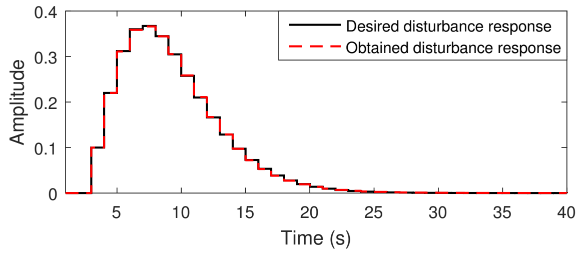

Figure 2: Comparison between desired and obtained closed-loop behavior, $Q_d(q)$ and $Q(q, \hat{\rho}_1)$ respectively.

3.2 Internal stability

The closed-loop system will be internally stable if the controller does not cancel out the non-minimum phase zeros of the plant with unstable poles.

Theorem 2 (Szita and Sanathanan (1996)) *The controller $C_d(q)$ will be stable if and only if there exists polynomials $f(q)$ and $g(q)$, such that*

1. $nQ_d(q) = f(q)nG^+(q)$
2. $dQ_d(q)nG^-(q) = dG(q)f(q) + g(q)nG^+(q)$.

with

$$\begin{cases} \mathrm{Deg}[f(q)] = \mathrm{Deg}[dQ_d(q)] + \mathrm{Deg}[nG^-(q)] - \mathrm{Deg}[dG(q)] \\ \mathrm{Deg}[g(q)] \leq \mathrm{Deg}[dQ_d(q)] + 2\mathrm{Deg}[nG^-(q)] - \mathrm{Deg}[dG(q)] \end{cases}$$

where $nG^-(q)$ and $nG^+(q)$ represent the minimum phase and non-minimum phase factors of $nG(q)$ respectively, i.e. $nG(q) = nG^-(q)nG^+(q)$. □

In case the conditions in Theorem 2 are satisfied, the ideal controller is given by

$$C_d(q) = \frac{g(q)}{f(q)nG^-(q)}.$$

In Szita and Sanathanan (1996), it is suggested that the order of $dQ_d(q)$ be chosen as

$$\mathrm{Deg}[dQ_d(q)] \geq 2\left\{\mathrm{Deg}[dG(q)] - \mathrm{Deg}[nG^-(q)]\right\} + l - 1 \tag{20}$$

where l again is the number of fixed zeros in $f(q)$, e.g. for step disturbance rejection a zero in 1 is fixed.

Once again, in order to assure internal stability for the ideal closed-loop, knowledge on the process' explicit model is needed. While the causality constraint can be solved in a data-based approach by performing a second experiment, we haven't found a similar data-based procedure to solve the stability constraint in order to define an appropriate reference model $Q_d(q)$.

Remark: In the reference tracking problem, a stable controller was obtained by simply adding the non-minimum phase zeros of $G(q)$ to $T_d(q)$ (Bazanella et al., 2012). It is possible to identify these zeros together with the controller parameters, without deriving a process model (Campestrini et al., 2011; Gonçalves da Silva, Campestrini and Bazanella, 2018).

Example 3 *Consider now a non-minimum phase plant*

$$G_2(q) = \frac{0.1(q-1.2)}{(q-0.8)(q-0.4)}. \tag{21}$$

In order to demonstrate the use of Theorem 2, we assume that the mathematical model $G_2(q)$ is known and therefore we are able to determine $Q_d(q)$ so that the ideal controller is not only causal but also stable.

In this example, using (20), we choose $\text{Deg}[dQ_d(q)] = 4$, and the structure of $Q_d(q)$ is given as

$$nQ_d(q) = (q-1.2)(q-1)(f_1 q - f_0) \tag{22}$$
$$dQ_d(q) = (q-0.6)^4 \tag{23}$$

where f_1 and f_0 represent the degrees of freedom in the numerator of $Q_d(q)$.

Solving the system in Theorem 2, the following reference model for disturbance is identified

$$Q_d(q) = \frac{0.1(q-1)(q-1.2)(q+0.825)}{(q-0.6)^4} \tag{24}$$

which results in a causal and stable ideal controller

$$C_d(q) = \frac{-10.25(q-0.405)(q-0.7901)}{(q+0.825)(q-1)}.$$

Using the VDFT method, with reference model for disturbance (24), the following PID controller is obtained:

$$C(q,\hat{\rho}_2) = \frac{-7.4392(q-0.6293)(q-0.7454)}{q(q-1)}, \tag{25}$$

which results in the closed-loop behavior seen in Figure 3.

Once again, although the linear parametrization of $C(q,\rho)$ in VDFT prevents it from achieving $C_d(q)$, Figure 3 shows a satisfactory matching between desired and obtained disturbance response.

In the example developed above, the model $G_2(q)$ is known. In a scenario in which $G_2(q)$ is not available and we do not intend obtaining this model, some parameters of the reference model $Q_d(q)$ should be let free to be identified together with the controller parameters, as explained in the next section.

4 Data-based solution

As seen above, choosing the reference model for disturbance can become a tricky task, specially when the mathematical model $G(q)$ is unknown.

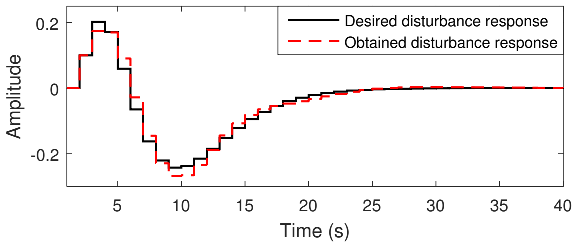

Figure 3: Comparison between desired and obtained closed-loop behavior, $Q_d(q)$ and $Q(q,\hat{\rho}_2)$ respectively.

Analyzing both Theorems 1 and 2, it can be seen that the choice of the numerator of $Q_d(q)$ is critical to assure causality and stability of the ideal controller. As an alternative to identifying the process' model and then employing Theorems 1 and 2, this work proposes a flexible formulation for data-based control tuning methods, in which part of the numerator of $Q_d(q)$ is loosened and identified in the method.

The idea will be here developed for the VDFT method, in which a flexible reference model for disturbance $Q_d(q,\eta)$ is employed, in a similar fashion as in Campestrini et al. (2011), where

$$Q_d(q,\eta) = \eta^T \bar{F}(q) \tag{26}$$

where $\eta = \begin{bmatrix} \eta_1 & \dots & \eta_m \end{bmatrix}^T$ is a vector of m parameters, and $\bar{F}(q)$, a m-vector of rational transfer functions. Therefore, part of the numerator of $Q_d(q,\eta)$ is let loose while the denominator is defined by the user in order to attain performance requirements.

4.1 *Structure of $Q_d(q,\eta)$*

To design the structure of $Q_d(q,\eta)$ in the flexible formulation, the following guidelines are derived:

1. Retrieve knowledge on the number of non-minimum phase zeros in $G(q)$, given by $\text{Deg}[nG^+(q)]$, and its relative degree $\Gamma[G(q)]$.

   Note that for minimum phase systems, $\text{Deg}[nG^+(q)] = 0$.

2. Choose the l fixed zeros in $nQ_d(q,\eta)$, according to the type of disturbance to be rejected, e.g. to reject step disturbances a zero should be fixed in 1.

3. Determine the order of the transfer function $Q_d(q,\eta)$:

$$\begin{aligned}\text{Deg}[dQ_d(q,\eta)] =& 2\{\Gamma[G(q)] + \text{Deg}[nG^+(q)]\} \\ &+ l - 1,\end{aligned} \tag{27}$$

   which is derived from (20), considering that $\text{Deg}[dG(q)] - \text{Deg}[nG^-(q)] = \Gamma[G(q)] + \text{Deg}[nG^+(q)]$.

4. Choose the poles from $Q_d(q,\eta)$ according to the desired dynamics, e.g. desired settling time for disturbance rejection.

5. Based on the order of $Q_d(q,\eta)$, determine the number of free parameters in $nQ_d(q,\eta)$ so that $\Gamma[Q_d(q,\eta)] = \Gamma[G(q)]$:

$$m = \text{Deg}[dQ_d(q,\eta)] - l - \Gamma[G(q)] + 1, \quad (28)$$

which comes from $m = \text{Deg}[f(q)] + \text{Deg}[nG^+(q)]$ and the definition of $\text{Deg}[f(q)]$ in Theorem 2.

Note that for minimum phase systems, for $C_d(q)$ to be causal, according to Theorem 1, $m = \Gamma[G(q)]$ and $\Gamma[Q_d(q)] = \Gamma[G(q)]$. Both conditions are compatible with guidelines in (27) and (28).

4.2 Flexible VDFT formulation

In order to identify both the controller parameters and the reference model $Q_d(q,\eta)$, the VDFT problem (Eckhard et al., 2018) is rewritten as

$$\min_{\rho,\eta} J^{VDf}(\rho,\eta)$$

$$J^{VDf}(\rho,\eta) \triangleq \sum_{t=1}^{N} \{K(q)[Q_d(q,\eta)(u(t) + C(q,\rho)y(t)) - y(t)]\}^2, \quad (29)$$

where $u(t)$ and $y(t)$ represent respectively the input and output data collected in an experiment and $K(q)$ is the filter used to approximate the shape of functions J^{VDf} and J^{DM}. In Eckhard et al. (2018), the practical choice $K(q) = Q_d(q)$ is suggested. The solution of (29) is given iteratively: the least squares problem is solved for η and ρ alternatively, for each iteration i, as in

$$\eta^i = \arg\min_{\eta} J^{VDf}(\rho^{i-1},\eta) \quad (30)$$

$$\rho^i = \arg\min_{\rho} J^{VDf}(\rho,\eta^i) \quad (31)$$

and ρ^0 is set with the parameters of the controller that is initially operating in the loop.

The following examples will be used to show how this strategy can be employed for data-based model reference control in order to obtain a closed-loop behavior close to the desired one.

5 Simulation results

Example 4 *Consider the plant $G_1(q)$ once again. It was seen that since $\Gamma[G_1(q)] = 2$, then $\Gamma[Q_d(q,\eta)] = 2$ and it is necessary that 2 parameters in the numerator of $Q_d(q,\eta)$ be manipulated so that the ideal controller is causal. Also, considering the guideline in* (27), $\text{Deg}[dQ_d(q,\eta)] = 4$.

Using the flexible formulation of VDFT, seen in (29), *with*

$$Q_d(q,\eta) = [\eta_1 \quad \eta_2] \begin{bmatrix} \frac{q(q-1)}{(q-0.6)^4} \\ \frac{(q-1)}{(q-0.6)^4} \end{bmatrix} \quad (32)$$

for a PID controller class, the following results are obtained:

$$Q_d(q,\hat{\eta}_3) = \frac{0.10087(q-1)(q-0.2241)}{(q-0.6)^4} \quad (33)$$

$$C(q,\hat{\rho}_3) = \frac{2.2355(q-0.7801)(q-0.3351)}{q(q-1)} \quad (34)$$

which are very similar to the ones obtained in Example 2, except now a single batch of closed-loop data was used to tune the parameters of the controller and of the reference model, without using the step response information. The closed-loop behavior can be seen in Figure 4.

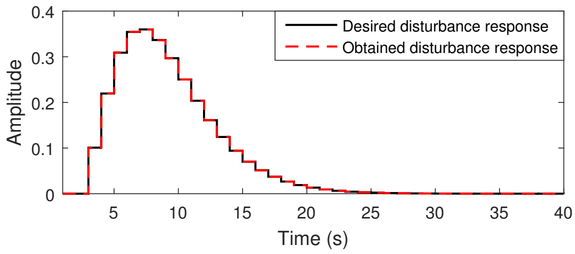

Figure 4: Comparison between desired and obtained closed-loop behavior, $Q_d(q,\hat{\eta}_3)$ and $Q(q,\hat{\rho}_3)$ respectively.

Example 5 *Consider the non-minimum phase plant $G_2(q)$ in Example 3. Assume that the following information is available:* $\text{Deg}[nG^+(q)] = 1$ *and* $\Gamma[G(q)] = 1$, *and let a zero of $Q_d(q,\eta)$ be fixed in 1 for step disturbance rejection. According to* (27), *the degree of the denominator of $Q_d(q,\eta)$ is given as*

$$\text{Deg}[dQ_d(q)] = 4 \quad (35)$$

and the number of free parameters in the numerator of $Q_d(q)$, from (28), *is chosen as*

$$m = 3. \quad (36)$$

Therefore, the flexible reference model for disturbance is chosen with two flexible zeros as in

$$Q_d(q,\eta) = \begin{bmatrix} \eta_1 & \eta_2 & \eta_3 \end{bmatrix} \begin{bmatrix} \frac{q^2(q-1)}{(q-0.6)^4} \\ \frac{q(q-1)}{(q-0.6)^4} \\ \frac{(q-1)}{(q-0.6)^4} \end{bmatrix}. \quad (37)$$

Using the flexible VDFT method with the reference model in (37) *and a PID controller structure, we obtain*

$$Q_d(q,\hat{\eta}_4) = \frac{0.068235(q-1)(q-1.206)(q+1.675)}{(q-0.6)^4} \quad (38)$$

$$C(q, \hat{\rho}_4) = \frac{-7.2039(q - 0.7381)(q - 0.6287)}{q(q - 1)} \quad (39)$$

which resemble the results from Example 3, specially the tuned controller parameters. It is worth noticing that in Example 3, the process' model was assumed to be known for the reference model choice, whereas now this choice is embedded in the control design procedure as an identification problem, where one batch of collected data is used and only knowledge on the relative degree and on the number of non-minimum phase zeros of $G_2(q)$ is necessary.

The obtained closed-loop behavior compared with the desired one can be seen in Figure 5.

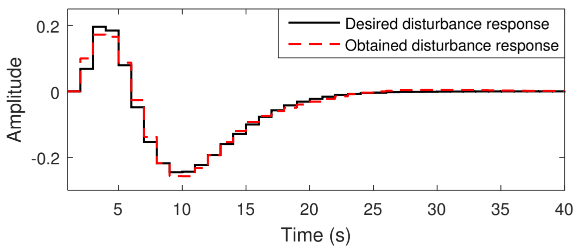

Figure 5: Comparison between desired and obtained closed-loop behavior, $Q_d(q, \hat{\eta}_4)$ and $Q(q, \hat{\rho}_4)$ respectively.

6 Conclusion

In this work, we explored the design of the reference model within a data-driven control framework for disturbance rejection. Contrary to the analog problem of reference tracking, the choice of a desired transfer function for disturbance can be quite challenging as it often demands knowledge on the process' model. To cope with this problem, a data-based strategy was adapted aiming to improve the choice of the reference model for disturbance by adding some degrees of freedom to its numerator. Some examples are presented in order to show the advantages of properly choosing the reference model in the convergence of the control method.

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