DOI: 10.48011/asba.v2i1.1012

An off-line output feedback RMPC-LPV applied to an inverted pendulum using relaxed LMI procedures

Luis Carvalho * **Marcus V. S. Costa** ** **Walber M. Lima** **
Elenilson V. Fortes ***

* *Electrical Engineering Graduate Program - PPGEE- UFERSA (e-mail: luiscarvalho9000@gmail.com).*
** *Department of Engineering, UFERSA (e-mail: marcus.costa@ufersa.edu.br; walber@ufersa.edu.br).*
*** *Department of Academic Areas, IFG, Jataí - GO, Brazil, (e-mail: elenilson.fortes@ifg.edu.br).*

Abstract: This paper aims an off-line output-feedback robust model predictive control (RMPC) using linear matrix inequalities (LMIs) applied to the angle position control of an inverted pendulum modeled by Linear Parameters Varying (LPV) affine scheme. The presented methodology involves the an off-line RMPC-LPV state feedback, which the gains are LPV by LMIs and stored in the look-up table. Also, it is presented a robust observer design using LMIs that is ensured the feasibility of the output feedback stability. The control strategies presented in this study considers the online and off-line state space feedback and off-line feedback control design along with state observer. The comparative analysis of the numerical results and cost indices evidence the suitability of the proposed methodology and the advantages of output feedback RMPC-LPV in comparison with the typical approaches based in the same control design conditions.

Keywords: Inverted Pendulum Control; Output feedback RMPC-LPV; LPV-SS Control; Off-line RMPC approach.

1. INTRODUCTION

The inverted pendulum is a nonlinear system very known in the literature. Its characteristics are useful for control design of real systems, such as: missile launching, robotic arms, horizontal movement of a person walking, etc (Lundberg and Barton, 2010). The main goal of this control design is to keep the pendulum rod in the equilibrium point accord to the movement of its base (Ogata, 2011)

In order to reduce the negative effects originated by non-linearity disturbances, the Linear Parameter Varying (LPV) model is a useful contouring solution (Caigny et al., 2009; Scherer, 2001), because it ensures the linearity and maintains the nonlinear characteristics of the plant, becoming easier the control design (Kajiwara et al., 1999; Koelewijn et al., 2018).

In addition, researches involving the off-line robust model predictive control (RMPC) via linear matrix inequalities (LMIs) have increased in the last years (Costa et al., 2017; Zheng et al., 2018; Moradi et al., 2018; Hu and Ding, 2018). Besides, studies of the RMPC applied to inverted pendulum could be highlighted in this context (Yue et al., 2018; Jung and Wen, 2004; Nam et al., 2019; Watson et al., 2019). Moreover, Park et al. (2011), Bumroongsri (2014), Longge and Yan (2017), and Ping (2017) presented applications of RMPC-LPV in their research fields, emphasizing the efficiency of this method as robust control design.

Initially proposed by Kothare et al. (1996), the RMPC-LMI consists to solve the robust control stability problem based on quadratic cost index by LMI optimization for each sample time. An off-line procedure of this RMPC was presented by Wan and Kothare (2002), which the systems with high computational cost could be controlled by a set of static gains known as look-up table.

Based on this background, this study aims the RMPC-LPV technique of Wada et al. (2006) using the LMI procedure of Cuzzola et al. (2002) and applied to an inverted pendulum modeled by LPV affine. The proposed control strategies in this study considers the following approaches:

(1) the online state feedback based on Kothare et al. (1996);
(2) the off-line state feedback with lookup-table conform to Wan and Kothare (2003);
(3) the off-line control method with observer design formulation of Wan and Kothare (2003).

Hence, the main contributions proposed in this paper are:

(1) the output feedback RMPC-LPV via LMI approach;
(2) the LMI relaxed procedure for LPV state feedback law and the observer design;
(3) the using of stability ellipsoids analysis for RMPC of Wan and Kothare (2003) considering the LPV emphasis to a lookup-table of state feedback gains;

(5) the observer design method proposed insures the feasibility of the robust input-output stability considering the LPV system condition;
(6) the efficiency of presented control method through the comparative analysis of the response time and performance indices, IAE, ISE, ITAE and ITSE.

This paper is organized as follows: Section 2 presents the problem statement of RMPC-LPV and the observer design. Section 3 explains the algorithm method of the proposed control strategy. Section 4 elucidates the LPV affine model of inverted pendulum along with the numerical example and analysis of simulated results. Lastly, the conclusions discusses the contributions shown in this study.

Remark: The symbol (*) represents the symmetrical block in the matrices as follow:

$$\begin{bmatrix} A & B^T \\ B & C \end{bmatrix} = \begin{bmatrix} A & * \\ B & C \end{bmatrix}$$

2. PROBLEM STATEMENT

Consider the following state space LPV affine model defined by

$$\begin{aligned} x(k+1) &= A(\alpha)\,x(k) + B(\alpha)\,u(k) \\ y(k) &= Cx(k), \end{aligned} \tag{1}$$

where $A(\alpha) = A_0 + \sum_{m=1}^{n} \alpha_m A_m$, $B(\alpha) = B_0 + \sum_{m=1}^{n} \alpha_m B_m$, $x(k)$ are the states, $u(k)$ is the control signal, $y(k)$ is the output signal and C is a static output array. Consider also the estimated state space model defined by

$$\begin{aligned} \hat{x}(k+1) &= A_0\hat{x}(k) + B_0u(k) + L_P(y(k) - C\hat{x}(k)) \\ y(k) &= C\hat{x}(k), \end{aligned} \tag{2}$$

which the observed state model is linear invariant at discrete time. The sate feedback control law of the RMPC-LPV is defined by min-max optimization problem of Kothare et al. (1996), given by

$$\min_{u(k)} \max_{\Omega} J_\infty(k) \tag{3}$$

which

$$J_\infty(k) = \sum_{i=0}^{\infty} \left[x(k+i|k)^T Q_c x(k+i|k) + u(k+i|k)^T R_c u(k+i|k) \right] \tag{4}$$

where $\Omega = [A(\alpha)\ B(\alpha)]$ is the LPV convex hull of $\alpha \in [\underline{\alpha}\ \bar{\alpha}]$, $Q_c = Q_c^T \geq 0$ and $R_c = R_c^T \geq 0$ are the weighting matrices defined by designer.

In order to obtain the feedback control law LPV defined by $u(k) = F(\alpha)\,x(k)$, where $F(\alpha)$ is the LPV feedback gain, the solution of the objective function is satisfied by following LMIs procedures (Cuzzola et al., 2002; Wada et al., 2006):

$$\begin{bmatrix} G + G^T - Q_j & * & * & * \\ \Gamma_c(\alpha) & Q_j & * & * \\ Q_c^{1/2}G & 0 & \gamma I & * \\ R_c^{1/2}Y & 0 & 0 & \gamma I \end{bmatrix} \geqslant 0, \tag{5}$$

$$\begin{bmatrix} 1 & * \\ x(k|k) & Q_j \end{bmatrix} \geqslant 0,\ Q_j > 0, \tag{6}$$

where $G \geq 0, > 0$, G non-symmetric and $\Gamma_c(\alpha) = A(\alpha)\,G + B(\alpha)\,Y(\alpha)$.

Adopting the procedures proposed by Briat (2015) and Boyd et al. (1994), the gain $F(k) = YG^{-1}$ used by Cuzzola et al. (2002) and Wada et al. (2006) it can be modeled using the following LPV structure:

$$F(\alpha) = Y_0G^{-1} + \sum_{m=1}^{n} \alpha_m Y_m G^{-1}. \tag{7}$$

Evidencing G^{-1} in (7) follows:

$$F(\alpha) = \left(Y_0 + \sum_{m=1}^{n} \alpha_m Y_m \right) G^{-1} \tag{8}$$

So, $Y(\alpha) = Y_0 + \sum_{m=1}^{n} \alpha_m Y_m$ is the LPV affine formulation of the feedback control gain, $F(\alpha) = Y(\alpha)\,G^{-1}$.

2.1 Input and Output Constraints

According to Cuzzola et al. (2002), the input and output constraints are sentenced by, respectively:

$$\begin{gathered} \begin{bmatrix} X & Y(\alpha) \\ * & G + G^T - Q_j \end{bmatrix} \geqslant 0, \\ With\ X_{rr} \leqslant u_{r,max}^2,\ r = 1, 2, 3, \ldots, n_u \end{gathered} \tag{9}$$

$$\begin{gathered} \begin{bmatrix} G + G^T - Q_j & * \\ C\Gamma_c(\alpha) & Z \end{bmatrix} \geqslant 0, \\ With\ Z_{rr} \leqslant y_{r,max}^2,\ r = 1, 2, 3, \ldots, n_y \end{gathered} \tag{10}$$

2.2 The relaxed observer design

Applying the relaxation procedure of Cuzzola et al. (2002) to state observer design formulated by Wan and Kothare (2002) become

$$\begin{bmatrix} \rho^2(G_o + G_o^T - Q_j) & * \\ G_oA_0 + Y_oC & Q_j \end{bmatrix} \geqslant 0 \tag{11}$$

$$\begin{bmatrix} G_o + G_o^T - Q_j & A_{poly,i,j}G_o \\ * & Q_j \end{bmatrix} > 0, \tag{12}$$

where $\rho \in [0\ 1]$ is the decay rate of the estimator design, and $A_{poly,i,j}$ is given by:

$$A_{poly,i,j} = \begin{bmatrix} A(\alpha) & B(\alpha)\,F(\alpha) \\ L_pC & A_0 + B_0F_i - L_PC \end{bmatrix}, \tag{13}$$

which the observer gain is defined by $L_P = G_o^{-1}Y_o$, for $i = 1, 2, 3, \ldots, N$ and $j = 1, 2, 3, \ldots, \mathcal{L}$, where $\mathcal{L}$ is the number of vertices and N is the samples used for building of look-up table.

3. OFF-LINE ALGORITHM

For the off-line robust constrained state feedback MPC, given an initial condition (X_{Set}) which generates a sequence of minimizers, γ_i, Q_i G_i, Y_i, X_i and Z_i, for $i = 1, 2, 3, \ldots, N$ and $j = 1, 2, 3, \ldots, \mathcal{L}$, subjected to (3), (5), (6), (9), and (10). The sequence of the off-line algorithm is as follows:

Set $i := 1$, then follow the steps:

(1) Evaluate the minimizers, γ_i, Q_i, G_i, Y_i, X_i and Z_i,with an additional constraint $G_{i-1} > G_i$, $G_i > Q_j$ and store, G_i^{-1}, F_i, X_i and Y_i in a look-up table.
(2) If $i < N$ choose a state x_{i+1} satisfying $||x_{i+1}||^2_{G_i^{-1}} \leq ||x_{i+1}||^2_{Q_j^{-1}} \leq 1$.
Add $i := i + 1$ and return to Step 1.
(3) Calculate $F(\alpha) = Y(\alpha)G^{-1}$.
(4) Apply the control law $u(k) = F(\alpha)x(k)$.

4. SIMULATION RESULTS AND DISCUSSION

In this section, the proposed RMPC-LPV technique is applied to the inverted pendulum model in three distinct approaches: Online feedback control design, Off-line feedback control design, and Off-line feedback control design with state observer.

4.1 Inverted pendulum: State-space LPV modeling

According to Teixeira et al. (2000) and Xiao-Jun Ma et al. (1998), the non-linear model of an inverted pendulum in a car illustrated in Fig. 1 can be described as:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= [\, -f_1(M+m)x_2 - m^2l^2x_2^2(sen\ x_1)cos\ x_1 \\
&\quad + f_0ml\ x_4cos\ x_1 + (M+m)mgl(sen\ x_1) \\
&\quad - ml(cos\ x_1)u\,]\,\frac{1}{\Delta} \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= [\, f_1mlx_2cos\ x_1 + (J+ml^2)mlx_2^2(sen\ x_1) \\
&\quad - f_0(J+ml^2)x_4 - m^2gl^2(sen\ x_1)cos\ x_1 \\
&\quad + (J+ml^2)u\,]\,\frac{1}{\Delta}
\end{aligned} \tag{14}$$

where $\Delta = \left[(M+m)(J+ml^2) - m^2l^2cos^2\ x_1\right]$.

Linearizing (14) and using the state-space LPV affine modeling, follows

$$\begin{aligned}
x(k+1) &= (A_0 + \alpha A_1)x\,(k) + Bu\,(k) \\
y(k) &= Cx\,(k)
\end{aligned} \tag{15}$$

where $m = 1 + \alpha$, and

$$A_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{M+1}{Ml}g & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{1}{M}g & 0 & 0 & 0 \end{bmatrix},\ A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{1}{Ml}g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{1}{M}g & 0 & 0 & 0 \end{bmatrix}$$

, which

$$B = \left[0\ -\frac{1}{Ml}\ 0\ \frac{1}{M}\right]^T. \tag{16}$$

The design parameters used to inverted pendulum control design are: Rod length ($l = 0,304\ m$); Car mass ($M = 1,3282\ Kg$); Pendulum mass ($m = 0,22\ Kg$), and gravity

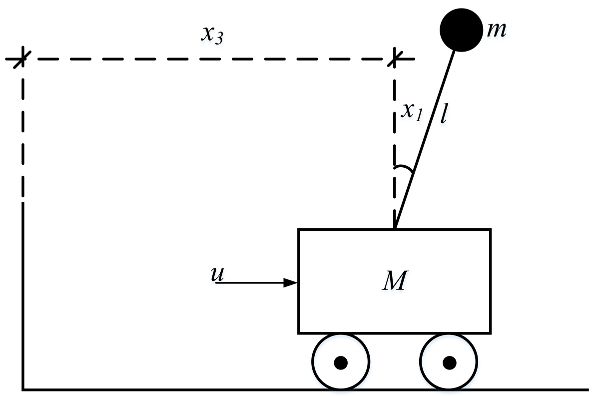


Figure 1. Inverted pendulum (Aguirre et al., 2007).

($g = 9,81\ m/s^2$). The model in (15) is discretized using a sampling time of $T = 0.10$ s. Therefore, $A_d = (I + TA_0) + \alpha TA_1$ and $B_d = TB$. The numerical model of (16) are

$$A_0 = \begin{bmatrix} 1 & 0.1 & 0 & 0 \\ 5.6566 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.1 \\ -0.7386 & 0 & 0 & 1 \end{bmatrix}, \tag{17}$$

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 2.4296 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.7386 & 0 & 0 & 0 \end{bmatrix}, \tag{18}$$

$$B = [0\ -0.2477\ 0\ 0.0753]^T \tag{19}$$

For the numerical implementation of RMPC-LPV applied to an inverted pendulum model, the input constraint defined is $u_{max} = 100\ N$, the weighting matrices are $R_c = 1$, $Q_c = I_{4x4}$, and $X_{Set} = [1\ 0.5\ 0.3\ 0.2\ 0.15\ 0.1\ 0.07\ 0.05\ 0.035\ 0.01]$, conform values presented by Wan and Kothare (2002).

The initial state of simulation is given by $x(0) = [0.96\ 0\ 0\ 0]^T$, where the initial value of $x_1(0)$ is equivalent to 55° from the equilibrium point. The LPV parameters are limited in the range given by $\alpha \in [0.22\ 1.10]$.

4.2 Analysis of results

The observer gain obtained from (11) and (2.2) is given by

$$L_P = [1.9802\ 16.2231\ 0.0010\ -0.5053]^T$$

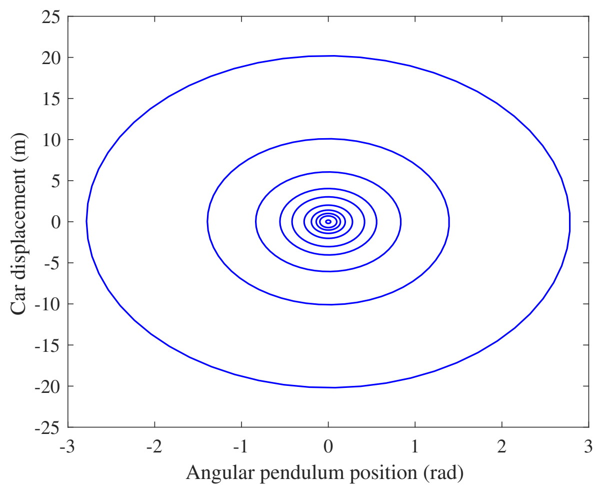
with decay rate of $\rho = 0.10$. The LPV feedback gains presented in Table 1 are used for RMPC-LPV off-line and off-line with observer design.

Figures 2a and 2b show the stability ellipsoids for off-line algorithm design from matrix G^{-1}, based on the vector X_{set} and Table 1, which will stabilize the system for each random value of $\alpha \in [\underline{\alpha}\ \bar{\alpha}]$, using the methodology of Wan and Kothare (2002).

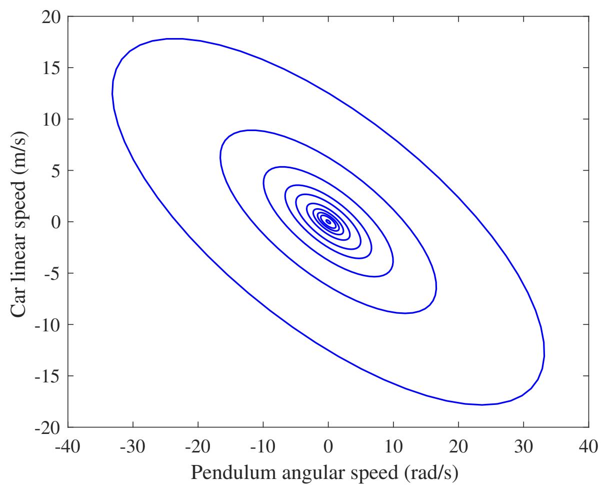
The feedback matrix is an ellipsoidal set in the plane designed by the interaction of optimization process from the recursive RMPC method, as described by Kothare et al. (1996), that those matrices are stored in a look-up-table and the designer can choose the better result, that usually is the last value obtained by the algorithm.

Table 1: Look-up Table

N	α		$F(\alpha) = F_0 + \alpha F_1$
1	0.822	F_0	$+$[49.617 6.660 0.644 1.690]
		F_1	$-$[$-$9.700 0.038 0.013 0.033]
2	0.708	F_0	$+$[49.497 6.651 0.641 1.682]
		F_1	$-$[$-$9.844 0.023 0.008 0.020]
3	0.225	F_0	$+$[49.175 6.628 0.633 1.662]
		F_1	$+$[10.215 0.007 0.002 0.006]
4	0.473	F_0	$+$[49.107 6.623 0.631 1.658]
		F_1	$+$[10.291 0.012 0.004 0.010]
5	0.551	F_0	$+$[49.019 6.617 0.629 1.653]
		F_1	$+$[10.390 0.019 0.006 0.016]
6	0.348	F_0	$+$[48, 978 6, 616 0.628 1.652]
		F_1	$+$[10.425 0.013 0.004 0.011]
7	0.285	F_0	$+$[48.762 6.601 0.623 1.639]
		F_1	$+$[10.669 0.029 0.010 0, 025]
8	0.625	F_0	$+$[49.007 6.620 0.630 1.656]
		F_1	$-$[$-$10.368 0.004 0.001 0.003]
9	0.545	F_0	$+$[48.950 6.614 0.629 1.652]
		F_1	$+$[10.429 0.008 0.002 0.007]
10	0.946	F_0	$+$[49.208 6.625 0.638 1.671]
		F_1	$-$[$-$09.987 0.028 0.011 0.025]



(a) Relate angular pendulum position to car linear displacement.



(b) Relate pendulum angular speed to car linear speed.

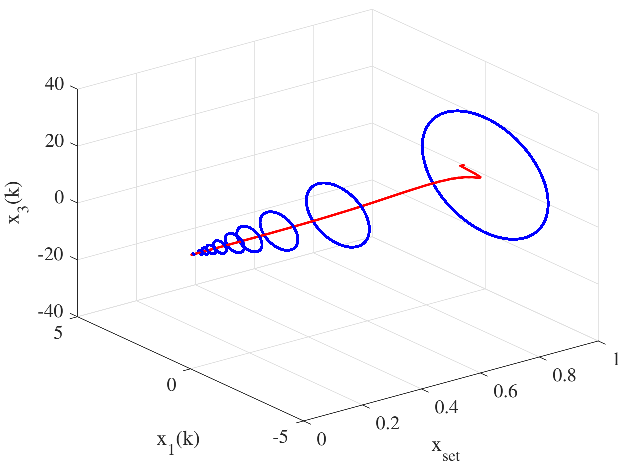
Figure 2. Off-line stability ellipsoids.

Figures 3a and 3b present the average time for the off-line algorithm. It is observed that the response time of system keeps inside of ellipsoids of stability region, conform demonstrated by (Kothare et al., 1996).

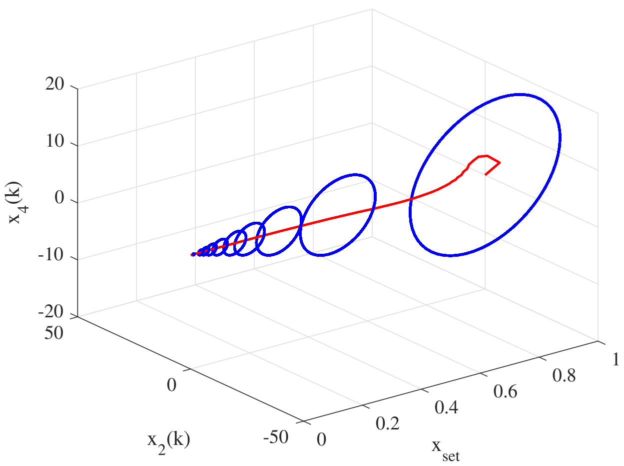
Figure 4 illustrates the state behavior of the plant. $x_1(k)$ and $x_2(k)$ are position and angular speed of the rod in relation to its equilibrium point, respectively. $x_3(k)$ and $x_4(k)$ are the position and linear speed of the car. Therefore, both the figures show that the all the approaches of RMPC are stable at response time. Figure 5 shows the output and control signal of the system in the response time.

The results presented in Figure 5 evidence similar performances. Also, note that the design of the designed observer ensures the stability in comparison with off-line state feedback.

Furthermore, the RMPC-LPV proposed methodology shows a feasible response to the pendulum's mass, randomly varying between the nominal value and fifteen times this value.



(a) Interaction between states $x_3(k)$, $x_1(k)$ and X_{set}



(b) Interaction between states $x_4(k)$, $x_2(k)$ and X_{set}

Figure 3. Optimization process of RMPC LPV off-line.

Considering the RMPC performance indices, it is adopted the approach of Carmo et al. (2012). Table 2 shows the respective indices for each implemented RMPC. It is observed that the off-line output feedback RMPC technique present the best performance in comparison with the other control designs. Therefore, the results obtained by off-line output feedback demonstrates that the adding of robust observer design can enhance the general performance at response time and the off-line algorithm ensures the feasibility of optimization.

Table 2: Performance indices

	On-line	Off-line	Off-line observer
IAE	0.0462	0.5290	0.0437
ISE	0.0192	0.0212	0.0178
ITAE	0.0063	0.0063	0.0045
ITSE	0.0005	0.0006	0.0004

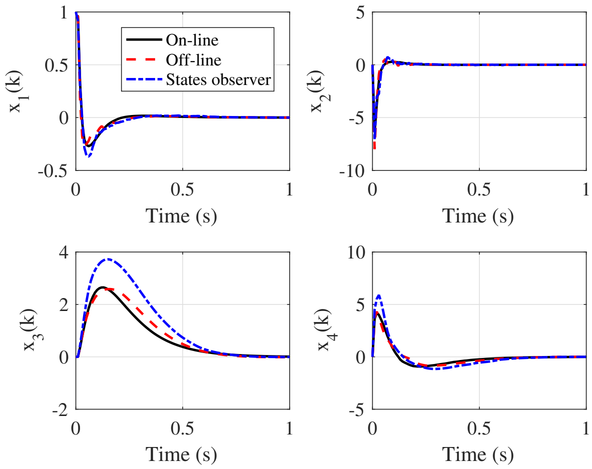


Figure 4. Time response for each states, $\alpha \in [m,\ 5m]$.

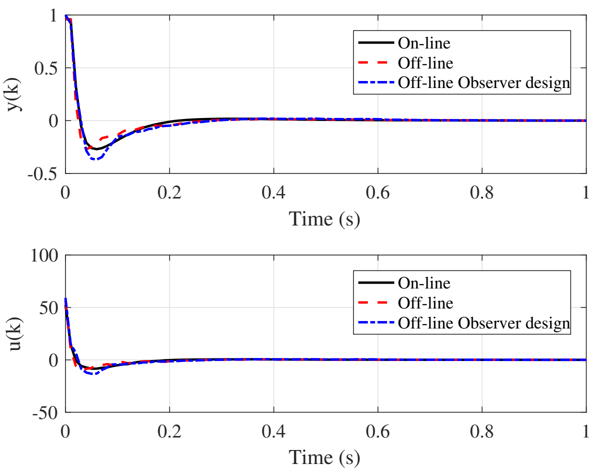


Figure 5. Time response of $y(k)$ and $u(k)$ for $\alpha \in [m,\ 5m]$.

5. CONCLUSIONS

In face of the obtained results, the RMPC control techniques presented similar response time. However, the off-line output feedback RMPC technique showed the best performance indices in comparison with other approaches displayed at long of study. Furthermore, the robust observer design for output feedback control ensures a significant performance along with the recurrent feasibility optimization procedure inside of the off-line algorithm. Therefore, the proposed study showed that the off-line output feedback control RMPC-LPV is robust, provided by stability ellipsoids theory, and obtains better results when compared with other RMPC techniques using the same design control specifications. This study proposes as future work the possibility of application the proposed method in an experimental setup as a next step of this research.

ACKNOWLEDGMENTS

This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001. The authors would like to thank the Federal Rural University of the Semi-Arid Region (UFERSA) and the Research Group on Automation and Control (GEAC) (financial provided by PIG00009-2018 project) for their support and support in the production of this work.

REFERENCES

Aguirre, L., Bruciapaglia, A., and Miyagi, P.E e Piqueira, J. (2007). *Enciclopédia de automática: controle e automação*. Blucher.

Boyd, S., El Ghaoui, L., Feron, E., and Balakrishnan, V. (1994). *Linear Matrix Inequalities in System and Control Theory*, volume 15 of *Studies in Applied Mathematics*. SIAM, Philadelphia, PA.

Briat, C. (2015). Introduction to lpv systems. *Linear Parameter-Varying and Time-Delay Systems*. doi:10.1007/978-3-662-44050-6_1.

Bumroongsri, P. (2014). An offline formulation of mpc for lpv systems using linear matrix inequalities. *Journal of Applied Mathematics*, 2014, 1–13. doi:10.1155/2014/786351.

Caigny, J.D., Camino, J.F., Oliveira, R.C., Peres, P.L., and Swevers, J. (2009). Gain-scheduled h-control for discrete-time polytopic lpv systems using homogeneous polynomially parameter-dependent lyapunov functions. *IFAC Proceedings Volumes*, 42(6), 19 – 24. doi:https://doi.org/10.3182/20090616-3-IL-2002.00004. 6th IFAC Symposium on Robust Control Design.

Carmo, M.J., JR., L.A., Oliveira, A., Panoeiro, N., and Rocha, K. (2012). Índices não intrusivos utilizados no ensino de controle com técnicas de identificação em malha aberta.

Costa, M., Reis, F., Campos, J., Nogueira, F., and Almeida, O. (2017). Robust mpc-lmi controller applied to three state switching cell boost converter. *Eletrônica de Potência*, 22, 81–90. doi:10.18618/REP.2017.1.2652.

Cuzzola, F.A., Geromel, J.C., and Morari, M. (2002). An improved approach for constrained robust model predictive control. *Automatica*, 38(7), 1183 – 1189. doi:https://doi.org/10.1016/S0005-1098(02)00012-2.

Hu, J. and Ding, B. (2018). An efficient offline implementation for output feedback min-max mpc: offline min-max mpc. *International Journal of Robust and Nonlinear Control*, 29. doi:10.1002/rnc.4401.

Jung, S. and Wen, J.T. (2004). Nonlinear Model Predictive Control for the Swing-Up of a Rotary Inverted Pendulum . *Journal of Dynamic Systems, Measurement, and Control*, 126(3), 666–673. doi:10.1115/1.1789541.

Kajiwara, H., Apkarian, P., and Gahinet, P. (1999). Lpv techniques for control of an inverted pendulum. *IEEE Control Systems*, 19(1), 44–54. doi:10.1109/37.745767.

Koelewijn, P., Cisneros, P., Werner, H., and Toth, R. (2018). Lpv control of a gyroscope with inverted pendulum attachment. *IFAC-PapersOnLine*, 51(26), 49 – 54. 2nd IFAC Workshop on Linear Parameter Varying Systems LPVS 2018.

Kothare, M., Balakrishnan, V., and Morai, M. (1996). Robust constrained model predictive control using linear matrix inequalities. *Automatica*, 32, 440 – 444 vol.1. doi:10.1109/ACC.1994.751775.

Longge, Z. and Yan, Y. (2017). Robust shrinking ellipsoid model predictive control for linear parameter varying system. *PLoS ONE*, 12. doi:10.1371/journal.pone.0178625.

Lundberg, K.H. and Barton, T.W. (2010). History of inverted-pendulum systems. volume 42, 131 – 135. doi:https://doi.org/10.3182/20091021-3-JP-2009.00025. 8th IFAC Symposium on Advances in Control Education.

Moradi, S., Akbari, A., and Mirzaei, M. (2018). An offline lmi-based robust model predictive control of vehicle active suspension system with parameter uncertainty. *Transactions of the Institute of Measurement and Control*, 41. doi:10.1177/0142331218787599.

Nam, D.P., Quang, N.H., Van Huong, N., and Dong, N.M. (2019). Tube based robust model predictive control for an inverted pendulum via solving linear matrix inequalities. In *Proceedings of the 5th International Conference on Mechatronics and Robotics Engineering*, ICMRE'19, 69–72. doi:10.1145/3314493.3314503.

Ogata, K. (2011). *Engenharia de controle moderno*. PRENTICE HALL BRASIL.

Park, J.H., Kim, T.H., and Sugie, T. (2011). Output feedback model predictive control for lpv systems based on quasi-min-max algorithm. *Automatica*, 47(9), 2052 – 2058.

Ping, X. (2017). Output feedback robust mpc based on offline observer for lpv systems via quadratic boundedness: Output feedback robust mpc for lpv systems via quadratic boundedness. *Asian Journal of Control*, 19. doi:10.1002/asjc.1469.

Scherer, C. (2001). Lpv control and full block multipliers. *Automatica*, 37(3), 361 – 375. doi:https://doi.org/10.1016/S0005-1098(00)00176-X.

Teixeira, M., Pietrobom, H., and Assunção, E. (2000). Novos resultados sobre estabilidade e controle de sistemas não-lineares utilizando modelos fuzzy e lmi. 11.

Wada, N., Saito, K., and Saeki, M. (2006). Model predictive control for linear parameter varying systems using parameter dependent lyapunov function. *IEEE Transactions on Circuits and Systems II: Express Briefs*, 53(12), 1446–1450.

Wan, Z. and Kothare, M.V. (2002). Robust output feedback model predictive control using off-line linear matrix inequalities. *Journal of Process Control*, 12(7), 763 – 774. doi:https://doi.org/10.1016/S0959-1524(02)00003-3.

Wan, Z. and Kothare, M. (2003). An efficient off-line formulation of robust model predictive control using linear matrix inequalities. *Automatica*, 39(5), 837–846. doi:10.1016/S0005-1098(02)00174-7.

Watson, M.T., Gladwin, D.T., Prescott, T.J., and Conran, S.O. (2019). Dual-mode model predictive control of an omnidirectional wheeled inverted pendulum. 1–1. doi:10.1109/TMECH.2019.2943708.

Xiao-Jun Ma, Zeng-Qi Sun, and Yan-Yan He (1998). Analysis and design of fuzzy controller and fuzzy observer. *IEEE Transactions on Fuzzy Systems*, 6(1), 41–51. doi:10.1109/91.660807.

Yue, M., An, C., and Sun, J.Z. (2018). An efficient model predictive control for trajectory tracking of wheeled inverted pendulum vehicles with various physical constraints. volume 16, 265–274. doi:10.1007/s12555-016-0393-z.

Zheng, H., Zou, T., Hu, J., and Yu, H. (2018). An offline optimization and online table lookup strategy of two layer model predictive control. *IEEE Access*, PP, 1–1. doi:10.1109/access.2018.2862428.