STATE ESTIMATOR APPLICATION IN DC MOTORS OF A MOBILE ROBOT

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Abstract— Mobile robotics is a multidisciplinary field involving both computer science and engineering. In this application the exact knowledge of the position and speed of a vehicle is a fundamental problem that has been taking the attention of researchers and engineers. In this paper, we propose a method to identify the model and to estimate the speed of hobby DC motor based on a state estimator. The proposed method is evaluated with simulations and experimental results. The results show that performance state estimator is as good as an encoder system.

Keywords— Mobile Robotic, Mechatronics Systems and State Estimator.

Resumo— A robótica móvel é uma ciência multidisciplinar que engloba a ciência da computação e engenharias. Nessas aplicações o exato conhecimento de posição e velocidade de um robô é um problema fundamental que vem tomando a atenção de pesquisadores e engenheiros. Neste trabalho é proposto um método para identicar o modelo e estimar a velocidade de um motor de corrente contínua baseado em um estimador de estados. O método proposto é avaliado por simulação e em experimento. Os resultados demonstram que o desempenho do estimador de estados é tão bom quanto um sistema com encoder.

Palavras-chave— Robótica Móvel, Sistemas Mecatrônicos, Estimador de Estados.

1 Introduction

The study of mobile robots involves two disciplinary fields, computer science and engineering. Addressing the design of automated systems, it lies at the intersection of artificial intelligence, computational vision, and robotics (Dudek and Jenkin, 2010). The ability of mobile robots to move around autonomously in their environment determines the best possible applications of such robots: tasks that involve transportation, exploration, surveillance, guidance, inspection, etc. In particular, mobile robots are used for applications in environments that are inaccessible or hostile to humans. Examples are underwater robots, planetary rovers, or robots operating in contaminated environments (Nehmzow, 2003). Therefore, to navigate reliably in indoor environments, a mobile robot must know where it is (Dellaert et al., 1999). Exact knowledge of the position and speed of a vehicle is a fundamental problem in mobile robot applications. In search for a solution, researchers and engineers have developed a variety of systems, sensors, and techniques for mobile robot positioning (Borenstein et al., 1997).

For instance, in (Liang et al., 2016) it is proposed a leader-following formation control method for mobile robots with a directed tree topology, without the direct use of accurate global or relative position measurements. An adaptive observer is developed based on the feedback information from a perspective camera, the odometry and AHRS sensors. On the other hand, theoretical and practical results show that the performance of automatic control systems can often be improved by using some type of velocity feedback control techniques. However, it may not always be possible to measure velocities due to costs and noisy environment (Su et al., 2016). To solve this problem in (Okuyama et al., 2015) it is proposed a method that uses a low cost acquisition setup and uses a frequency domain analysis based on the Bode diagram technique. To improve the identification accuracy of a hobby DC motor. A new scheme is proposed to design an adaptive virtual velocity controller and torque control law. Meanwhile, a disturbance observer is applied to estimate the lumped disturbance to achieve the feedforward compensation in (Huang et al., 2016). Recently, a new topology with a robust and an efficient tracking controller based on model reference adaptive system (MRAS) technique is performed for mobile robot in order to get the accurate trajectory tracking performance is shown in (Dumlu and Ayten, 2017).

Motivated by (Okuyama et al., 2015), in this paper we propose a method to identify the model and to estimate the speed of hobby DC motor based on state estimator. The state estimator is developed to estimate the speed from the dc voltage and current motor. This solution has shown lower costs than encoder systems. The simulation and experimental validation showed that method can be used to estimate the speed with accuracy of a mobile robot system.

The remainder of this paper is organized as follows: in the next section (Section 2) we introduce the DC motor modeling and a review of the methods used in this paper. Then, in Section 3, we show the DC motor measurement problem and the solution proposed. In Section 4, we describe in detail the real DC motor modeling and show the experimental results illustrating the performance of the solution proposed. Finally, Section 5 contains the conclusion to this paper.

2 Preliminaries

2.1 Brushed DC motor Modelling

A DC motor is any of a class of electrical machines that converts direct current electrical power into mechanical power (Rizzoni, 2005). Therefore, all electric motors are governed by the laws of electromagnetism, and are subjected to essentially the same constraints imposed by the materials (copper, iron and insulation) from which they are made (Hughes and Drury, 2013). The brushed DC motor can be described to diagram as shown in the Figure 1 (Ogata, 2010).

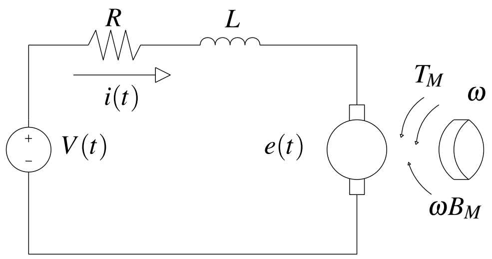

Figure 1: Schematic diagram of the DC motor

Where V is the armature voltage in (V), i is the armature current in (A), e is the electromotive force of the motor, T_M is the motor torque, $\omega(t)$ is the angular speed in (rad/s), R armature resistance in (Ω), L is the armature inductance in (H) and b is the coefficient of viscous friction.

According to the Kirchhoff's voltage law, the electrical equation of the DC motor is described as

$$V(t) = Ri(t) + e(t) + L\frac{di(t)}{dt}. \tag{1}$$

The electromotive force of the motor, e, is proportional to the angular velocity of the rotor in the motor, expressed as

$$e(t) = K_b\omega(t), \tag{2}$$

where K_b is the electromotive force constant. Moreover, the brushed DC motor generates a torque, T, proportional to the armature current, given as

$$T_M(t) = K_T i(t) \tag{3}$$

where K_T is the torque constant. Besides, if the DC motor is apllied to drive an external torque, $T_L(t)$ of payload then its mechanical behavior can be described as

$$J_M\frac{d\omega(t)}{dt}(t) + B_M\omega(t) = T_M(t) - T_L(t) \tag{4}$$

where J_M is the rotor moment of inertia in $(kg \cdot m^2)$ and B_M is the motor viscous frictional coefficient in $(N \cdot m \cdot s)$. Accordingly, based on (1) to (4), the dynamic of the brushed DC motor can be expressed as (Bolton, 2016)

$$\begin{aligned} V(t) &= Ri(t) + K_M\omega(t) + L\frac{di(t)}{dt} \\ T_L(t) &= K_M i(t) - J_M\frac{d\omega(t)}{dt}(t) - B_M\omega(t) \end{aligned} \tag{5}$$

where $K_M = K_b = K_T$.

2.2 State Estimator

The State Estimator is a tool responsible for estimating the states of a system when they are not accessible for direct connection or because sensing devices or transducers are not available or very expensive (Chen, 1999).
Considering a state-space equation:

$$\begin{aligned} \dot{x}(t) =& Ax(t) + Bu(t) \\ y(t) =& Cx(t) + Du(t) \end{aligned} \tag{6}$$

if we have information about the input $u(t)$, the output $y(t)$, and the parameters A, B, C and D, but don't have the state $x(t)$, we can duplicate the system into an open-loop state estimator, creating a estimation $\hat{x}(t)$, and using it as our state $x(t)$.

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) \tag{7}$$

However, there are disadvantages in using this the open-loop state estimator. To upgrade it, the closed-loop estimator will compare the output $y(t)$ with it estimation $(C\hat{x}(t) + Du(t))$, and multiply by a constant L. Therefore, the state estimator will be represented by:

$$\begin{aligned} \dot{\hat{x}}(t) =& A\hat{x}(t) + Bu(t) \\ &+ L\left(y(t) - C\hat{x}(t) - Du(t)\right) \end{aligned} \tag{8}$$

In addition, the Figure 2 (Chen, 1999) shows the closed-loop state estimator. To design the constant L, it is necessary to analyze the dynamics of the estimation error, given by (Chen, 1999):

$$e(t) = x(t) - \hat{x}(t) \tag{9}$$

$$\dot{e}(t) = (A - LC)\,e(t) \tag{10}$$

Thus, L must be chosen so that the eigenvalues of $(A - LC)$ have a negative real part, causing estimation error converges to zero.

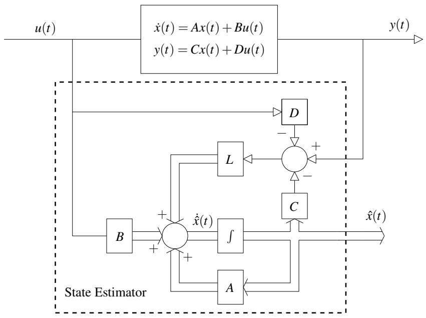

Figure 2: Closed-Loop State Estimator

2.3 Step Method Response

Process identification is a key element in process control and it needs some kinds of tests, such as step, pulse, pseudo-random binary sequence, sinusoidal or other deterministic signals (Bi et al., 1999), (Åström and Hägglund, 1995). The step response based methods are most commonly used for system identification, especially in process industries (Ahmed et al., 2007). Usually, this method is used to describe simplest parametric models of process dynamics as given by

$$G(s) = \frac{K}{1+s\tau}e^{-sL} \tag{11}$$

where K is the static gain, τ is the time constant and L is the dead time. This parameters can be determined graphically (Ljung, 1999) as shown in Figure 3. According to Åström and Hägglund the static gain (K) is obtained from the final steady-state level of the process output. On the other hand, the dead time (L) is the time between the start step at the intercept of the tangent to step response that has the largest slope with the time axes or horizontal axes (see Figure 3). Lastly, the time constant (τ) have different ways to be determined. One method determines τ from the distance AB, where B is the time when the step response has reached the value $0.63K$. Another procedure determines τ from the distance AC, where C is the time when the tangent intersects the line $s(t) = K$. More details about step method response are available in (Åström and Hägglund, 1995, p. 11).

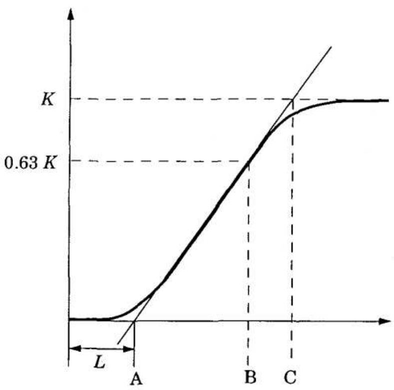

Figure 3: System Response

Once the model system is describes as (11), we can rewrite the model on the time domain. To do so, we applied (11) at the Inverse Laplace Transform (Ogata, 2010), this results in

$$\dot{x}(t) = -\frac{1}{\tau}x(t) + \frac{K}{\tau}u(t) \tag{12}$$

where $x(t)$ is the state of the system and $u(t)$ is the control signal.

2.4 Least Square Method

Nowadays, the least square method is widely used to find or estimate the numerical values of the parameters to fit a function to a set of data and to characterize the statistical properties of estimates (Miller, 2006). The least square is a method used to solve overdetermined linear systems, given by:

$$\begin{aligned} y_1 &= f(x_1, \hat{\theta}) \\ \vdots &= \quad \vdots \\ y_N &= f(x_N, \hat{\theta}) \end{aligned} \tag{13}$$

Where $\mathbf{x} = [x_{11}, x_{12} \ldots x_{1n}]$, $\hat{\theta} \in \mathbb{R}^n$ is estimated parameter and $N > n$. This amounts to minimizing the expression:

$$\xi = \sum_i \left[y_i - \mathbf{x}^T\hat{\theta}\right]^2 \tag{14}$$

where ξ is the estimated error. Once, $N > n$ the system (13) can be rewritten as

$$\mathbf{y} = \mathbf{x}^T\hat{\theta} + \xi. \tag{15}$$

Where $\mathbf{y} \in \mathbb{R}^N$, $\mathbf{x} \in \mathbb{R}^{N\times n}$ and $\xi \in \mathbb{R}^N$. The solution of the optimazation problem (14) is given by (Aguirre, 2007):

$$\hat{\theta} = [\mathbf{x}^T\mathbf{x}]^{-1}\mathbf{x}^T\mathbf{y} \tag{16}$$

3 Problem

3.1 DC motor Measurement Problem

Generally, mechatronics systems including DC motors need to monitor motors rotation speed. A simple solution to this problem is to use a rotatory encoder attached to the motor axis or attached to the motor gearbox. The function of the encoder is to translate the motor rotation speed to a digital signal that can be easily read (see (Ellin and Dolsak, 2008) for more details). An encoder is normally composed by two parts, a mechanical and an electronic one. This last implies that an encoder can occupy a significant space in the hardware, have a non negligible weigh, it must be energized implying a battery power consumption, and it may even need a regular maintenance.

In order to reduce costs, weigh, power consumption and simplify the robot maintenance described below, we propose to remove the encoder and estimate the motor rotation speed which is the principal aim of this work.

3.2 System Used for Implementation

For the purpose mentioned below, we assumed two conditions: the system (5) can be described as a first order system, once the electric pole is faster than the mechanical pole (Okuyama et al., 2015). On the other hand, we assume that the load torque, T_L, is null (Patané, 2008). Therefore, observing these assumptions, we can simplify the system (5) as:

$$K_M i(t) = J_M \frac{d\omega(t)}{dt}(t) + B_M \omega(t)$$
$$V(t) = Ri(t) + K_M \omega(t). \tag{17}$$

The system (17) can be described in the state space form, to do so the electric equation present in (17), can be rewritten as:

$$i(t) = \frac{V(t) - K_M \omega(t)}{R}, \tag{18}$$

applying (18) in the mechanical equation present in (17) and after some straightforward manipulation, we obtain

$$\dot{\omega}(t) = -\frac{K_M^2 + B_M R}{R J_M}\omega(t) + \frac{K_M}{R J_M} V(t). \tag{19}$$

Moreover, from (18) and (19) we have:

$$\dot{x}(t) = \left[-\frac{K_M^2 + B_M R}{R J_M}\right] x(t) + \left[\frac{K_M}{R J_M}\right] u(t) \tag{20a}$$
$$y(t) = -\frac{K_M}{R} x(t) + \frac{1}{R} u(t) \tag{20b}$$

where $x(t) = \omega(t)$ is the state of the system, $u(t) = V(t)$ is the control signal and $y(t) = i(t)$ is the output of the system.

Once, we have knowledge about the input and the output of the system (20a)-(20b), it is possible to design a state estimator to the system as design shown in the Section 2.2. Therefore, the state estimator of the system (20a)-(20b) can be described as:

$$\dot{\hat{x}}(t) = \left[-\frac{K_M^2 + B_M R}{R J_M}\right] \hat{x}(t) + \left[\frac{K_M}{R J_M}\right] u(t) + L\left(y(t) + \frac{K_M}{R}\hat{x}(t) - \frac{1}{R}u(t)\right) \tag{21}$$

4 Experimental Results

In this section we propose a synthesis for the state estimator developed in Section 2.2. First, we obtain the numerical values of the model from Eq.(21) using the tools mentioned in Sections 2.3-2.4. Then, the estimator synthesis and validation are performed for a differential mobile robot estimating the angular velocity from the current and voltage of the DC motor.

4.1 The Hardware

In order to evaluate the state estimator of Eq.(21), we intend to implement the topology described in Figure 2 on the differential mobile robot shown in Figure 4. This robot is propelled by two brushed DC micromotors with 298 : 1 gearbox coupled to each backward wheel. There is an omnidirectional wheel at the front-side to guarantee a third support. Both brushed DC micro-motors are each equipped with a magnetic quadrature encoder. This encoder has the characteristic to offers 3576 counts per revolution (CPR). Besides, the system is also featured with a vision system, two current sensors (INA219), an accelerometer, an ultrasonic sensor and a limit switch. Both engines and all the instruments are controlled and actuated by a Raspberry Pi 3 as master controller and an Arduino Mini as slave controller. To program the master system we use *Python* language and *C* language to program the slave system. The systems communicate with each other using the I_2C protocol and the robot is powered by a *LiPo* (Lithium Polymer) battery with $11{,}1V$ and $2200mAH$.

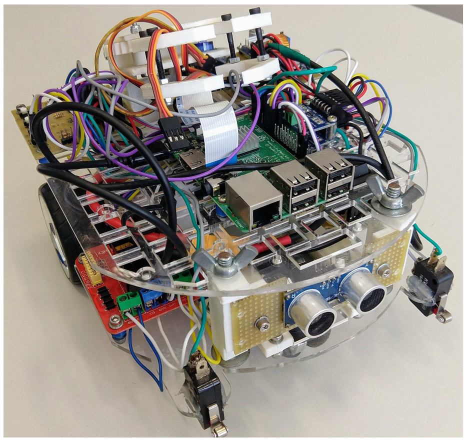

Figure 4: The differential mobile robot.

4.2 Modelling System

The brushed DC micro-motor can be modeled through the structure described in the Section 2.1. However, the datasheet supplied by the vendor does not provide all the parameters needed in Eqs.(20a)-(20b). To solve this problem, we firstly identify the Eq.(20a) unknown parameters applying the step response method introduced in Section 2.3 and we obtain the following dynamic equation:

$$\dot{x}(t) = -2.857x(t) + 4.8u(t). \tag{22}$$

The equivalent transfer function of this equation has a time constant $\tau = 0.35$s and a static gain $K = 1.68$. In order to validate the model obtained, we make an experimental test for angular speed setpoint equal to $15rad/s$. Around this setpoint, a sequence of relative steps $[-3, +3, 0]rad/s$ with $5s$ time windows length each was submitted to both, the model and the robot. The results of this test are shown in Fig. 5. The black line illustrates the setpoint reference, the red line illustrates the Eq.(22) response and the blue line illustrates the angular speed measured by the angular encoder attached to the brushed DC micro-motor of the differential robot shown in Fig. 4. We can see that the model satisfactorily reproduces the system behavior with some errors in the static gain.

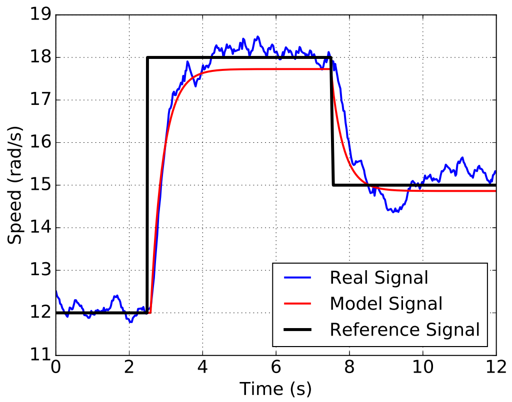

Figure 5: Experimental Model Validation

Secondly, to obtain the numerical values of the static equation EQ.(20b), we need to find K_M and R parameters. However, the datasheet of the DC micro-motor inform just an approximation of the armature resistance ($R \approx 15\Omega$). To obtain a better knowledge of these parameters we used the least-square method introduced in the Section 2.4. Applying (16) in (20b) for $x(t) \approx 15 \pm 3rad/s$, we obtain: $K_M = 0.31$ and $R = 15.81\Omega$. Finally, the complete model of the brushed DC micro-motor is given by:

$$\begin{aligned} \dot{x}(t) &= -2.857x(t) + 4.8u(t) \\ y(t) &= -0.019x(t) + 0.063u(t) \end{aligned} \tag{23}$$

4.3 Syntheses of the State Estimator

The state estimator must be tuned with respect to two criteria. Firstly, the eigenvalue of $(a - lc)$ must have a negative real part, so that the estimation error

$$\dot{e}(t) = \dot{x}(t) - \dot{\hat{x}}(t) = (a - lc)e(t) \tag{24}$$

is asymptotically stable and converges to zero. Thus:

$$a - lc < 0 \rightarrow l < \frac{c}{a} = 143$$

Secondly, the state estimator must be discretized and implemented in real time with a sampling time imposed by hardware limits. This implies that the time constant of the estimation error must follow the following inequality: $\tau_e > 10T_s$. Where T_s is the sample time, equal to 0.01 seconds and $\tau_e = -1/(a - lc)$ is the time constant of the state estimator. This leads to the second constraint for l, $l > -357$. Concluding, the constant l must be designed respecting the following limits: $-357 < l < 143$.

In this paper, we want the best compromise between noise filtering and convergence rate, experimentally we achieved the value of $l = -50$. Therefore, the state estimator of Eq. (21) can be rewritten as:

$$\dot{\hat{x}}(t) = -3.86\hat{x}(t) + 7.95u(t) - 50y(t) \tag{25}$$

4.4 State Estimator Validation

To verify the performance of the designed state estimator, we performed open-loop tests with the robot. In these tests, the robot navigates with predetermined control signals at speeds $\dot{\theta} \approx 15 \pm 3rad/s$, and, at the same time that the state estimator is running, the speed sensor is monitoring the angular velocity. Thus, it was possible to compare the real speed of the robot and the speed estimated by the state estimator. Figure 6 shows the reference in black, the encoder measurements in blue, and the speed estimation in red.

5 Conclusion

In this work we proposed a method to identify the model of a hobby DC motor and a state estimator to estimate the wheels angular speed for replacing the encoder attached to the motor. Simulations and experimental tests were performed to evaluate the quality of the estimation. We can conclude that the performance of the state estimator is as good as an encoder system. This makes a viable solution to the problem of monitoring the angular speed with the advantages of reducing space, weigh, energy consumption and maintenance. However, this solution shows the same problems as an encoder regarding wheel skating.

As a future work, the extension of the here proposed state estimator to monitor in real time some model parameters will be studied in order to augment the operational range.

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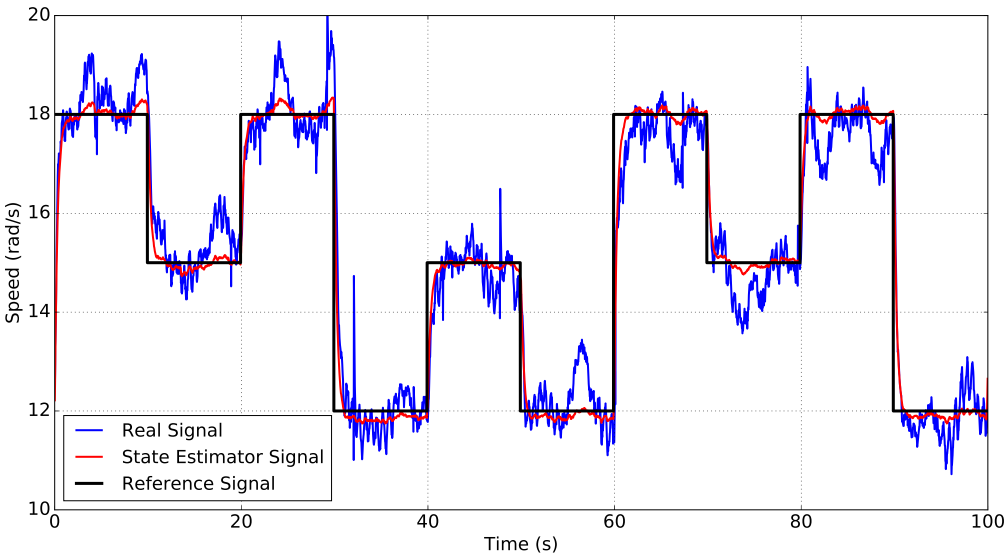

Figure 6: State Estimator Experimental Validation

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