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$\mathcal{H}_2$ gain-scheduled state-feedback synthesis conditions applied to a quadruple tank system

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Abstract: This article presents design conditions for gain-scheduled controllers with polynomial dependence on the parameters for LPV systems. The technique is used to solve the problem of controlling a quadruple tank considering variations in valve opening. First, the nonlinear modeling of the system as well as an analysis of its characteristics are presented, and then an approximation to the dynamics is proposed in terms of a LPV polynomial model. As main contribution, a sufficient condition based on LMIs is proposed for the design of gain-scheduled controllers by state feedback with performance criterion based on the $\mathcal{H}_2$ norm. The design condition is general in the sense of treating LPV models with an arbitrary number of parameters and degree of polynomial dependence. Simulations in three distinct scenarios are presented to illustrate the design methodology and the quality of the controller designed for the quadruple tank.

Keywords: Linear parameter-varying; Gain-scheduled; State-feedback control; LMIs; Quadruple tank.

1. INTRODUCTION

Systems with two-input two-output (TITO) are the prevalent category of multiple-input multiple-output (MIMO) systems in industry, because many real processes belong to this class of systems or because more complex systems can be decomposed in a TITO process with good decoupling among their inputs and outputs.

The quadruple tank (QT), proposed in (Johansson, 2000), is a TITO system extensively investigated due to its characteristics (as multivariable process, coupling and the positioning of the transmission zero). A schematic representation of this system is presented in Figure 1. The QT system presents a considerable coupling between channels and has an adjustable transmission zero that can be either minimum or non-minimum phase, depending on the opening of the valves. If the sum of the percentage openings regarding the lower tanks is greater than 1 ($\gamma_1 + \gamma_2$), the system features a minimum phase zero. Otherwise, the system has a non-minimum phase zero.

Various control techniques have been implemented in the QT system, such as decoupled PI, internal model control (IMC), model predictive control (MPC), sliding mode control, among others. In (Åström et al., 2002) a decoupled PI structure is considered together with a static decoupling. In (Husek, 2014) decentralized PI controllers are designed based on phase margin specifications. The work

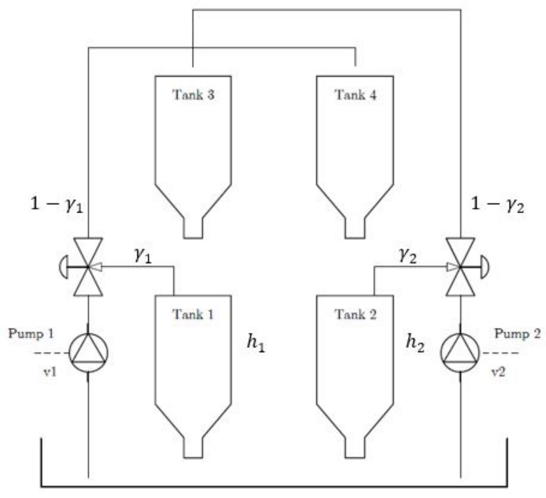

Figure 1. Schematic of a Quadruple Tank.

in (Gatzke et al., 2000) applies inner-outer factorization for non-minimum phase internal model control. A distributed model predictive control framework is proposed and applied to the quadruple tank in (Mercangöz and Doyle III, 2006), while in (Almurib et al., 2011) an offset-free model predictive control is addressed. In (Li and Zheng, 2014) an H_∞ loop-shaping is applied to the QT system and an analysis of the robustness of different control techniques was performed in reference (Vadigepalli et al., 2001), considering IMC (Internal Model Control), PI (proportional integral) and H_∞. Other robust control techniques were tested in the same plant, as in (Neves et al., 2016), which

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applies an LQG/LTR control, and in (Neves and Angélico, 2016), which works with dynamic decoupling and QFT control for uncoupling uncertainties.

Dynamic systems affected by time-varying parameters have attracted considerable interest in the last decades in academic researches as well in industrial applications (Hoffmann and Werner, 2015; Mohammadpour and Scherer, 2012; Palma et al., 2020b,a). One of the reasons for this fact is that linear time-invariant (LTI) approximations sometimes can lead to poor performance in closed-loop or even instability (Shamma and Athans, 1991). Moreover, one of the main appeals of *linear parameter-varying* (LPV) models is the ability to represent locally nonlinear systems with more accuracy than standard LTI models (Caigny et al., 2011; Tóth, 2010).

The purpose of this paper is to provide a new $\mathcal{H}_2$ gain-scheduled state-feedback synthesis conditions for continuous-time LPV systems, with application in a QT system. First, an LPV model is proposed to represent the QT system, having as time-varying parameters the percentage opening of the two valves. Then, design conditions solvable in terms of LMIs are provided as main contribution. The conditions allow to take into account bounded rates of variation for the time-varying parameters, which are specially suitable to deal with QT system. A numerical experiment is given to illustrate the approach.

2. MODELING

The quadruple tank is formed by four interconnected tanks. The system features two pumps (controllable inputs) that pour liquid through two valves, which divide the liquid flow between tanks 1 and 4 (valve 1), and tanks 2 and 3 (valve 2) (Johansson, 2000). A schematic drawing is shown in Figure 1.

The dynamic model is based on the mass balance in each tank (Neves et al., 2016). Considering the schematic presented in Figure 1, and applying mass conservation in each tank, it is obtained the nonlinear model

$$\begin{aligned}
\frac{dh_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1 v_1}{A_1},\\
\frac{dh_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2 v_2}{A_2},\\
\frac{dh_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2 v_2}{A_3},\\
\frac{dh_4}{dt} &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1 v_1}{A_4},
\end{aligned}$$

where, k_i and v_i are the gain and the input voltage, h_i, a_i and A_i are the liquid level, the output hole area and the section area of the tank i, respectively. Besides, γ_i represents the percentage opening of the i-th valve. The values of all parameters are informed in the Table 1.

When the system is linearized at an operation point (h_{op}, v_{op}), the following linear model is obtained

$$\dot{x}(t) = A(x(t) - h_{op}) + B(u(t) - v_{op}),$$

where, $h_{op} = [h_{1op}\ h_{2op}\ h_{3op}\ h_{4op}]^\top$ and $v_{op} = [v_{1op}\ v_{2op}]^\top$. Besides, the operation point has to satisfy the following system of equations

Table 1. Quadruple tank parameters.

	Parameter	value
g	Gravity acceleration [cm/s^2]	981
k_1,k_2	Pump constant [cm^3/s]	5.5556
a_1,a_2	Output hole area [cm^2]	0.15
a_3,a_4	Output hole area [cm^2]	0.071
A_1,A_2	Section area [cm^2]	32.1699
A_3,A_4	Section area [cm^2]	28.2743

$$\begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & \frac{(1-\gamma_2)k_2}{A_1} \\ \frac{(1-\gamma_1)k_1}{A_2} & \frac{\gamma_2 k_2}{A_2} \end{bmatrix} \begin{bmatrix} v_{1op} \\ v_{2op} \end{bmatrix} = \begin{bmatrix} \frac{a_1\sqrt{2gh_{1op}}}{A_1} \\ \frac{a_2\sqrt{2gh_{2op}}}{A_2} \end{bmatrix}, \quad (1)$$

and

$$h_{3op} = \frac{((1-\gamma_2)k_2)^2}{2a_3^2 g} v_{2op}^2, \quad (2)$$

$$h_{4op} = \frac{((1-\gamma_1)k_1)^2}{2a_4^2 g} v_{1op}^2. \quad (3)$$

Thus, it is possible to determine the operation point of the interest variables (h_{1op} and h_{2op}), by computing v_{op} using the Eq. (1), and then computing h_{3op} and h_{4op} using the Eqs. (2) - (3).

The operation of the tank depends on the opening of the valves, as shown in (Johansson, 2000). Depending on the sum $\gamma_1 + \gamma_2$, its operation has certain characteristics. For instance, if $\gamma_1 + \gamma_2 = 1$, the system is singular.

To show the influence of γ_1 and γ_2 on the system, a simulation is performed. We set $\gamma_1 = \gamma_2 = 0.65$ and the voltages applied to the pumps were considered necessary to maintain $h_1 = 10$cm and $h_2 = 10$cm, that, from Eq. (6), are equal to $v_1 = v_2 = 3.7819$ V. Then, from Eqs. (7) - (8), one can see that $h_{3op} = h_{4op} = 5.4677$cm. The simulation presents the system in the situation previously described, then (at the instant $t = 20s$) γ_2 was changed from 0.65 to 0.7, resulting in the output shown in Figure 2.

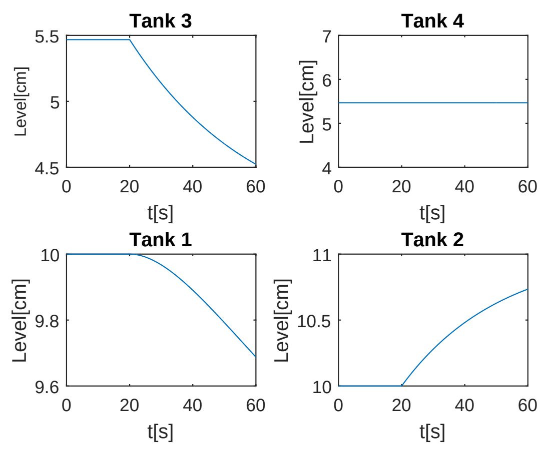

Figure 2. Tank levels considering a small disturbance in γ_2.

As can be seen, a small variation in γ_2 causes a significant difference in the outputs. This behavior can happen in practical situations due to several factors, for example, it is possible that the valve starts to clog, an error occurs in the valve opening control, or the valve has a mechanical break, among many other factors. For this reason, a time-varying control law, adapted online according to the values of γ_1 and γ_2, is considered to control the quadruple tank system.

2.1 LPV model

The LPV model considers γ_1 and γ_2 as time-varying parameters. Thus, considering the operation point [1]

$$h_{op} = [10\ 10\ 5.4677\ 5.4677]^\top,$$
$$v_{op} = [3.7819\ 3.7819]^\top,$$

the matrices of the linear system approximation are

$$A = \begin{bmatrix} -0.033 & 0 & 0.021 & 0 \\ 0 & -0.033 & 0 & 0.021 \\ 0 & 0 & -0.024 & 0 \\ 0 & 0 & 0 & -0.024 \end{bmatrix}, \tag{4}$$

$$B(\gamma_1, \gamma_2) = \begin{bmatrix} 0.173\gamma_1 & 0 \\ 0 & 0.173\gamma_2 \\ 0 & 0.197(1-\gamma_2) \\ 0.197(1-\gamma_1) & 0 \end{bmatrix}. \tag{5}$$

The LPV approximation, calculated in this form, has only the matrix $B(\gamma_1, \gamma_2)$ with time-varying entries, being written as

$$B(\gamma_1, \gamma_2) = B_0 + \gamma_1 B_1 + \gamma_2 B_2.$$

3. CONTROLLER SYNTHESIS

Although the motivation of the proposed control approach is the QT system, the synthesis conditions are formulated for a general continuous-time LPV system with an arbitrary number of time-varying parameters.

Consider the following LPV system

$$\begin{aligned} \dot{x} &= A(\rho(t))x + B_u(\rho(t))u + B_w(\rho(t))w, \\ z &= C(\rho(t))x + D_u(\rho(t))u, \end{aligned} \tag{6}$$

where $x(t) \in \mathbb{R}^{n_x}$, $u(t) \in \mathbb{R}^{n_u}$, $w(t) \in \mathbb{R}^{n_w}$ and $z(t) \in \mathbb{R}^{n_z}$ denote, respectively, the state, control input, exogenous input and output vectors. Besides, $\rho(t)$ is a vector of bounded time-varying parameters and matrices $A(\rho(t))$, $B(\rho(t))$, $B_w(\rho(t))$, $C(\rho(t))$ and $D(\rho(t))$ depend polynomially on $\rho(t)$.

The parameters $\rho(t) = [\rho_1(t), \ldots, \rho_N(t)]$ and their time-derivatives are assumed bounded in the form

$$\underline{a}_i \leq \rho_i(t) \leq \overline{a}_i$$
$$\underline{b}_i \leq \dot{\rho}_i(t) \leq \overline{b}_i$$

such that $\rho(t)$ belongs to the hyperrectangle Θ and $\dot{\rho}(t)$ belongs to the hyperrectangle Γ to all $t \geq 0$ (Apkarian and Adams, 1998).

Next theorem presents synthesis conditions formulated in terms of parameter-dependent LMIs for the design of $K(\rho(t))$ associated to the state-feedback control law $u(t) = K(\rho(t))x(t)$.The notation $\mathrm{He}(M)$, is used to indicate $M + M^\top$.

Theorem 1. Let $\xi \in (-1, 1)$ be a given scalar. If there exist parameter-dependent matrices $W(\rho) = W(\rho)^\top$, $H(\rho) = H(\rho)^\top$, $Y(\rho)$, $X(\rho)$ and $Z(\rho)$ and a scalar $\mu > 0$ such that the following parameter-dependent LMIs

$$\mu > Tr(H(\rho)) \tag{7}$$

$$\begin{bmatrix} H(\rho) & B_w(\rho(t))^\top \\ \star & W(\rho) \end{bmatrix} > 0, \tag{8}$$

$$\begin{bmatrix} \Xi(1,1) & \Xi(1,2) & \Xi(1,3) & \bar{V}(\rho) \\ \star & \Xi(2,2) & \xi\Xi(1,3) & -\bar{V}(\rho) \\ \star & \star & -I & 0 \\ \star & \star & \star & -\mathrm{He}(X(\rho)) \end{bmatrix} < 0, \tag{9}$$

where

$$\Xi(1,1) = W(\rho) + \mathrm{He}(\tilde{A}(\rho)Y(\rho) + B_u(\rho)Z(\rho)), \tag{10}$$
$$\begin{aligned} \Xi(1,2) &= \xi(\tilde{A}(\rho)Y(\rho) + B_u(\rho)Z(\rho)) \\ &\quad -Y(\rho)^\top \hat{A}(\rho)^\top - Z(\rho)^\top B_u(\rho)^\top, \\ \Xi(1,3) &= Y(\rho)^\top C(\rho)^\top + Z^\top(\rho) D_u(\rho)^\top \\ \Xi(2,2) &= -W(\rho) - \xi \mathrm{He}(\hat{A}(\rho)Y(\rho) + B_u(\rho)Z(\rho)), \end{aligned}$$
$$\hat{A}(\rho) = A(\rho) + \frac{1}{2}I, \tag{11}$$
$$\tilde{A}(\rho) = A(\rho)) - \frac{1}{2}I, \tag{12}$$
$$\bar{V}(\rho) = -\dot{W}(\rho) + \frac{1}{2}X(\rho), \tag{13}$$

hold for all $\rho \in \Theta$ and $\dot{\rho} \in \Gamma$, then $K(\rho) = Z(\rho)Y(\rho)^{-1}$ is a robustly stabilizing parameter-dependent state-feedback gain and μ is an $\mathcal{H}_2$ guaranteed cost for system (6) in closed-loop.

Proof. First it is shown that ξ can be constrained to the range $(-1, 1)$ without loss of generality [2] . Multiply (9) on the left by $\mathcal{B}_\perp^\top$ and on the right by $\mathcal{B}_\perp$, with

$$\mathcal{B}_\perp = \begin{bmatrix} \xi I & 0 & 0 \\ -I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix},$$

yielding

$$\begin{bmatrix} (\xi^2-1)W & 0 & (\xi+1)\bar{V} \\ 0 & -\gamma I & 0 \\ (\xi+1)\bar{V}^\top & 0 & -X - X^\top \end{bmatrix} < 0,$$

which is feasible only if $(\xi^2 - 1)W$ is negative definite. As W is positive definite, then it is necessary that $-1 < \xi < 1$.

As next step, multiply (9) on the left by $\mathcal{A}_\perp^T$ and on the right by $\mathcal{A}_\perp$, with

$$\mathcal{A}_\perp = \begin{bmatrix} \hat{A}^\top + K^\top B_u^\top & C^\top + K^\top D_u^\top & 0 \\ \tilde{A}^\top + K^\top B_u^\top & C^\top + K^\top D_u^\top & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix},$$

which provides

[1] The operation point was calculated considering average values of γ_1 and γ_2, i.e., on average, $\gamma_1 = 0.65$ and $\gamma_2 = 0.65$.

[2] The dependence of the variables on ρ is omitted to save space.

$$\begin{bmatrix} \hat{A}W\hat{A}^\top - \tilde{A}W\tilde{A}^\top & \star & \star \\ \bar{C}W(\hat{A}-\tilde{A})^\top & -I & \star \\ \bar{V}^\top(\hat{A}-\tilde{A})^\top & 0 & -\mathrm{He}(X) \end{bmatrix} < 0.$$

Using the definitions given in (11)-(13), the previous inequality can be rewritten in the form

$$\begin{bmatrix} WA_{cl}^\top + A_{cl}W & WC_{cl}^\top & -\dot{W} + \frac{1}{2}X \\ CW_{cl} & -I & 0 \\ -\dot{W} + \frac{1}{2}X^\top & 0 & -X - X^\top \end{bmatrix} < 0, \tag{14}$$

where, $A_{cl} = A + B_u K$, $C_{cl} = C + D_u K$.

Multiplying last inequality on the left by $\mathcal{C}_\perp^T$ and on the right by $\mathcal{C}_\perp$, with

$$\mathcal{C}_\perp = \begin{bmatrix} I & 0 \\ 0 & I \\ \frac{1}{2}I & 0 \end{bmatrix} < 0,$$

provides

$$\begin{bmatrix} WA_{cl}^\top + A_{cl}W - \dot{W} & WC_{cl}^\top \\ C_{cl}W & -I \end{bmatrix} < 0, \tag{15}$$

which can be recognized as the observability Grammian formulated in terms of an inequality for a continuous-time LPV system (Sznaier, 1999). Conditions (15), (7) and (8) assures that $K(\rho) = Z(\rho)Y(\rho)^{-1}$ is a stabilizing gain and μ is an $\mathcal{H}_2$ guaranteed cost for the closed-loop system.

Finally, to show that matrix Y is invertible, note that feasibility (9) assures that $\Xi(1,1) < 0$, which implies that

$$W + \mathrm{He}((\tilde{A} + B_u K)Y) < 0 \Rightarrow$$
$$\Rightarrow \mathrm{He}((\tilde{A} + B_u K)Y) < -W < 0$$

As a consequence, Y has full rank. □

The novelty of the synthesis conditions of Theorem 1 is that the scalar parameter ξ is constrained to the range (-1,1). Other conditions from the literature (Xie, 2005; Tognetti et al., 2010), which also employ the search on a scalar to improve the guaranteed costs, do not present bounds for the scalar. This is an advantage of the proposed condition. Besides, to the best of the authors' knowledge, the treatment of the time-derivative term is new. This technical novelty is necessary to provide the bounds for ξ.

The conditions of Theorem 1 are parameter-dependent LMIs, which are not numerically tractable. However, imposing polynomial structures of fixed degree for the optimization variables, it is possible to find solutions by applying sufficient polynomial positivity tests for the resulting polynomial matrix inequalities. Note that the control gain will be a rational function of ρ, depending on the degrees of the variables $Z(\rho)$ and $Y(\rho)$. In the experiment presented next, all optimization variables were fixed as polynomials of degree one on both γ_1 and γ_2. The tasks of programming and solving the inequalities can be performed with the parsers Yalmip and ROLMIP (Agulhari et al., 2019), and a SDP solver, like Mosek (Andersen and Andersen, 2000).

Back to the tank quadruple problem, using the matrices given in (4), (5) and

$$C = \mathrm{diag}(0.5\ 0.5\ 10^{-4}\ 10^{-4}),$$
$$D_u = \mathrm{diag}\left(\frac{1}{12}\ \frac{1}{12}\right),$$

and considering that

$$0.55 \le \gamma_1 \le 0.75, \quad 0.55 \le \gamma_2 \le 0.75, \tag{16}$$
$$-0.05 \le \dot{\gamma}_1 \le 0.05, \quad -0.05 \le \dot{\gamma}_2 \le 0.05, \tag{17}$$

it is possible to construct the matrices given in (14) considering $\rho_1 = \gamma_1$ and $\rho_2 = \gamma_2$. Applying the condition of Theorem 1 with a scalar search among the values

$$\xi = [\,-0.9\ -0.8\ \ldots\ 0.8\ 0.9\,],$$

a guaranteed cost of $\mu = 1.0035$ with $\xi = 0$ has been obtained. The resulting gain is given by

$$K(\rho_1,\rho_2) = Z(\rho_1,\rho_2)G(\rho_1,\rho_2)^{-1},$$

and must computed in real-time once the values of ρ_1 and ρ_2 are available.

4. NUMERICAL RESULTS

To evaluate the performance of the proposed gain-scheduled control strategy, the same conditions of the previous simulation were considered, but assuming that γ_i, $i = 1, 2$ are time-varying in the form

$$\gamma_1(t) = 0.65 + 0.1\sin(0.1t),$$
$$\gamma_2(t) = 0.65 - 0.1\sin(0.1t),$$

which satisfy the bounds informed in (16) and (17).

The first simulation considers only the variations of γ_i with null initial condition and no external disturbances. The levels of the tanks and the control effort are presented in Figures 3 and 4, respectively.

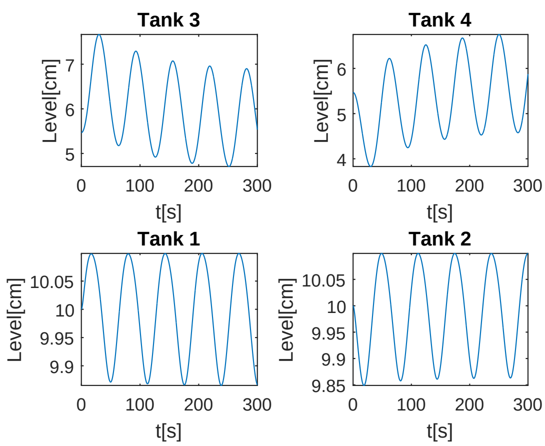

Figure 3. Tank levels for time-varying γ_1 and γ_2.

As next experiment, the system is tested with the following initial conditions

$$h_{ic} = h_{op} + [4\ -4\ 0\ 0]^\top.$$

The results are shown in Figures 5 and 6.

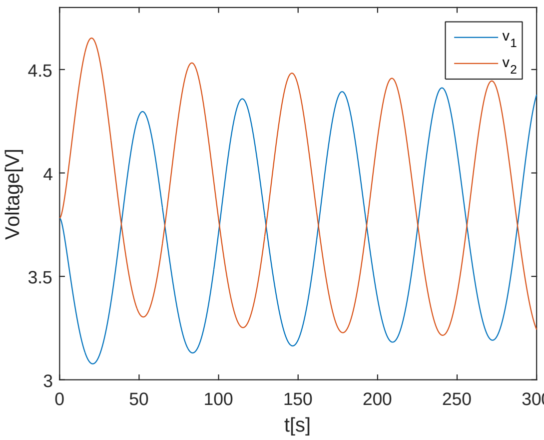

Figure 4. Control effort for time-varying γ_1 and γ_2.

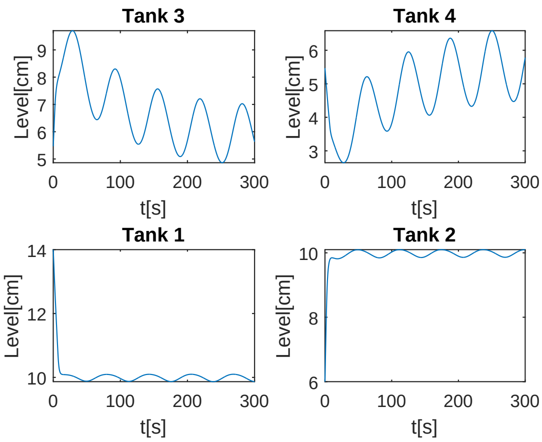

Figure 5. Tank levels for time-varying γ_1 and γ_2 and initial condition $h_1 = h_{1op} + 4$ and $h_2 = h_{2op} - 4$.

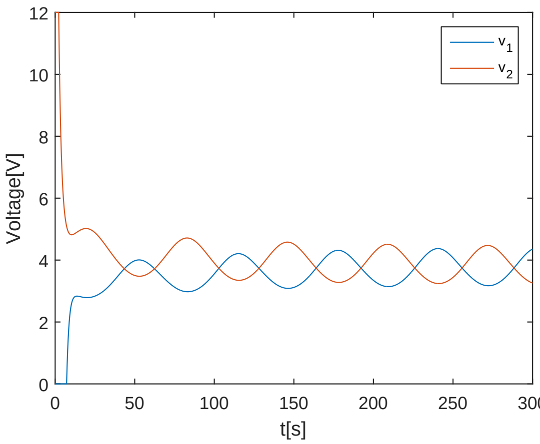

Figure 6. Control effort for time-varying γ_1 and γ_2 and initial condition $h_1 = h_{1op} + 4$ and $h_2 = h_{2op} - 4$.

Finally, consider the existence of an external disturbance in the plant output, as shown in Figure 7. The results are presented in Figures 8 and 9.

As can be observed, in all simulations the proposed LPV controller presented a good performance, keeping the output at the desired operation point, even with variations in the inlet flows, when the system started with non zero initial conditions and with external disturbances.

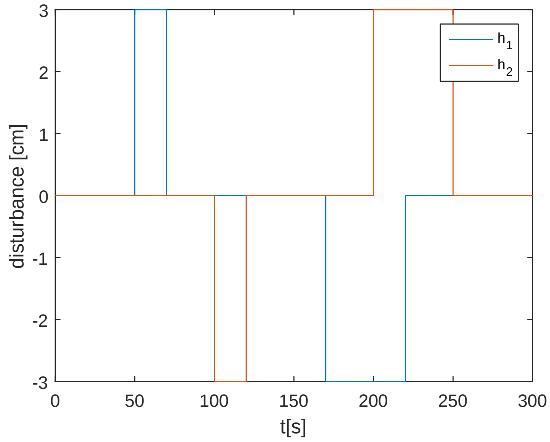

Figure 7. Additional disturbance in the plant output.

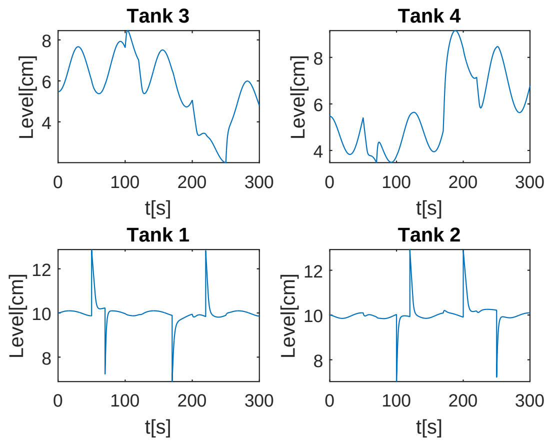

Figure 8. Tank levels for time-varying γ_1 and γ_2 with additional disturbance in the plant output.

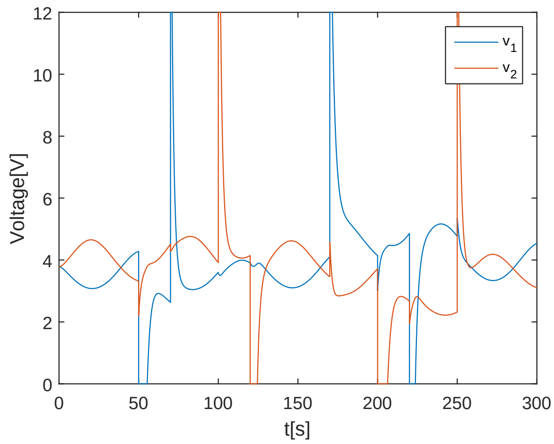

Figure 9. Control effort for time-varying γ_1 and γ_2 with additional disturbance in the plant output.

5. CONCLUSION

This paper presented a model approximation that resulted in a pure LPV representation for the quadruple tank. This model considers the percentage opening of the two valves as bounded time-varying parameters with bounded rates of variation.

Then a gain-scheduled $\mathcal{H}_2$ state-feedback control synthesis condition was proposed to address the LPV model used to represent the plant. The controller showed good results for γ_i varying within the assumed limits.

The proposed synthesis conditions are not limited to the QT model, treating any LPV model with polynomial dependence on the time-varying parameters.

Future work includes the practical experiment and the model extension to obtain a complete LPV model, without the approximation of the average operating point.

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