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# Nonlinear Model Approximation Methods for Off-road Vehicle Path Tracking with MPC ⋆

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**Abstract:** This paper presents a comparative study between two different approaches for dynamic systems based on Taylor series and based on a double-integrator, and their influence in path tracking control using model predictive controller (MPC) with the view to solve the path tracking problem of an autonomous off-road vehicle. A physical-mathematical model, including the celebrated tire model known as "Magic Formula", is modified and linearized to control the steering of the vehicle considering different reference trajectory inputs. Simulation results are considered satisfactory, showing that the model linearized considering Taylor series presented better results compared to the double-integrator which indicates that the prediction of the plant output is driven close to the reference. Moreover, the results show that the implemented control strategies for path tracking are adequate for applications in off-road autonomous vehicles.

*Keywords:* Path tracking, linearization models, model predictive control, off-road autonomous vehicles.

## 1. INTRODUCTION

With the science and technology advance, autonomous vehicles are coming into view (Lin et al., 2019). Autonomous vehicles are those in which some aspects of a safety-critical control function (e.g., steering, braking) occur without a direct driver interference increasing safety and improving energy efficiency (NHTSA, 2013; Sun et al., 2018; Li et al., 2018). One of the most important topics of autonomous vehicles research has been the path tracking problem, which refers to following a desired path accurately by controlling the speed and/or steering of the vehicle by means of designing control techniques (Raffo et al., 2009; Sun et al., 2012).

Path tracking of off-road vehicles is a complex task since these vehicles are subjected to high level of slip due to the rough and curvy terrains as well as sudden terrain changes that can lead to significant difference between the real and predicted trajectory. Thus, tire-soil interaction models, such as those proposed by (Bekker, 1962) and (Wong and Reece, 1967a,b) became reference in studies related to off-road vehicles but the celebrated "Magic Formula", firstly proposed by (Bakker et al., 1987), is one of the most used tire models even for a general off-road analysis. Here, a dynamic model is combined with a kinematic model considering the "Magic Formula" tire model to design the controller.

However, as these models are nonlinear, it is common to linearize the states in order to design trajectory tracking controllers. Thus, two different approaches are proposed and compared in this work in order to simplify the vehicle dynamic model. First, a method based on the expansion of nonlinear function into a Taylor series is developed. This expansion occurs about a point of equilibrium operation maintaining only the linear term. Recent research has highlighted the potential of the linearization based on Taylor series as Gidlewski and Zardecki (2016) that investigated the vehicle motion control due to the linearization of lateral dynamics. Similarly, Snider (2009) linearized the lateral dynamic of a vehicle considering the traditional "bicycle model" with Taylor series approach. Linear time-varying model of lateral dynamics based on Taylor expansion to design control algorithms for path trajectory of autonomous vehicle are developed by Shen et al. (2017); Lin et al. (2019). Second, the dynamic system is simplified into a double integrator plant, which is one of the most fundamental systems in control applications (Rao and Bernstein, 2001). Different works have used double integrator as simplification to dynamic model as Cabecinhas and Silvestre (2019) that proposed a nonlinear controller based on a double integrator simplified model for path tracking of an autonomous vessel. A containment control for multiple autonomous vehicles, modelled as a double integrator, was studied by Cao et al. (2011). Qian (2016), to design a path tracking controller, simplified the vehicle dynamics by a double integrator taking into account tire force to limit the lateral acceleration. George and O'Brien (2004) developed a strategy control for lateral vehicle dynamics using a double integrator model.

In order to solve the path tracking problem, it is required to design a path tracking controller that aims to minimize

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the difference between the desired path and the vehicle trajectory. Thus, a model predictive control (MPC) is proposed in this paper. This technique is widely used to path tracking control since it uses the physical-mathematical model of a vehicle in order to predict a future situation over a finite horizon (Ataei et al., 2020). Considering off-road vehicles, different works have been used dynamic and kinematic models with slip assumption in order to design a MPC controller as Lenain et al. (2005); Lenain et al. (2007) that used a kinematic with two slip angles to develop a MPC controller which presented satisfactory results even though the slip estimation were considered a problem because the absence of a more robust method in their modelling. Raffo et al. (2009) applied MPC techniques in an autonomous Baja vehicle considering different paths input in a dynamic model of the vehicle. The results were considered satisfactory after practical experiments to validate the simulation data. Shen et al. (2017) applied a MPC-based path tracking control in a ground vehicle over three different types of roads: wet and dry asphalt pavement, and ice-covered soil. Li et al. (2019) proposed a nonlinear model predictive controller for path tracking of a ground vehicle considering the Magic Formula.

The aim of this work is to provide a contribution towards nonlinear model approximation techniques and their influence in path tracking control of autonomous vehicles. Therefore, this paper presents a comparative study between two different approaches based on Taylor series and double-integrator considering different reference trajectories. Moreover, the presented results are analysed and compared in the view of two different metrics: $R^2$ and RMSE.

The remainder of this paper is organized as follows: In section II, the bicycle dynamic model and the kinematic techniques are described and discussed. In section III, the scheme of MPC is detailed. Numerical results are shown in Section IV, where the proposed approaches are applied to design the MPC controller to path tracking and then, they are tested and compared according to different road paths. Finally, in Section V we write the conclusion of this paper and point future research directions.

## 2. NONLINEAR STATE-SPACE MODEL

A rigid vehicle is assumed to move on a flat surface considering a constant longitudinal velocity and small steer angles, so that the planar forces on the tire surface act at the wheel center (Fig. 1). Thus, the following equations describe the Newton-Euler equations of motion for a rigid vehicle in the local coordinate frame

$$F_y(t) = m\ddot{y}(t) + m\dot{\psi}(t)v_x\,, \tag{1}$$

$$M_z(t) = I_z\ddot{\psi}(t)\,, \tag{2}$$

where $F_y$ is the total lateral force, $m$ is the vehicle mass, $y$ is the position in the local coordinate frame, $\psi$ is the vehicle yaw angle, $v_x$ is the constant longitudinal velocity, $M_z$ is the yaw moment, $I_z$ is the moment mass relative to the vertical axis and $t$ is the time.

Considering a small steer angle $\delta_f$, the total force can be approximated by

$$F_y \approx F_{yf} + F_{yr}\,, \tag{3}$$

$$M_z \approx aF_{yf} - bF_{yr}\,, \tag{4}$$

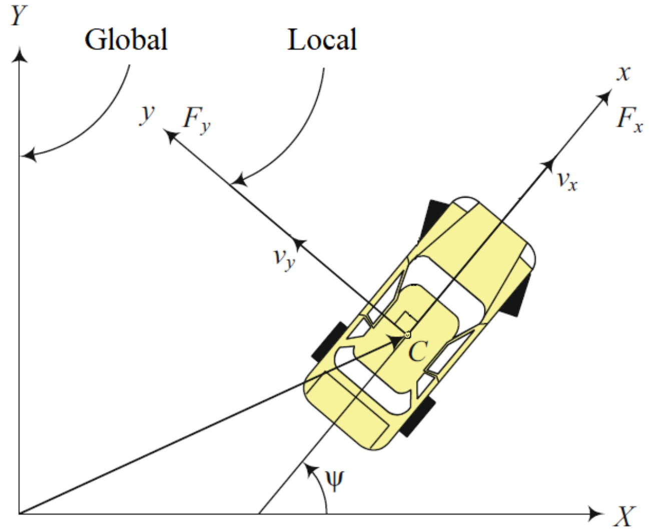


Figure 1. Global and local coordinate systems.

where $F_{yf}$ and $F_{yr}$ are the lateral forces on the tireprint of the front and rear tires, respectively; $a$ and $b$ are the distance from center of gravity (CG) to front axle and rear axle, respectively.

The tire model which is actually most well established is based on the work of Bakker et al. (1987). The tire model is known as "Magic Formula" since there is no particular physical basis for equations chosen. Then, the "Magic Formula" is not considered a predictive model but is commonly used to represent tire force over different operating conditions (Blundell and Harty, 2004; Jazar, 2017). In this work, the third version of the "Magic Formula" is used for lateral forces

$$F_y(\alpha) = Dsin\{Ctan^{-1}[B(\alpha+s_h) - E(B(\alpha+s_h) - tan^{-1}(B(\alpha+s_h)))]\} + s_v\,, \tag{5}$$

where $\alpha$ is the sideslip angle. The sideslip angle is important for off-road vehicles since a high slip may lead to the vehicle instability. Sideslip angles for the front ($\alpha_1$) and rear ($\alpha_2$) tires can be determined respectively by

$$\alpha_1 = -\frac{\dot{y} + a\dot{\psi}}{v_x} + \delta_f\,, \tag{6}$$

$$\alpha_2 = -\frac{\dot{y} - b\dot{\psi}}{v_x}\,. \tag{7}$$

The other parameters which are listed on Table 1, depend on the vertical force $F_z$ and the camber angle $\gamma$.

Table 1. Magic Formula Parameters (Blundell and Harty, 2004)

| Parameter |
|---|
| $D = \mu F_z$ |
| $\mu = (a_1F_z + a_2)(1 - a_{15}\gamma^2)$ |
| $BCD = a_3sin(2tan^{-1}(F_z/a_4))(1 - a_5\lvert\gamma\rvert)$ |
| $C = a_0$ |
| $E = (a_6F_z + a_7)(1 - (a_{16}\gamma + a_{17})sgn(\alpha + s_h))$ |
| $B = BCD/CD$ |
| $s_h = a_8F_z + a_9 + a_{10}\gamma$ |
| $s_v = a_{11}F_z + a_{12} + (a_{13}F_z^2 + a_{14}Fz)\gamma$ |

The real trajectory $Y(X)$ in the global coordinates can be determined by integrating the following equations

$$\dot{X}(t) = v_xcos(\psi(t)) - \dot{y}(t)sin(\psi(t))\,, \tag{8}$$

$$\dot{Y}(t) = v_x sin(\psi(t)) - \dot{y}(t)cos(\psi(t))\,, \tag{9}$$

Consequently, we have

$$\ddot{Y}(t) = v_x\dot{\psi}(t) + \ddot{y}(t)\,. \tag{10}$$

Finally, reorganizing (10) and substituting them in (1) and (2), the dynamic equations considering kinematic aspects can be obtained:

$$m\ddot{Y}(t) = F_y(t)\,, \tag{11}$$

$$I_z\ddot{\psi}(t) = M_z(t)\,. \tag{12}$$

In terms of a nonlinear system with the form $\dot{\mathrm{x}} = f(x,u)$, where $\mathrm{x} = [\mathrm{Y}, \psi, \dot{\mathrm{Y}}, \dot{\psi}]$ is the state vector and $\mathrm{u} = \delta_\mathrm{f}$ is the input signal of the plant, we have:

$$\dot{x} = \begin{bmatrix} \dot{Y} \\ \dot{\psi} \\ \ddot{Y} \\ \ddot{\psi} \end{bmatrix}\,. \tag{13}$$

In this paper two different methods were used in order to simplify the nonlinear model of the vehicle. The first method is based on expanding the Taylor series around an equilibrium point of operation (see Section 2.7 of Ogata (2010) for details). If a given dynamic system works around an equilibrium point and if the signals involved are considered small, then it is possible to linearize the dynamic system. Once the nonlinear system in the form $\dot{\mathrm{x}} = f(x,u)$ in (13) determined, the linearization of the system around the following equilibrium can be obtained considering the input signal $\mathrm{u}^* = 0$,

$$\left.\begin{bmatrix} \dot{Y} \\ \dot{\psi} \\ \ddot{Y} \\ \ddot{\psi} \end{bmatrix}\right|_* = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\,.$$

The linearized model can be described as

$$\delta\dot{x} = A\delta x + B\delta x\,, \tag{14}$$

where $\delta\mathrm{x} = \mathrm{x} - \mathrm{x}^*$ and $\delta\mathrm{u} = \mathrm{u} - \mathrm{u}^*$ are the disturbance variables; $A$ and $B$ are Jacobian matrices related to the state vector and input signal around the equilibrium point.

The second method is simple and consists in simplifying the system using a double integrator with velocity dependent gain. This choice was made by empirically observing the lateral displacement by harmonic excitations, as it was found similar to the dynamics of double integrators. In order to set the gain parameter, simulations were carried out in such conditions and the matching was made by trial and error.

## 3. MODEL PREDICTIVE CONTROLLER

Model predictive controller (MPC) is an optimal control algorithm that is widely used to predict, on a specific finite time-horizon, the future states of a dynamic system (Amer et al., 2016). MPC uses a discrete-time model in order to predict the future states:

$$\begin{aligned} x(k+1) &= A_d x(k) + B_d u(k)\,, \\ y(k) &= C_d(k)\,, \end{aligned} \tag{15}$$

where $x(k)$ is the state vector, $y(k)$ is the output vector, $u(k)$ is the input vector; $A_d$, $B_d$ and $C_d$ are the state-space matrices of the vehicle model in discrete-time form.

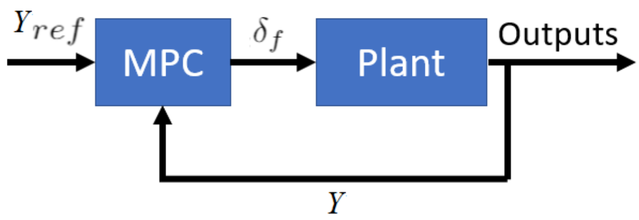


Figure 2. Scheme of the MPC controller without disturbance.

Based on the predicted states, the control action is determined by minimizing the following cost function:

$$J(k) = \sum_{n=1}^{N_p} \parallel y_{ref}(k+n) - y(k+n) \parallel_Q^2 + \sum_{n=0}^{N_c-1} \parallel u(k+n) \parallel_R^2\,, \tag{16}$$

where $y_{ref}$ and $y$ are the output reference and predicted, respectively, $N_p$ is the predictive horizon, $N_c$ is the control horizon, $Q$ and $R$ are weighting matrices regarding the tracking errors and the control input efforts, respectively.

The control scheme for the path tracking system is shown in Fig. 2, where the inputs of the controller are the reference trajectory ($Y_{ref}$) and the trajectory predicted ($Y$) in order to determine a specific steer angle ($\delta_f$) which works as input for the plant.

## 4. NUMERICAL RESULTS

In this section, the MPC controller designed by both the Taylor series and double integrator is applied in cases with different trajectories input in order to verify the effectively of each proposed method. It is important to state that both methods are used to design the MPC controller. After that, the controller is applied to the nonlinear system.

Before starting the tests, it is essential to ensure that the system is controllable and observable. It then follows that

$$\begin{aligned} \text{rank } Q_c &= \text{rank } [B\ AB\ A^2B] = 3\,, \\ \text{rank } Q_o &= \text{rank } [C\ CA\ CA^2] = 4\,, \end{aligned} \tag{17}$$

where $Q_c$ and $Q_o$ are matrices that have full rank indicating an observable and controllable dynamic system.

For the MPC controller implementation, the simulation step, predictive horizon and control horizon are 0.01s, 12 steps ans 2 steps, respectively. For vehicle control problems this horizon is commonly applied, since the dynamics response of the vehicle are sufficiently fast enough to be noted if occurs some disturbance (Beal, 2011).

The vehicle simulation parameters are listed in Table 2 and are based on a “Baja” prototype which are similar to dune buggies. Regarding the sensing of the state variables, it was considered that all four states at global reference (position, velocity, yaw angle and angular velocity) can be monitored on the vehicle. However, for the MPC controller, only lateral position state is required. The problem was solved by using MATLAB/Simulink software version R2019b.

The coefficients $a_n$ for the “Magic Formula” were provided by Dunlop Tyres using in-house software to adjust the parameters. The data (Table 3) is available in Blundell and Harty (2004).

Substituting all parameters in the linearized model of the system, we can obtain the linearized matrices that will be used by Taylor Series technique as follows

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -11.097 & -12.039 \\ 0 & 0 & -0.184 & -27.991 \end{bmatrix} ; B = \begin{bmatrix} 0 \\ 0 \\ 53.580 \\ 224.811 \end{bmatrix} . \quad (18)$$

In order to compare the simulation results, two commonly metrics based on the error between the reference signal and predicted data are used in this paper: Root Mean Squared Error (RMSE) and Multiple Correlation Coefficient ($R^2$). These metrics are formulated as

$$RMSE = \sqrt{\frac{1}{N}\sum_{t=1}^{N}[y(t) - \hat{y}(t)]^2} , \quad (19)$$

$$R^2 = 1 - \frac{\sum_{t=1}^{N}[y(t) - \hat{y}(t)]^2}{\sum_{t=1}^{N}[y(t) - y_{mean}(t)]^2} , \quad (20)$$

where $y$ is the reference signal, $\hat{y}$ is the predicted data, $y_{mean}$ is the mean value of the reference signal, and $N$ is the length of the data vector.

The poles location related to lateral position of the linearized system based on Taylor series are located on the left half of the s-plane ($p_1 = 0$; $p_2 = 0$; $p_3 = -10.97$ and $p_3 = -28.12$) being so, no pole is unstable. Two repeated poles are located at the origin which is a indicative of the potential use of the double integrator in the study. Different tests changing some parameters such as vehicle mass, vehicle inertia, location of the center of gravity were made and it was observed that the system maintained the stable situation.

We show in the following, comparative tests and their impacts in a MPC controller for path tracking considering: step input signal (Section 4.1); lane-change input signal (Section 4.2); sinusoidal input signal (Section 4.3).

Table 2. Table of Vehicle Parameters

| Parameter | Description | Value | Unit |
|---|---|---|---|
| m | Mass of the vehicle | 200 | kg |
| $I_z$ | Yaw moment of inertia | 42.9 | kg.m$^2$ |
| l | Wheelbase | 1.5 | m |
| a | Distance front axle to CG | 0.9 | m |
| b | Distance rear axle to CG | 0.6 | m |
| vx | Longitudinal velocity | 12 | m/s |

Table 3. Coefficients from Magic Formula (Blundell and Harty, 2004)

| Parameter | Value | Parameter | Value |
|---|---|---|---|
| $a_0$ | 1.0337 | $a_9$ | 0 |
| $a_1$ | -0.2245e-5 | $a_{10}$ | 0 |
| $a_2$ | 0.8 | $a_{11}$ | 0 |
| $a_3$ | 0.6040e5 | $a_{12}$ | 0 |
| $a_4$ | 0.8777e4 | $a_{13}$ | 0 |
| $a_5$ | 0 | $a_{14}$ | 0 |
| $a_6$ | 0,4581e-4 | $a_{15}$ | 0 |
| $a_7$ | 0,4682 | $a_{16}$ | 0 |
| $a_8$ | 0 | $a_{17}$ | 0 |

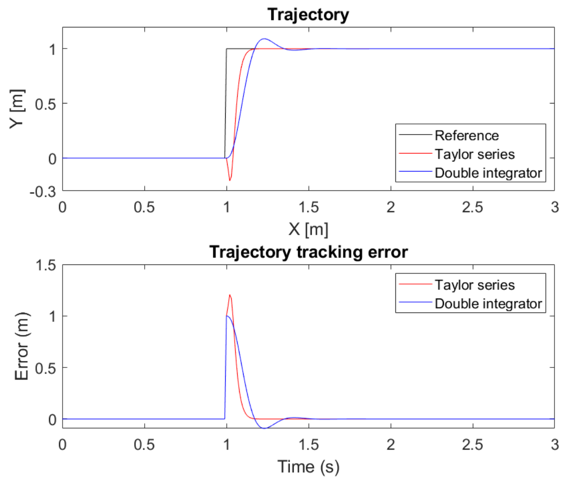


Figure 3. Trajectory and trajectory tracking error - unit step function.

### 4.1 Step Input Signal

For the first test, a unit step trajectory is used, starting at 1 s. As shown in Fig. 3, the trajectory is followed faster by the system linearized by Taylor series than that in which a double integrator is applied, even though the case with Taylor series presents some deviation in negative direction that do not impact the final result. Consequently, the trajectory tracking error for Taylor series starts bigger than that presented for the double integrator. However, after achieving the maximum value, the error starts to decrease faster than that for the double integrator and the steady state is achieved. Regardless of the method used to simplify the dynamic system, MPC acts in a receding horizon obtaining new response frequently whilst controllers, as for example those based on linear quadratic regulator (LQR) type, use the optimal solution (resulted from a optimization process) for the entire time horizon. Thus, MPC controller commonly spent less time to obtain an optimal solution.

The results can be corroborated by the control action (Fig. 4) and Metric results (Table 4). The system linearized by Taylor series presented faster results, even though, at the beginning of the motion, the system decreases into negative values. Considering $R^2$ and RMSE metrics, we can note that the Taylor series presented the best results although double integrator results are similar.

Table 4. Unit Step Signal Results

| Technique | $R^2$ | RMSE |
|---|---|---|
| Taylor series | 0.9107 | 0.1195 |
| Double integrator | 0.9020 | 0.1252 |

### 4.2 Lane-Change Input Signal

The second test performed considered the off-road vehicle during a lane-change manoeuvre starting at 2 s.

It can be seen in Fig. 5 that the off-road vehicle follows the track with good performance even though during the single-lane maneuver an trajectory error appears for both methods. However, Taylor series performed better than

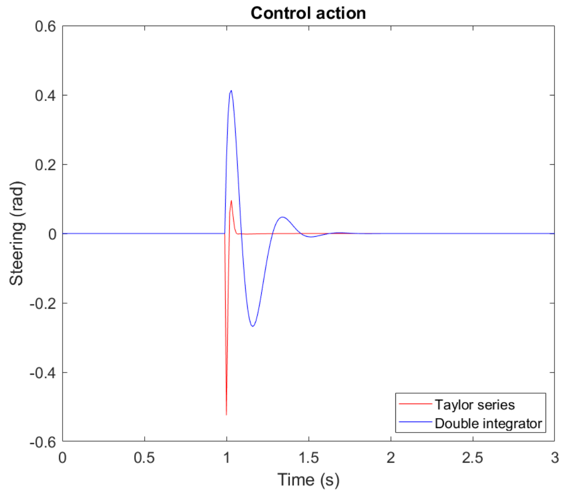


Figure 4. Control action - unit step function.

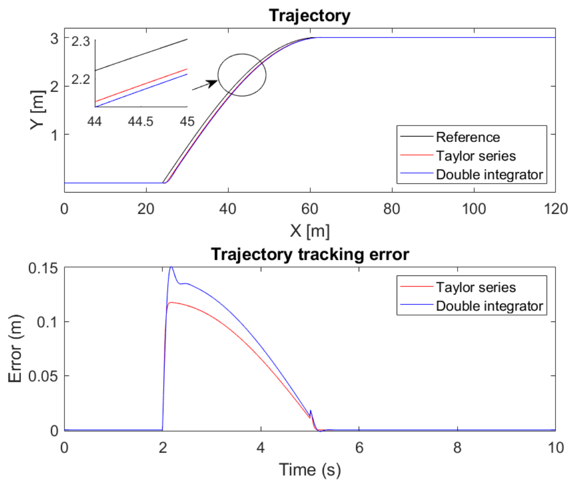


Figure 5. Trajectory and trajectory tracking error - lane-change signal.

double integrator, as observed with the previous test, since it produces a smaller difference between the reference trajectory and the predicted trajectory, which confirms that this method of linearization is doing well.

In Fig. 5, two peak points can be identified, one comes from the controller response after the lane change starts and the following is caused by the controller response at the end of the lane change in order to complete the single-lane change maneuver.

The results can either be corroborated by the metric results (Table 5). Although the results with double integrator presented good metric results, Taylor series linearization performed better in this case with $R^2$ and RMSE metric values very close to the ideal.

Table 5. Single Lane Change Signal Results

| Technique | $R^2$ | RMSE |
|---|---|---|
| Taylor series | 0.9986 | 0.0464 |
| Double integrator | 0.9981 | 0.0546 |

The control action from the controller can be observed in Fig. 6 and indicates that the vehicle steering is constant

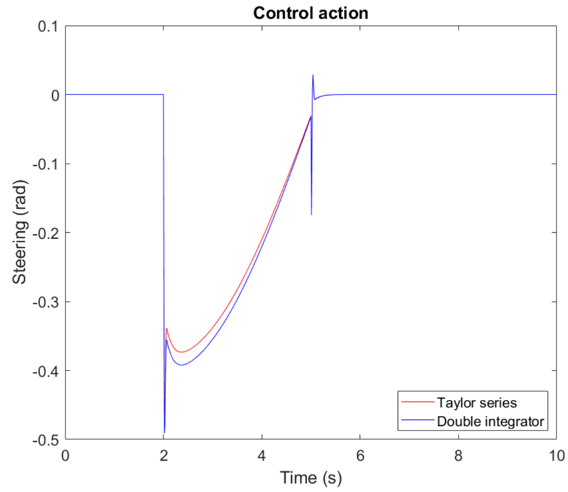


Figure 6. Control action - lane-change signal.

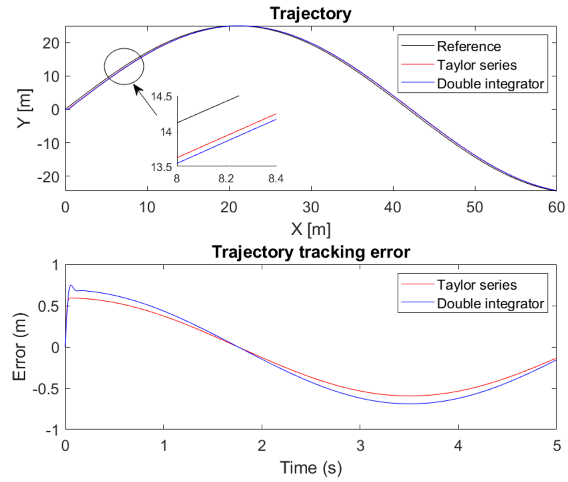


Figure 7. Trajectory and trajectory tracking error - sinusoidal signal (amplitude 25 m).

until the beginning of the maneuver. After that, the maneuver occurs indicating that for Taylor series a smaller steering angle is necessary to path tracking if compared with the system composed by a double integrator. Finally, the steering angle remains zero until the end of the simulation which indicates the end of the lane-change maneuver.

### *4.3 Sinusoidal Input Signal*

For this simulation, the off-road vehicle performed a manoeuvre in a form of a sinusoidal function (amplitude of 25 m and angular frequency of 0.3 rad/s).

It can be noted for this test that both Taylor series and double integrator responses (Table 7) are close to each other. However, the trajectory tracking error increases if compared to the previous test. This occurs mainly due to the linearization process which simplify the dynamic model and, consequently, affects the MPC controller performance. Despite that both methods presented good responses for path tracking as seen in tests before.

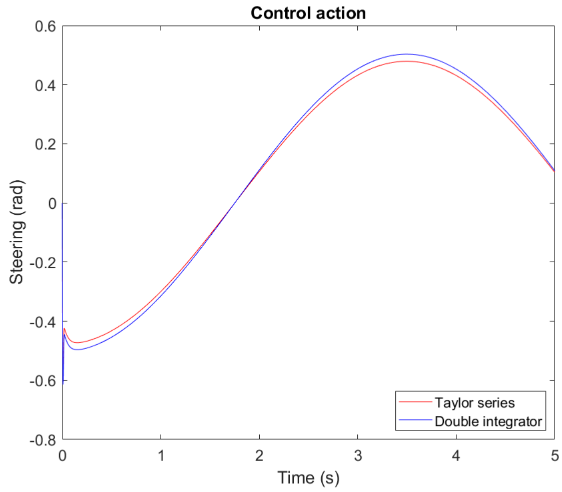


Figure 8. Control action - sinusoidal signal (amplitude 25 m).

Although double integrator performed well in both metrics ($R^2$ and RMSE) (Table 6), Taylor series performed even better. $R^2$ metric results are the best results considering all tests performed. However, for RMSE metric the results are higher than the other performed tests. This is due to the sum of significant quadratic errors which increase the metric value.

The control action from the controller can be observed in Fig. 8 and indicates that the vehicle steering is similar to the path followed. As noted in the previous test, the steering angle presented by the system linearized by Taylor series is smaller if compared with the system composed by a double integrator which indicates a faster and precisely path tracking.

## 5. CONCLUSIONS

The present work dealt with the application of Taylor series and double integrator to design a model predictive controller in order to solve the path tracking problem of an autonomous off-road. The linearization based on Taylor series is applied in a physical-mathematical model which considers dynamic and kinematic characteristics of the vehicle. The model still considers the well-known "Magic Formula" tire model.

Three different tests considering different trajectories were performed: unit step, single lane-change, and sinusoidal signal. $R^2$ and RMSE metrics were established to compare the reference trajectory and that performed by the system with a determined controller. Based on the results obtained we conclude that the linearization based on Taylor series performed better for all tests if compared with double integrator in order to provide an effective means to solve the path tracking problem of an autonomous off-road vehicle. However, it is important to point out that a double integrator can be used for a preliminary test, since it presented good results for all tests.

Table 6. Sinusoidal Signal Results

| Technique | $R^2$ | RMSE |
|---|---|---|
| Taylor series | 0.9993 | 0.4283 |
| Double integrator | 0.9990 | 0.4977 |

In the future we shall focus on implement a nonlinear model predictive model (NMPC) to enhance the accuracy of the control system; improve the dynamic model adding a more complex tire model that take into account tire properties, terrain properties and other nonlinearities looking for a more realistic setting for the system as Wong and Reece (1967a,b). After that, we will try to validate the numerical experiments herein conducted.

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