

ROBUST MIMO SMITH PREDICTOR TUNING VIA CONVEX OPTIMIZATION

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Abstract— The basic principles of a multivariable Smith Predictor controller tuning method for stable processes is presented. Originally developed for PID tuning, this automatic method formulates the tuning procedure as a convex optimization problem, where convergence to a local minimum is guaranteed. It can be readily extended in many ways to more complex applications. In this paper, besides adapting the algorithm to a Smith Predictor controller, an additional constraint regarding disturbance rejection is added, and its robustness is increased via polytopic approach. The method is tested on simulated multivariable processes with constant transport delays.

Keywords— Smith Predictor, convex optimization, robust control, polytopic approach.

Resumo— Os princípios básicos de um método de sintonia de controladores multivariáveis baseados em Preditor de Smith para processos estáveis são apresentados. Originalmente desenvolvido para sintonia de controladores PID, este método automático formula o procedimento de sintonia como um problema de otimização convexa, onde a convergência para um mínimo local é garantida. Ele pode ser facilmente estendido de várias maneiras, para aplicações mais complexas. Neste trabalho, além de adaptar o algoritmo para um controlador baseado no Preditor de Smith, uma restrição adicional relacionada à rejeição de perturbações é adicionada, e sua robustez é melhorada através de abordagem politópica. O método é testado em processos multivariáveis simulados com atraso constante.

Palavras-chave— Preditor de Smith, otimização convexa, controle robusto, abordagem politópica.

1 Introduction

Many processes in industry (e.g. heating, manufacturing supply chain, some chemical and flow processes) present dead-times in their dynamics, which may be caused by the flow of information, energy or mass, among other reasons. For them, every action executed in the manipulated variable of the process will only affect the controlled variable after the process dead-time. Because of this, analysing and designing controllers for dead-time systems is more difficult.

For processes with significant dead-time, the Smith Predictor - presented at the end of the 1950s in Smith (1957) - is the automatic control strategy most widely used in practice, specially because of its simplicity. Being a predictive controller, the Smith Predictor includes a model of the process in the structure of the controller, in order to cope with the dead-time.

Multiple-input multiple-output (MIMO) processes with reasonably well decoupled dynamics can be controlled through single-input single-output (SISO) controllers. When this is not the case, MIMO controller design should be used. The controller parameters to tune then become matrices. As the number of process inputs and outputs increases, it becomes very difficult, if not impossible, to tune the controller manually.

The researches on automatic tuning techniques did start some decades ago, linked to adaptive control strategies. Some examples are the self-tuning techniques for SISO processes with

PID controller parameters obtained as functions of identified process parameters (Warwick and Kang (1998), Yamamoto et al. (1999), He and Xu (2008)).

The advance of computational power allowed the development of automatic tuning techniques, with recent algorithms capable of handling hundreds of constraints in the same optimization problem. SISO PID tuning is obtained via heuristic optimization in Mercorelli (2015), optimized pole placement in Ciganek et al. (2015), Fuzzy pole placement in Dey and Ayyagari (2016), genetic algorithms in de Castro et al. (2016) and Puchta et al. (2016), data-driven design in Tesch et al. (2016) and Tanaskovic et al. (2015), not to mention other techniques.

When it comes to MIMO controller tuning, the number of available techniques reduce drastically. MIMO PID is obtained via convex optimization in Boyd et al. (2016) and extremum seeking in Oliveira et al. (2014). Finally, a MIMO Smith Predictor tuning technique via convex optimization is presented in Nicoletti and Karimi (2016).

The main idea of this paper is to propose a practical and easily comprehensible method for automatic robust tuning of MIMO Smith Predictor controllers, with constraint satisfaction. As it relies on convex optimization algorithms, which can solve practical high-dimension problems in a reliable and efficient way (Hindi (2004)), this method would be of great interest for application on processes with high number of inputs and/or outputs, where controller tuning might be consid-

ered a challenge.

The rest of the paper is organized as follows. The original MIMO PID tuning method is briefly described in Section 2. Section 3 describes the extension of this method to MIMO Smith Predictor controllers, the inclusion of a new constraint related to disturbance rejection, and the improvement of its robustness via polytopic approach. Finally, simulation examples are presented in Section 4.

2 PID design via convex optimization

In the PID design method proposed by Boyd et al. (2016), process and controller are connected within a classical feedback control loop as shown in Figure 1, where r is the reference input, e is the tracking error and y is the output.

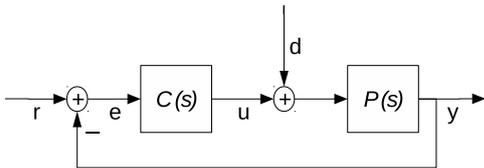


Figure 1: Classical feedback controller.

P is the process transfer function and the primary controller C used is a parallel PID given by Equation 1,

$$C(s) = K_P + \frac{1}{s}K_I + \frac{s}{1 + \tau s}K_D \quad (1)$$

where $K_P, K_I, K_D \in R^{m \times p}$ are the proportional, integral and derivative gain matrices, respectively.

It consists of a simple, automated and practical tuning method, that relies initially on the following assumptions:

- The MIMO process P is linear time-invariant and has m inputs (actuators) and p outputs (sensors).
- $p \leq m$
- Process transfer function P is known.
- Process is stable and strictly proper, that is, $P(s) \rightarrow 0$ as $s \rightarrow \infty$.
- The process may include transport delay.
- Inputs and outputs are already scaled¹.
- Controller parameter $\tau > 0$ is the derivative action time constant, which is assumed to be fixed and accordingly chosen.

¹Normalization is not performed in this work, but it can be done.

According to Boyd et al. (2016), upon further development, some assumptions, among others, might be relaxed and other objectives reached:

- Other objectives and constraints could be used (e.g. main optimization objective could be disturbance rejection, instead of reference tracking).
- Other closed-loop transfer functions could be used.
- Other variations of linearly parametrized controllers and PID's could be used.
- Method could be applied on unstable plants, as long as the initial controller stabilized and also satisfied the constraints.
- Process with $p > m$ (more outputs than actuators) could be controlled, nonetheless without perfect static tracking.
- Robustness to plant variations could be achieved through polytopic approach.

Some of these ideas were explored in this paper, in order to achieve practical and robust tuning of predictive controllers for stable linear processes, specially the ones with large dead-times and higher dimensions (high number of inputs and/or outputs).

3 Smith Predictor design via convex optimization

In this section, the method proposed in Boyd et al. (2016) is extended to a predictive control strategy. Control architectures based on predictors, as the Smith Predictor, represent generally a natural choice to processes with significant dead-time (transport delay). Figure 2 shows the architecture for MIMO processes with multiple dead-times (Normey-Rico and Camacho (2007)), used throughout this paper.

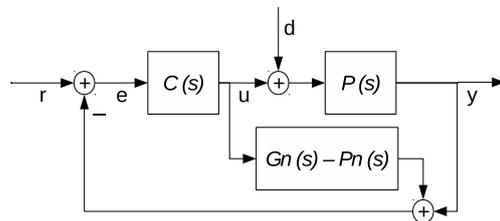


Figure 2: Smith Predictor controller.

The following closed loop transfer functions are obtained, with $P_n(s)$ representing the model of the process with multiple dead-times, and G_n representing the process P_n without dead-time (as in the *full DTC fast model* presented in Santos et al. (2014)):

- From reference input r to tracking error e :
 $S = (I + G_n C)^{-1}$
- From reference input r to output y :
 $T = P_n C (I + G_n C)^{-1}$
- From reference input r to control signal u :
 $Q = C (I + G_n C)^{-1}$
- From disturbance d to tracking error e ²:
 $R = -P_n (I + C G_n)^{-1}$

Restrictions are imposed to guarantee closed-loop stability and reduced control effort. The objective is to attain the best possible low-frequency sensitivity S , which means $\|P_n(0)K_I\|^{-1}$ will be minimized. The design problem then becomes:

$$\begin{aligned} & \text{minimize} && \|P_n(0)K_I\|^{-1} \\ & \text{subject to} && \|S\|_\infty \leq S_{max}, \\ & && \|T\|_\infty \leq T_{max}, \\ & && \|Q\|_\infty \leq Q_{max}, \\ & && \|R\|_\infty \leq R_{max}, \end{aligned} \quad (2)$$

Design parameters S_{max} , T_{max} , Q_{max} and R_{max} must be provided. The variables to be found are the coefficient matrices K_P , K_I and K_D . *This problem is not convex.*

Then, the constraints are expressed as semi-infinite constraints (e.g. $\|S(i\omega)\| \leq S_{max}$), consisting of an infinite number of constraints (one for each $\omega \geq 0$). They can be handled by choosing a reasonable finite (but large) set of frequencies samples $0 < \omega_1 < \dots < \omega_k$, and replacing the semi-infinite constraints with the finite set of constraints at each of the given frequencies (e.g. $\|S(i\omega_k)\| = \|S_k\| \leq S_{max}, k = 1, \dots, N$). Figure 3 shows an example of closed-loop transfer function T respecting T_{max} over all discrete frequencies.

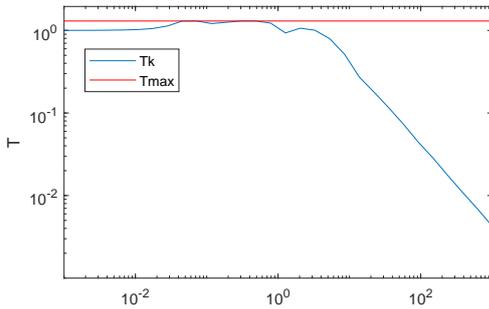


Figure 3: Constraint $\|T\|_\infty \leq T_{max}$.

The optimization computational effort then grows linearly with N , which permits that a large – but reasonable – value of N be chosen. The

²The transfer function R might include a robustness filter (Normey-Rico and Camacho (2009)) and allow the control of unstable processes, which will be subject of future developments.

frequency sampling must be fine enough to catch any rapid changes in the closed-loop transfer function with frequency, and also cover an appropriate range. The sampled problem, with $4 \times N$ constraints, becomes:

$$\begin{aligned} & \text{minimize} && \|P_n(0)K_I\|^{-1} \\ & \text{subject to} && \|S_k\| \leq S_{max}, \\ & && \|T_k\| \leq T_{max}, \\ & && \|Q_k\| \leq Q_{max}, \\ & && \|R_k\| \leq R_{max}, \\ & && k = 1, \dots, N \end{aligned} \quad (3)$$

This problem is then converted to quadratic matrix inequalities (QMI), where new variables are introduced, the objective function and every constraint has the form $Z^*Z \succeq Y^*Y$, and both Z and Y are affine functions of the variables:

$$\begin{aligned} \min \|P_n(0)K_I\|^{-1} \Rightarrow & Z = P_n(0)K_I \\ & Y = tI \\ \|S_k\| \leq S_{max} \Rightarrow & Z = I + G_{nk}C_k \\ & Y_s = (1/S_{max})I \\ \|T_k\| \leq T_{max} \Rightarrow & Z = I + G_{nk}C_k \\ & Y_t = (1/T_{max})P_{nk}C_k \\ \|Q_k\| \leq Q_{max} \Rightarrow & Z = I + G_{nk}C_k \\ & Y_q = (1/Q_{max})C_k \\ \|R_k\| \leq R_{max} \Rightarrow & Z_r = I + C_kG_{nk} \\ & Y_r = (1/R_{max})P_{nk} \end{aligned} \quad (4)$$

The QMI problem has the form:

$$\begin{aligned} & \text{maximize} && t \\ & \text{subject to} && Z_k^*Z_k \succeq Y_k^*Y_k \\ & && k = 1, \dots, M \end{aligned} \quad (5)$$

with $M = 4 \times N + 1$, and where Z^* denotes the Hermitian conjugate transpose of matrix Z .

The QMI is already convex in Y . Introducing an arbitrary matrix \tilde{Z} , the matrix inequality $Z^*\tilde{Z} + \tilde{Z}^*Z - \tilde{Z}^*\tilde{Z} \succeq Y^*Y$, which is convex in (Z, Y) , represents a convex restriction of the QMI obtained at the point \tilde{Z} .

The problem finally becomes:

$$\begin{aligned} & \text{maximize} && t \\ & \text{subject to} && \begin{bmatrix} Z_k^*\tilde{Z}_k + \tilde{Z}_k^*Z_k - \tilde{Z}_k^*\tilde{Z}_k & Y_k^* \\ Y_k & I \end{bmatrix} \succeq 0 \\ & && k = 1, \dots, M \end{aligned} \quad (6)$$

This problem has linear objective and LMI constraints, and so it is a semidefinite program (SDP). The optimization algorithm is initialized with values of K_P , K_I and K_D , and at each iteration the LMI restrictions are formed using the current value of Z_k as Z_k^* . The iterations can stop when not much progress is being made. Convergence to a local minimum is guaranteed, as the iterates are all feasible, there is closed-loop stability

since $\|Q\|_\infty$ is finite, the objective is nonincreasing and nonnegative.

3.1 Robust design via polytopic approach

Up to this moment, the controller tuning method has not taken into account process uncertainties (e.g. variations on transport delay, static gain or response time). If these variations are considerable, the dynamics of the process nominal model P_n used for the controller tuning becomes much different than that of the real plant. As a consequence, the response of the real plant in closed loop may violate the constraints or even become unstable.

To prevent this, several process models P_L may be provided with the nominal model for the optimization problem (6). The number of constraints increase linearly, as now $M = (4 \times N \times L) + 1$. But if a solution is found, it will be valid for all processes located within the polytope formed with P_L on its vertices.

To resume, instead of designing the controller based only on the nominal plant model, the design will require that the constraints hold also for several or many plausible values of the plant transfer function.

4 Numerical examples

In this section, the developments described in Section 3 are applied to classic MIMO processes. The simulations were performed in Matlab, with the optimization algorithm using the convex framework CVX and the SDPT3 solver.

4.1 Wood-Berry distillation column

The Wood-Berry binary distillation column, described in Wood and Berry (1973), is a two-input two-output stable process with quite coupled dynamics. The process transfer function is

$$P(s) = \begin{bmatrix} \frac{12.8e^{-1s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21.0s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.2s+1} \end{bmatrix}$$

It must be mentioned that the same benchmarking process used in the previous developments was used, to ease the comparisons.³

The design parameters used were:

$$S_{max} = 1.4, T_{max} = 1.4, Q_{max} = 0.738$$

$$\tau = 0.3, N = 300, \omega \in [10^{-3}, 10^3] \text{ rad/s}$$

³As this initial development was based on a PID-tuning method, this process does not present a significant dead-time, where the Smith Predictor structure would be more useful. The application on processes with higher dead-times will be presented later in this paper.

4.1.1 Smith Predictor tuning

The PID tuning found in Boyd et al. (2016) was:

$$K_P = \begin{bmatrix} 0.1750 & -0.0470 \\ -0.0751 & -0.0709 \end{bmatrix}$$

$$K_I = \begin{bmatrix} 0.0913 & -0.0345 \\ 0.0402 & -0.0328 \end{bmatrix}$$

$$K_D = \begin{bmatrix} 0.1601 & -0.0051 \\ 0.0201 & -0.1768 \end{bmatrix}$$

The algorithm converged in 7 iterations and took 18 seconds to run in our standard computer with an Intel Core I7 processor.

The same design parameters were applied to the Smith Predictor tuning, and the results obtained were:

$$K_P = \begin{bmatrix} 0.6127 & -0.1596 \\ -0.5277 & -0.6242 \end{bmatrix}$$

$$K_I = \begin{bmatrix} 0.1243 & 0.0240 \\ 0.0667 & -0.0088 \end{bmatrix}$$

$$K_D = \begin{bmatrix} -0.0118 & -0.0352 \\ 0.1443 & 0.0001 \end{bmatrix}$$

The algorithm converged in 12 iterations and took 30 seconds to run.

Figures 4 and 5 compare the responses of the PID and the Smith Predictor. They do not show a great difference in the performance because, as mentioned earlier, the process does not present a significant dead-time. This example was intentionally chosen, in order to validate the algorithm after the modifications in the closed loop transfer functions.

4.1.2 Disturbance rejection

Figures 6 and 7 compare the responses of the Smith Predictor with different values of the constraint R_{max} . They show how the response to a disturbance may vary without modifying significantly the reference tracking (as S_{max} and T_{max} were kept constant).

4.1.3 Robustness to process uncertainties

The robustness to process uncertainties was evaluated under the following significant variations:

- +100% on the process dead-times;
- $\pm 20\%$ on the process static gains;
- $\pm 70\%$ on the process time constants.

Figure 8 shows the step response, in closed loop, of the Wood-Berry process under the above mentioned variations, but controlled by a Smith Predictor tuned considering only the nominal process. It is an example of how constraints - T_{max} in

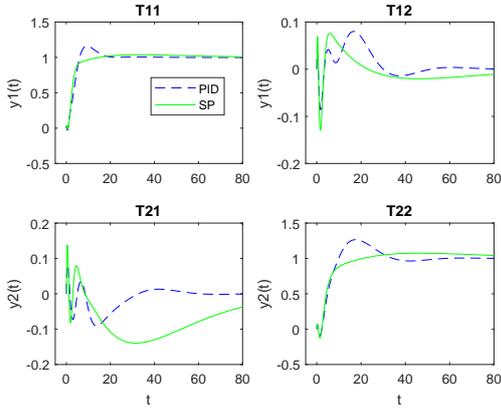


Figure 4: PID and Smith Predictor output.

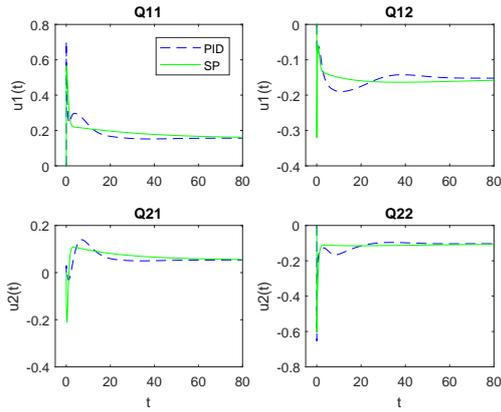


Figure 5: PID and Smith Predictor control signal.

this case - could be violated in some cases. Some variations could even cause unstable behavior.

The results obtained for a Smith Predictor tuning via polytopic approach (with $S_{max} = 1.4$ and $T_{max} = 1.4$) were:

$$K_P = \begin{bmatrix} 0.4386 & 0.0142 \\ -0.5699 & -0.6132 \end{bmatrix}$$

$$K_I = \begin{bmatrix} 0.0525 & 0.0141 \\ 0.0288 & -0.0017 \end{bmatrix}$$

$$K_D = \begin{bmatrix} 0.0435 & -0.1038 \\ 0.1661 & 0.0291 \end{bmatrix}$$

The algorithm converged in 13 iterations and took 170 seconds to run, much longer compared to previous tunings because the number of constraints in the optimization problem is multiplied by the number of uncertainties (five in this case).

Figure 9 shows the step response, in closed loop, of the Wood-Berry process under variations controlled by the polytopic Smith Predictor. It can be seen that, the controller not only stabilizes the system, but respects the constraint T_{max} (as long as the other constraints).

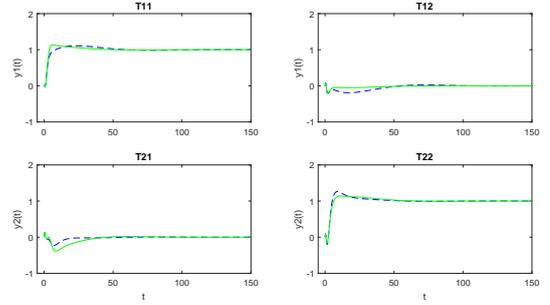


Figure 6: Reference tracking for different R_{max} .

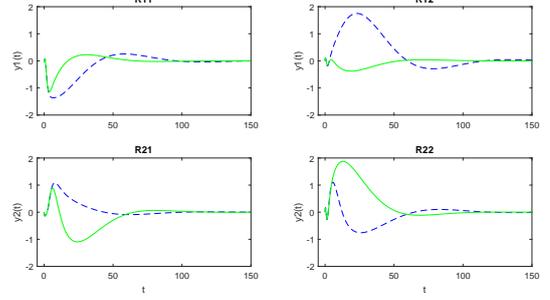


Figure 7: Disturbance rejection for different R_{max} .

4.2 Shell process 2×3

The industrial Shell problem, described in Rao and Chidambaram (2006), presents a bigger control challenge, as it is a highly coupled system with large dead-times. This heavy oil fractionator is a process mostly used in the petrochemical industry. The simplified 2×3 stable process transfer function, with augmented dead-times⁴ and time scale in minutes, is

$$P(s) = \begin{bmatrix} \frac{4.05e^{-81s}}{50s+1} & \frac{1.77e^{-84s}}{60s+1} & \frac{5.88e^{-81s}}{50s+1} \\ \frac{5.39e^{-54s}}{50s+1} & \frac{5.72e^{-42s}}{60s+1} & \frac{6.90e^{-45s}}{40s+1} \end{bmatrix}$$

The same process was simulated in Nicoletti and Karimi (2016), using a method similar to the one presented in this paper. In resume, their method also designs Smith Predictor controllers via convex optimization, respecting H_∞ robust performance, but prioritizing controllers that decouple the MIMO system.

The simulation considered a possible dead-time variation of 20%, and the design parameters used were:

$$S_{max} = 1.2, T_{max} = 1.2, Q_{max} = 0.5, R_{max} = 5.0$$

$$\tau = 5.0, N = 200, \omega \in [10^{-4}, 10^1] \text{ rad/min}$$

⁴In this case, just a part of the process is simulated, and the dead-times are augmented to emphasize the effect of the compensator.

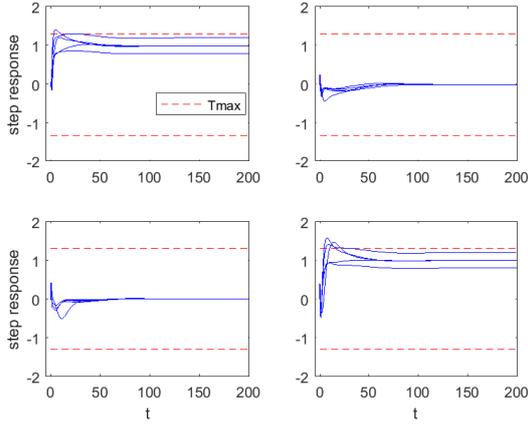


Figure 8: Step response of processes with uncertainties and constraint T_{max} .

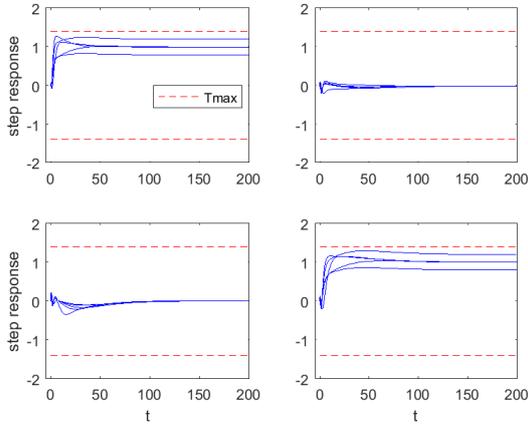


Figure 9: Step response of robust Smith Predictor.

The Smith Predictor tuning obtained in this paper was:

$$K_P = \begin{bmatrix} 0.1328 & 0.0217 \\ -0.3028 & 0.2757 \\ 0.2902 & -0.0114 \end{bmatrix}$$

$$K_I = \begin{bmatrix} 0.0016 & 0.0006 \\ -0.0049 & 0.0042 \\ 0.0037 & -0.0005 \end{bmatrix}$$

$$K_D = \begin{bmatrix} 0.7481 & 1.2513 \\ 0.2559 & 0.5820 \\ -0.0004 & 0.1903 \end{bmatrix}$$

Figures 10 and 11 compare the responses obtained in this paper and in Nicoletti and Karimi (2016) (with a PI as primary controller). The results are very similar, but the way they were obtained differ in some terms. The method proposed in this paper does not focus on decoupling the system, but obtain the same results without specifying desired closed-loop dynamics, with a simple cost function and defining closed-loop constraints more straightforwardly.

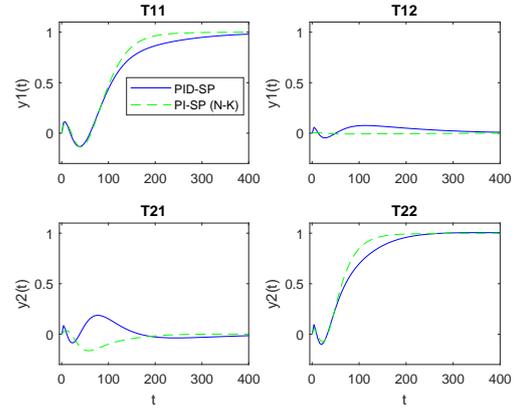


Figure 10: Smith Predictor reference tracking.

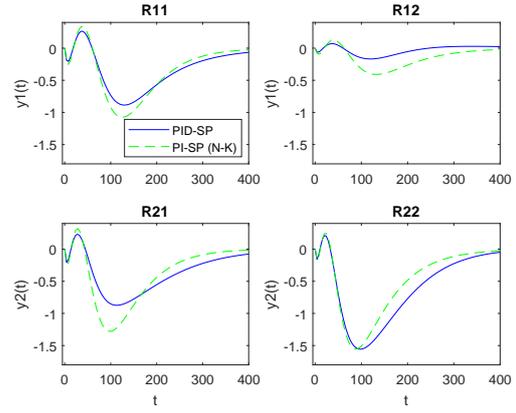


Figure 11: Smith Predictor disturbance rejection.

4.3 Shell process 7×3

A more detailed transfer function of the Shell process, with 3 inputs, 7 outputs and smaller dead-times (Prett and Morari (1987)), is represented below⁵.

$$P(s) = \begin{bmatrix} \frac{4.05e^{-27s}}{50s+1} & \frac{1.77e^{-28s}}{60s+1} & \frac{5.88e^{-27s}}{50s+1} \\ \frac{5.39e^{-18s}}{50s+1} & \frac{5.72e^{-14s}}{60s+1} & \frac{6.90e^{-15s}}{40s+1} \\ \frac{3.66e^{-2s}}{9s+1} & \frac{1.65e^{-20s}}{30s+1} & \frac{5.53e^{-2s}}{40s+1} \\ \frac{5.92e^{-11s}}{12s+1} & \frac{2.54e^{-12s}}{27s+1} & \frac{8.10e^{-2s}}{20s+1} \\ \frac{4.13e^{-5s}}{8s+1} & \frac{2.38e^{-7s}}{19s+1} & \frac{6.23e^{-2s}}{10s+1} \\ \frac{4.06e^{-8s}}{13s+1} & \frac{4.18e^{-4s}}{33s+1} & \frac{6.53e^{-1s}}{9s+1} \\ \frac{4.38e^{-20s}}{33s+1} & \frac{4.42e^{-22s}}{44s+1} & \frac{7.20}{19s+1} \end{bmatrix}$$

⁵The measurable disturbances are not considered in this paper.

The design parameters used were:

$$S_{max} = 1.2, T_{max} = 1.2, Q_{max} = 0.5, R_{max} = 10.0$$

$$\tau = 0.5, N = 50, \omega \in [10^{-4}, 10^4] \text{ rad/min}$$

The Smith Predictor tuning obtained was:

$$K_P = \begin{bmatrix} 0.0662 & -0.0591 & 0.0837 \\ 0.0184 & 0.2702 & 0.0280 \\ 0.2216 & -0.0867 & 0.0074 \\ 0.2539 & -0.1396 & 0.0807 \\ 0.0985 & -0.0469 & 0.1140 \\ -0.0564 & 0.1363 & 0.1509 \\ -0.1045 & 0.1553 & 0.1632 \end{bmatrix}^T$$

$$K_I = 10^{-3} \times \begin{bmatrix} -0.1326 & -0.0245 & 0.0788 \\ 0.0513 & 0.0267 & -0.0563 \\ 0.0533 & 0.0318 & -0.0363 \\ 0.0669 & -0.0419 & 0.0202 \\ -0.0660 & 0.0083 & 0.0051 \\ -0.0158 & 0.0227 & -0.0270 \\ -0.0371 & 0.0240 & 0.0645 \end{bmatrix}^T$$

$$K_D = \begin{bmatrix} -0.0262 & 0.0308 & -0.0312 \\ 0.0023 & -0.1167 & 0.0112 \\ -0.0619 & 0.0472 & 0.0026 \\ -0.0731 & 0.0749 & -0.0070 \\ -0.0044 & 0.0310 & -0.0030 \\ 0.0474 & -0.0568 & -0.0054 \\ 0.0576 & -0.0662 & -0.0365 \end{bmatrix}^T$$

In this case, as the number of outputs is higher than the number of actuators ($p > m$), perfect static tracking cannot be achieved. The following reference intervals are provided:

- $y_1, y_2 \in [0, 0.5]$
- $y_3, y_4, y_5, y_6 \in [-0.5, 0.5]$
- $y_7 \in [-0.5, 0]$

Figure 12 shows that the Smith Predictor obtained maintains the outputs within the desired intervals.

5 Conclusions

This paper presented an automated tuning algorithm for controllers based on the Smith Predictor structure. The algorithm relies on a convex optimization problem to obtain the PID matrices K_P , K_I and K_D for linear stable MIMO processes, with or without multiple dead-times, respecting given constraints.

The algorithm was extended to include constraints on the disturbance rejection, and to be more robust to process uncertainties (by means of polytopic approach).

The algorithm was applied on two simulated benchmarking processes: the Wood-Berry distillation column and the Shell fractionator. Compared to a previous method, similar performance

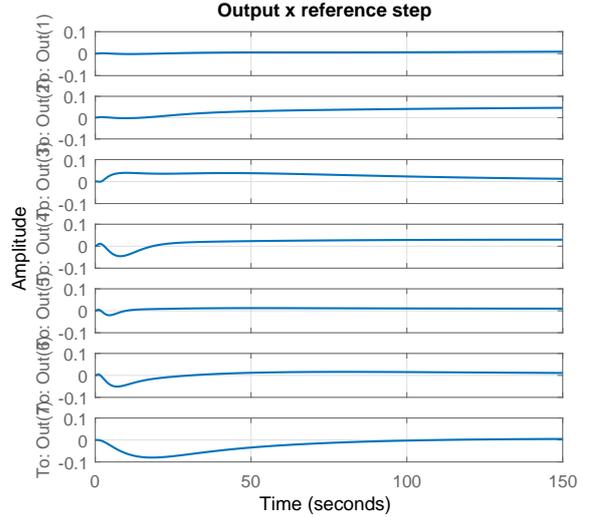


Figure 12: Smith Predictor reference tracking.

and robustness results were obtained in a more simple way.

It is important to mention that, once the tuning algorithm is created, it is quite easy to adapt it to different processes and to some different control architectures. As an automatic tuning method, it is also very useful to processes with high dimensions (high number of inputs and/or outputs).

The use of Model Predictive Control (MPC) on MIMO processes with large dead-times is expected to present better performance than with the use of Smith Predictor controllers. Nevertheless, MPC requires considerable computational effort and its implementation in low-level Programmable Logic Controllers (PLC) may be challenging. By working with a Smith Predictor automatically tuned, the idea is to leave the low-level automation as simple as possible, and put the complexity of the tuning procedure - either online or offline - on higher automation levels, running at longer cycles.

The extension to unstable processes, and to processes with variable dead-times, will be studied in future works.

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