OUTPUT-FEEDBACK CONTROLLED-INVARIANT POLYHEDRA FOR CONSTRAINED UNCERTAIN LINEAR SYSTEMS

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Abstract— This paper presents a proposal of a robust output feedback controller to perform reference tracking, ensuring that the system complies with the constraints despite the effect of bounded disturbances and parameter uncertainties. The proposed technique is based on the calculation of invariant sets to ensure constraints satisfaction and a model update to reduce the tracking error of constant reference, through a system of inequalities. The numerical simulation results illustrate the efficiency of the proposed method.

Keywords— Output Feedback, Robust Control, Invariant Sets, Reference Tracking, Constraints.

Resumo— Este artigo apresenta a proposta de um controlador por realimentação de saída robusto para realizar o rastreamento de referência, garantindo que o sistema obedeça todas as restrições apesar do efeito de perturbações limitadas e incertezas nos parâmetros. A técnica proposta é baseada no calculo de conjuntos invariantes para garantir a satisfação das restrições e uma atualização no modelo para reduzir o erro do rastreamento de referência constante, através de um sistema de inequações. O resultado da simulação numérica demonstra a eficiência do método proposto.

Palavras-chave— Realimentação de Saída, Controle Robusto, Conjuntos Invariantes, Rastreamento de Referência, Restrições.

1 Introduction

A mathematical model of dynamic systems can be obtained by physical analysis of the process or by using systems identification techniques. Modeling is performed with the objective of representing the most relevant aspects of the dynamic behavior in a precise way, but it may have uncertainties. These uncertainties can be originated from various sources like modeling errors, parameter variations, non-modeled dynamics, etc (Kluever, 2015).

To accommodate the model uncertainties, robust controllers are used. Those have the capability of ensuring the performance and the stability of the system in face of uncertainties.

Beyond model uncertainties, another important characteristics on most of the practical applications, is the existence of constraints in the control, states and output variables. Sometimes these constraints are purposely placed with the objective of reducing the use of energy, minimizing resource utilization and to ensure that some variables will not go beyond critical values, for example so that a tank does not overflow.

The reference tracking problem for constrained systems is more complex, because it requires a controller that can lead system output to the reference values without violating the constraints (Blanchini and Miani, 2000).

The positive invariance approach has been used to solve these constrained control problems, in particular, using the invariant polyhedra concept, taking into consideration that linear constraints can be mathematically translated into the polyhedral form (Blanchini and Miani, 2015). A positive invariant set has the property that for any initial condition belonging to the set, the state trajectory will remain inside the same set all the time, ensuring that the system constraints will not be violated.

There is a rich literature on set-invariance under state feedback. On the other hand, few works have considered invariance under output feedback. In (Dórea, 2009) output-feedback-invariant ($OFCI$) sets were defined as sets where the state trajectories can be confined through a computed suitable sequence of control actions, which are based only in the measured outputs of the system. In (Artstein and Rakovic, 2011), set dynamics use past output measurements to reduce the set of possible states associated to the measurements.

The present work has the purpose of obtaining a solution to the problem of robust reference tracking of constrained linear systems via output feedback. To this end, conditions under which a given polyhedron is $OFCI$ with respect to the uncertain system are derived. The existence of such a set guarantees robust constraint satisfaction. In addition, in order to reduce the tracking error, a model update procedure is proposed, relying on the resolution of a set of linear inequalities.

Many works of constant reference tracking under constraints can be found in the Model Predictive control framework (Limon et al., 2008), most of them considering full state feedback and models without uncertainties. The works dealing with output feedback make use, in general, of previously designed linear state observers which can lead to small sets of admissible initial states (Dórea, 2009). In this work we propose an output-feedback solution for linear discrete-time systems that ensures constraint satisfaction, provides a large set of admissible initial states and tries to reduce as much as possible the tracking error, despite the uncertainties in the model.

This work is organized in 4 sessions: Introduction, Controlled-Δ-Invariant Sets, Simulations and Conclusion. The Controlled-Δ-Invariante sets session, the method proposed to achieve robust reference tracking under constraints is presented. Numerical simulations illustrating the effectiveness of the proposed approach are presented in the Simulation session. At last, Conclusions are drawn on the obtained results and future developments on the subject are proposed.

2 Controlled-Δ-Invariant Sets

2.1 Constrained Uncertain Linear Systems

Consider the following linear system subjected to parametric uncertainties described by equation (1):

$$\begin{cases} x(k+1) & = A(\underline{\alpha})x(k) + B(\underline{\beta})u(k) \\ y(k) & = Cx(k) \end{cases} \tag{1}$$

Uncertain matrices $A(\underline{\alpha})$ and $B(\underline{\beta})$ are linear functions of their parameters, so that:

$$A(\underline{\alpha}) = A_o + \alpha_1 A_1 + \cdots + \alpha_p A_p = A_o + \Delta A(\underline{\alpha}) \tag{2}$$

$$B(\underline{\beta}) = B_o + \beta_1 B_1 + \cdots + \beta_q B_q = B_o + \Delta B(\underline{\beta}) \tag{3}$$

The uncertain parameters $\underline{\alpha} \in \Re^p$ e $\underline{\beta} \in \Re^q$ belong to the ranges:

$$-\underline{\alpha}_M \leq \underline{\alpha} \leq \underline{\alpha}_M \tag{4}$$

$$-\underline{\beta}_M \leq \underline{\beta} \leq \underline{\beta}_M \tag{5}$$

The equations (4) and (5) define the upper and lower limits for each of the uncertain elements of $A(\underline{\alpha})$ and $B(\underline{\beta})$, which allows us to define two hypercubes, Δ_A and Δ_B:

$$\Delta_A \triangleq \left\{ A \in \Re^{n\times n}; \sum_{i=1}^{\eta_A} \xi_i A^i, \sum_{i=1}^{\eta_A} \xi_i = 1, \xi_i \geq 0 \right\} \tag{6}$$

$$\Delta_B \triangleq \left\{ B \in \Re^{n\times m}; \sum_{j=1}^{\eta_B} \mu_j B^j, \sum_{j=1}^{\eta_B} \mu_j = 1, \mu_j \geq 0 \right\} \tag{7}$$

Where $\eta_A = 2^p$ and $\eta_B = 2^q$. Then, $A(\underline{\alpha}) \in \Delta_A$ and $B(\underline{\beta}) \in \Delta_B$.

This system is subjected to state and control constraints:

$$\begin{cases} x(k) \in \Omega = \{x : Gx \leq 1\} \\ u(k) \in \nu = \{u : Uu \leq 1\} \end{cases} \tag{8}$$

It is desired to calculated a control sequence $u(k), k = 0, 1 \ldots$, such that:

$$\begin{cases} x(k) \in \Omega, \forall \alpha, \beta \ satisfying \ (4) \ and \ (5) \\ u(k) \in \nu \end{cases} \tag{9}$$

$$\lim_{k\to\infty} y(k) = r \tag{10}$$

Where r is the constant reference to be tracked.

2.2 Controlled Invariant Set

A subset in the state space is said to be positive invariant if all trajectories originated from a state in this subset remain in the same set. (Blanchini, 1994) defines the invariant sets for systems with uncertain models as follows:

Definition 1 *Given a set Ω where $\Omega \subset \Re^n$, Ω is said controlled Δ-invariant for the systems shown in the equations (1), (6), (7) if, $\forall x \in \Omega$, there exists a control vector, $u \in \nu$, such that $Ax + Bu \in \lambda\Omega, \forall A \in \Delta_A, \forall B \in \Delta_B$, with $0 \leq \lambda \leq 1$. Parameter λ stands for the contraction rate of the invariant set.*

The one-step admissible set is given by:

$$\zeta(\Omega, \Delta) = \{x \in \Re^n; \exists u \in \Re^m; Ax + Bu \in \Omega, \forall A \in \Delta_A, \forall B \in \Delta_B\} \tag{11}$$

In the polyhedral case, $\Omega = \{x : Gx \leq 1\}$. Consider the following matrices:

$$(GA)^\Delta = \begin{bmatrix} GA^1 \\ GA^2 \\ \vdots \\ GA^{\eta_A} \\ GA^1 \\ \vdots \\ GA^{\eta_A} \\ \vdots \\ GA^1 \\ \vdots \\ GA^{\eta_A} \end{bmatrix}, (GB)^\Delta = \begin{bmatrix} GB^1 \\ GB^1 \\ \vdots \\ GB^1 \\ GB^2 \\ \vdots \\ GB^2 \\ \vdots \\ GB^{\eta_B} \\ \vdots \\ GB^{\eta_B} \end{bmatrix}, \rho^\Delta = \begin{bmatrix} \rho \\ \rho \\ \vdots \\ \rho \\ \rho \\ \vdots \\ \rho \\ \vdots \\ \rho \\ \vdots \\ \rho \end{bmatrix} \tag{12}$$

Where, $G \in \Re^{g\times n}; (GA)^\Delta \in \Re^{(\eta_A\eta_B g)\times n}; (GB)^\Delta \in \Re^{(\eta_A\eta_B g)\times m}; \rho^\Delta \in \Re^{\eta_A\eta_B g}$.

Thus, the one-step admissible set is given by:

$$\zeta(\{x : Gx \leq 1\}, \Delta) = \left\{x; \exists u; (GA)^{\Delta}x + (GB)^{\Delta}u \leq \rho^{\Delta}\right\} \quad (13)$$

A control input that ensures the constraints satisfaction can be characterized as follows:

$$\begin{bmatrix} (GB)^{\Delta} \\ U \end{bmatrix} u(k) \leq \begin{bmatrix} \rho^{\Delta} - (GA)^{\Delta}x(k) \\ 1 \end{bmatrix} \quad (14)$$

2.3 Output feedback control

Not every system that is invariant controlled via state feedback is also via output feedback. However, its possible to discover if a set is output-feedback controlled invariant ($OFCI$).

Consider the system shown in the equation (1) and the set of admissible outputs associated with Ω:

$$Y(\Omega) = \{y : y = Cx \ for \ x \in \Omega\} \quad (15)$$

(Dórea, 2009) proposes a definition of $OFCI$ for system with known models. It is possible to extend this definition for uncertain models.

Definition 2 *Given a set* Ω *where* $\Omega \subset \Re^n$, Ω *is said to be output-feedback controlled-*Δ*-invariant (OFCI) with respect to the system in (1), if,* $\forall y \in Y(\Omega), \exists u \in \nu : A(\underline{\alpha})x + B(\underline{\beta})u \in \lambda\Omega, \forall x \in \Omega : Cx = y, \forall \alpha, \beta$ *satisfying* (4) *and* (5), $0 \leq \lambda \leq 1$.

Consider the closed and convex polyhedral sets containing the origin:

$$\Omega = \{x : Gx \leq 1\}, \nu = \{u : Uu \leq 1\} \quad (16)$$

Where $G \in \Re^{g \times n}$ and $V \in \Re^{v \times m}$. As matrix C in (1) is not uncertain, the set of admissible outputs, that is a closed and convex polyhedra that contains the origin, is given by:

$$Y(\Omega) = \{y : y = Cx \ for \ x : Gx \leq 1\} \quad (17)$$

Given the convex structure of Δ_A and Δ_B (6),(7), it is clear that Ω is $OFCI$ with contraction rate λ if, and only if:

$$\begin{aligned} &\forall y \in Y(\Omega), \exists u : G(A(\underline{\alpha})x + B(\underline{\beta})u) \leq \lambda 1, \\ &Uu \leq 1, \forall x : Cx = y, Gx \leq 1 \\ &\forall A(\underline{\alpha}) \in \Delta_A, B(\underline{\beta}) \in \Delta_B \end{aligned} \quad (18)$$

Define now the vectors ξ^{Δ} and ϕ^{Δ}, given by:

$$\begin{aligned} \xi_j^{\Delta *}(y) = arg \ max (GA)_j^{\Delta} x \\ subject \ to \ Gx \leq 1 \\ Cx = y \end{aligned} \quad (19)$$

$$\phi_j^{\Delta}(y) = (GA)_j^{\Delta} \xi_j^{\Delta *}(y) \quad (20)$$

Considering $\phi_{\eta_B}^{\Delta}(y) = \begin{bmatrix} \phi^{\Delta}(y) \\ \vdots \\ \phi^{\Delta}(y) \end{bmatrix}_{\eta_B \times 1}$ and $\xi_{\eta_B}^{\Delta}(y) = \begin{bmatrix} \xi^{\Delta}(y) \\ \vdots \\ \xi^{\Delta}(y) \end{bmatrix}_{\eta_B \times 1}$, that is, vectors of η_B repetitions of vector $\phi^{\Delta}(y)$ and the vector $\xi^{\Delta}(y)$.

Since the same input u must work $\forall x$ in Ω consistent with the output u, the worst case x can be computed row by row. Hence, the equation (18) is equivalent to:

$$\forall y \in Y(\Omega), \exists u : \begin{bmatrix} \phi_{\eta_B}^{\Delta}(y) \\ 0 \end{bmatrix} + \begin{bmatrix} (GB)^{\Delta} \\ U \end{bmatrix} u \leq \begin{bmatrix} \lambda 1 \\ 1 \end{bmatrix} \quad (21)$$

Consider now the following *pointed polyhedral cone*:

$$\Gamma = \left\{ \begin{bmatrix} t \\ w \end{bmatrix} \in \Re^{\eta_B g + v} : t, w \geq 0, [t^T w^T] \begin{bmatrix} (GB)^{\Delta} \\ U \end{bmatrix} = 0 \right\} \quad (22)$$

where $\left\{ \begin{bmatrix} T_i & W_i \end{bmatrix}^T, i = 1, \ldots, n_r \right\}$ form a minimal generating set of Γ. Since Γ is pointed, the elements of the minimal generating set are the extreme rays of Γ. So, any vector $\begin{bmatrix} t \\ w \end{bmatrix} \in \Gamma$ can be written as a positive linear combination of vectors $\begin{bmatrix} T_i & W_i \end{bmatrix}^T$.

Using Farkas' Lemma it is possible to write the equation (21) as: (Dórea, 2009)

$$\begin{bmatrix} T_i & W_i \end{bmatrix} \begin{bmatrix} \phi_{\eta_B}^{\Delta}(y) \\ 0 \end{bmatrix} \leq \begin{bmatrix} T_i & W_i \end{bmatrix} \begin{bmatrix} \lambda 1 \\ 1 \end{bmatrix}, \quad \forall y \in Y(\Omega), \forall i = 1, \cdots, n_r. \quad (23)$$

The following theorem establishes necessary and sufficient conditions for output feedback controlled invariance under model uncertainties:

Theorem 1 *The Polyhedral set* $\Omega = \{x : Gx \leq 1\}$ *is* $OFCI$ *with contraction rate* λ *if, and only if,* $\forall i = 1, \ldots, n_r$*:*

$$\sum_{j=1}^{\eta_B g} T_{ij} (GA)_j^{\Delta} \xi_{\eta_B j}^{\Delta} \leq (\sum_{j=1}^{\eta_B g} T_{ij}(\lambda)) + W_i 1, \quad (24)$$

$$\begin{aligned} \forall y, \xi_{\eta_B}^{\Delta}, j = 1, 2, \ldots, \eta_B g : G_{\eta_B} \xi_{\eta_B j}^{\Delta} \leq 1, \\ -G_{\eta_B} \xi_{\eta_B j}^{\Delta} + y \leq 1 \end{aligned} \quad (25)$$

The proof follows the same lines of (Dórea, 2009).

Proof: (if) Since (24) holds for all $y, \xi^{\Delta}_{\eta_B j}, j = 1, 2, \ldots, \eta_B g$ satisfying (25), then it clearly also does for $y, \xi^{\Delta *}_{\eta_B j}(y)$ in (19). Therefore, from (19) and (20), $\forall y \in Y(\Omega)$, condition (23) is satisfied, which proves that Ω is OFCI.

(only if) Assume, by contradiction, that $\sum_{j=1}^{\eta_B g} T_{ij}(GA)^{\Delta}_j \xi^{\Delta}_{\eta_B j} > (\sum_{j=1}^{\eta_B g} T_{ij}(\lambda)) + W_i 1$ for given i and $\xi^{\Delta}_{\eta_B j}$ satisfying (25). Then, from (19) and the fact that $T_ij \geq 0$, one has: $\sum_{j=1}^{\eta_B g} T_{ij}(GA)^{\Delta}_j \xi^{\Delta *}_{\eta_B j}(y) > (\sum_{j=1}^{\eta_B g} T_{ij}(\lambda)) + W_i 1$, which violates the necessary and sufficient condition (23), and proves that conditions (24), (25) are necessary as well. □

One can notice that such conditions are different from those presented in (Hempel et al., 2011) for linear parameter varying systems.

2.4 Constant Reference Tracking

Consider now a constant reference to be tracked, r. It is assumed that such a reference is admissible, that is, that the equilibrium point $(\overline{u}, \overline{x})$ corresponding to $y = r$ satisfies the state and control constraints.

In order to calculate the control signal (u) that leads the system output to the reference value we use the one-step ahead error given by $e(k+1) = r - y(k+1)$, where $y(k+1) = Cx(k+1) = C(Ax(k) + Bu(k))$. Thus:

$$e(k+1) = r - CAx(k) - CBu(k) \tag{26}$$

Since the system described in (1) has uncertainties in A and B, we use as initial value the nominal values, A_o and B_o ((2) and (3)) to evaluate the one-step ahead tracking error.

We assume we are given an $OFCI$ polyhedron contained in the set of state constraints. Even though no method is available to directly compute such a set in the general case, we point out that if the system is open-loop robustly stable, the maximal positive invariant set contained in the set of constraints is clearly $OFCI$, as long as $u = 0$ belongs to the set of input constraints. If the system is not open-loop stable, the following strategy can be used: Compute the maximal controlled-invariant set contained in the set of constraints and check if it is $OFCI$ If not, compute a stabilizing state feedback and compute the maximal positive invariant set with respect to the prestabilized system. The computation of maximal invariant sets can be performed using the algorithms proposed in (Dórea and Hennet, 1999).

Since it is not possible to obtain the values of the state variables then the worst case of these variables is estimated using the equations (19) and (20). In addition to these vectors we define another two vectors, γ_o^+ and γ_o^-.

Consider the following constraint $|Cx(k+1) - r| \leq e$, written in terms of the nominal model $x(k+1) = A_o x(k) + B_o u(k)$, resulting in:

$$\begin{cases} CA_o x(k) + CB_o u(k) \leq r + e \\ -CA_o x(k) - CB_o u(k) \leq r - e \end{cases} \tag{27}$$

Just like in (19),(20), it is possible to calculate the vector $\gamma(y(k))$ that represents the worst case for $\begin{bmatrix} CA_o \\ -CA_o \end{bmatrix} x(k)$, row by row:

$$\kappa_{jo}(y) = arg\ max \begin{bmatrix} CA_o \\ -CA_o \end{bmatrix}_j x \tag{28}$$
$$subject\ to\ Gx \leq 1$$
$$Cx = y$$

Thus, we obtain:

$$\gamma_o^+(y) = CA_o \kappa_{1o}(y) \tag{29}$$

$$\gamma_o^-(y) = -CA_o \kappa_{2o}(y) \tag{30}$$

State and control constraints 21 and tracking error bound e 27 can be written as the following set of inequalities:

$$\begin{bmatrix} (GB)^{\Delta} & 0 \\ U & 0 \\ CB_o & -1 \\ -CB_o & -1 \end{bmatrix} \begin{bmatrix} u(k) \\ e \end{bmatrix} \leq \begin{bmatrix} 1 - \phi^{\Delta}_{\eta_B} \\ 1 \\ r - \gamma_o^- \\ -r - \gamma_o^+ \end{bmatrix} \tag{31}$$

A control signal that ensures constraints satisfaction and minimizes the one-step ahead tracking error can be calculated by solving the following linear programming problem:

$$\begin{aligned} &\min_{u(k),e}\ e \\ &subject\ to \\ &\qquad (GB)^{\Delta} u(k) \leq 1 - \phi^{\Delta}_{\eta_B} \\ &\qquad Uu(k) \leq 1 \\ &\qquad CB_o u(k) - e \leq r - \gamma_o^- \\ &\qquad -CB_o u(k) - e \leq -r + \gamma_o^+ \end{aligned} \tag{32}$$

The solution of this problem guarantees robust constraint satisfaction. However, it does not guarantee robust reference tracking, for two reasons:

1) The state is not precisely known; 2) The system model is not precisely known.

In order to cope with the second difficulty we propose a model update method which recalculates the values of the parameters of A and B when it is detected that there is a difference between the nominal model output (y_o) and the real process measured output (y_m), that is, $|y_o - y_m| \geq \chi$, where χ is a given tolerance.

If the model was perfect we would have:

$$y_m(k) = Cx(k) \tag{33}$$

Where $y_m(k)$ is the measured output given by:

$$y_m(k) = CA(\underline{\alpha})x(k-1) + CB(\underline{\beta})u(k-1) \tag{34}$$

Since the state x is not known we use the estimate of the worst case $\kappa(y(k-1))$ (28).Thus we obtain the following equation:

$$y_m(k) = CA(\underline{\alpha})\kappa(y(k-1)) + CB(\underline{\beta})u(k-1) \tag{35}$$

In order for the system state to remain within the controlled invariant polyhedra, in the model update process, constraints must be added to ensure that the updated model matrices A and B belongs respectively to Δ_A and Δ_B (see equations (6) and (7)) that is $\underline{\alpha}$ and $\underline{\beta}$ are still limited by the equations (4) and (5). Then, parameters α and β that best match the measured output can be computed via the solution of the following linear programming problem:

$$\begin{aligned}
&\min_{\underline{\alpha},\underline{\beta},\epsilon} \ \epsilon \\
&\textit{subject to} \\
&\quad CA(\underline{\alpha})\kappa_1(y(k-1)) + CB(\underline{\beta})u(k-1) - y_m \le \epsilon \\
&\quad -CA(\underline{\alpha})\kappa_2(y(k-1)) - CB(\underline{\beta})u(k-1) + y_m \le \epsilon \\
&\quad -\underline{\alpha}_M \le \underline{\alpha} \le \underline{\alpha}_M \\
&\quad -\underline{\beta}_M \le \underline{\beta} \le \underline{\beta}_M
\end{aligned} \tag{36}$$

By minimizing the ϵ it is possible to obtain the values of $\underline{\alpha}$ and $\underline{\beta}$ that best match the measured output.

3 Simulations

Suppose that the matrices of the model are, $A_m = \begin{bmatrix} 0.9347+\alpha & 0.5194 \\ 0.33835 & 0.831 \end{bmatrix}$, $B_m = \begin{bmatrix} -1.4462 \\ -0.7012+\beta \end{bmatrix}$ and $C_m = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}$, where $-0.03 \le \alpha \le 0.03$ and $-0.05 \le \beta \le 0.05$ and the parameters of the real system are, $A = \begin{bmatrix} 0.9647 & 0.5194 \\ 0.33835 & 0.831 \end{bmatrix}$, $B = \begin{bmatrix} -1.4462 \\ -0.7512 \end{bmatrix}$ and $C = C_m$, so $\alpha = 0.03$ and $\beta = -0.05$. The system is subject to following states and control constraints:

$$|x_j(k)| \le 4, |u(k)| \le 1 \tag{37}$$

Calculating the λ-contractive controlled Δ-invariant polyhedra to the constraints with $\lambda = 0.99$, we obtain:

$$G = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \\ 0.1510 & 0.1597 \\ -0.1510 & -0.1597 \end{bmatrix}, \rho_f = \begin{bmatrix} 4 \\ 4 \\ 4 \\ 4 \\ 0.9604 \\ 0.9604 \end{bmatrix} \tag{38}$$

After calculating the Maximum Controlled Δ-Invariant Polyhedra it was determined from application of **Theorem 1** that the polyhedron is $OFCI$. Thus, the system is controllable as output feedback. In figures 1-3 are shown simulation results with $r = 3$ and a parameter variation at the instant $k = 10$, where $A = \begin{bmatrix} 0.9047 & 0.5194 \\ 0.33835 & 0.831 \end{bmatrix}$ and $B = \begin{bmatrix} -1.4462 \\ -0.6512 \end{bmatrix}$, that is $\alpha = -0.03$ and $\beta = 0.05$.

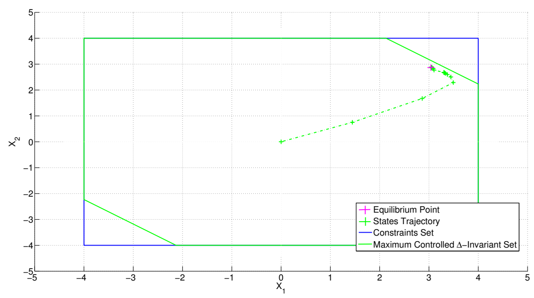

Figure 1: State Trajectory

In 1 is possible to see the state trajectory inside the invariant set, showing the respect of state constraints.

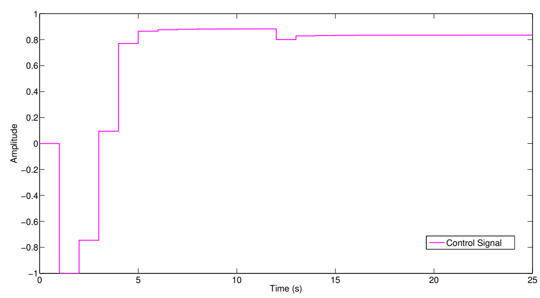

Figure 2: Control Signal

In 2 is noticeable that the control signal respects the constraints as well.

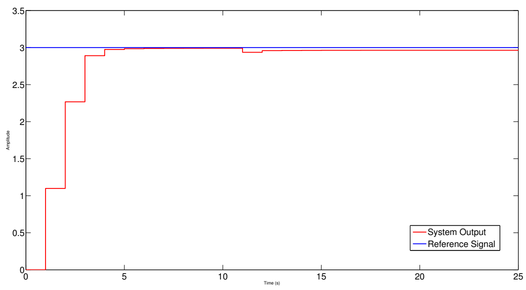

Figure 3: Reference Tracking

Observing the figure 3 we can see that with the parameter variation, the controller reduces the tracking error. However, the reference value is not reached, because the controller has to take the whole set of admissible states into account due to the model update procedure.

4 Conclusion

The main objective of this work was to propose a solution to the problem of robust reference tracking of constrained linear systems via output feedback. As it can be seen in the simulations results, it was possible to reduce the tracking error while still satisfying the constraints on state and control variables. To this end, the concept of output-feedback controlled-invariant sets was extended to uncertain systems and conditions were proposed to check this property for polyhedral sets.

As it could be observed in the simulations, the use of invariant sets is quite efficient in the control of constrained linear systems including the cases with uncertainties. Systems with incomplete state measurement and model uncertainties are hard to control, even so the proposed method resulted in the reduction of the tracking error and the satisfaction of the constraints. This method needs to use optimization techniques for the calculation of the control input and the model update of the system, which may require a great computational effort. Depending on the time for calculation, it may be impracticable to use this control procedure, especially for very fast processes.

In future works, the reduction of the tracking error will be investigated through the reduction of the set of admissible states associated to the measured output, using the ideas of set-valued observers (Shamma and Tu, 1999) and set-dynamics (Artstein and Rakovic, 2011).

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