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Gain-scheduled control of nonlinear sampled-data systems: A Wirtinger-based approach ⋆

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Abstract: This paper addresses the design of gain-scheduled state-feedback controllers for sampled-data nonlinear systems, aiming at the minimization of the $\mathcal{L}_2$-gain. A description of nonlinear systems based in polynomial quasi-linear parameter-varying models is employed. Sufficient conditions for the synthesis of sampled-data controllers are derived in terms of polynomial linear matrix inequalities, using Wirtinger's Inequality and considering Lyapunov-Krasovskii functionals. The designed controllers ensure both closed-loop stability and guaranteed $\mathcal{L}_2$-gain costs. The effectiveness of the proposed approach is assessed through numerical simulations.

Keywords: Nonlinear systems; Sampled-data control; Gain scheduling; $\mathcal{L}_2$-gain performance; Lyapunov-Krasovskii functional.

1. INTRODUCTION

When it comes to the digital implementation of controllers for sampled-data nonlinear systems, a key issue is the proper determination of the sampling times. Ideally, larger sampling times are desirable to save resources (microprocessors, sensors and data transfer, for instance) and to provide more economical solutions. On the other hand, the closed-loop systems must remain stable despite the size of the sampling intervals (Hooshmandi et al., 2018).

In the past years, gain-scheduled controllers have been largely used in the context of polynomial linear parameter-varying (PLPV) systems (Blanchini and Miani, 2003; de Caigny et al., 2010; Pandey and de Oliveira, 2019; Sadeghzadeh, 2019). Performance constraints can be additionally imposed for guaranteeing a desired behavior to the closed-loop system with the controller to be designed. Two commonly used performance criteria are the disturbance rejection (minimization of the $\mathcal{L}_2$-gain) and the minimization of the system energy (also known as $\mathcal{H}_2$ guaranteed cost) (Mohammadpour and Scherer, 2012; Briat, 2015).

Additionally, given that nonlinear systems exhibit some properties, such as smoothness, they can be cast to the framework of PLPV systems, being denoted as quasi-PLPV systems (Leith and Leithead, 2000; Rugh and Shamma, 2000; Lacerda et al., 2011; Hooshmandi et al., 2018; Rodrigues et al., 2018). Such denomination comes from the fact that the converted systems actually depend on their internal state variables, in spite of some other external signal. In other words, quasi-PLPV realizations hide the nonlinearities of the nonlinear systems among the scheduling parameters (Rotondo et al., 2013). As a result, the knowledge of the boundaries of both the range of values and the variation rates of such parameters is required. The main interest behind (quasi-)PLPV models lies in the possibility of applying powerful tools, usually dedicated to linear systems, to certify the stability and to synthesize controllers and filters for nonlinear systems (Rotondo et al., 2013).

On the other hand, according to Goebel et al. (2009), in a typical control scenario, a continuous-time system is controlled by means of a sampled-data controller. For instance, in the framework of state-feedback controllers, a controller samples the states of a nonlinear system, and computes the control signal, which is then held constant over two successive sampling instants. Hence, from the controller perspective, the nonlinear system to be controlled can be cast as a sampled-data system.

Different sampled-data control strategies for the control of quasi-PLPV systems are found in the scientific literature, which can be grouped in three major areas: emulation, approximate discretization, and direct sampled-data. The emulation approach designs a continuous-time controller, which is discretized for obtaining a sampled-data controller (Tóth et al., 2010). The downsides of such method are the need for considering the discretization error when discretizing the continuous-time controller, and the assumption that the scheduling parameters are held constant between two successive sampling instants. In the approximate discretization method, a stabilizing sampled-data controller is designed for the approximate discrete-time quasi-PLPV model (Lam and Zhou, 2008; de Caigny et al., 2010). In this concept, not only the time dependence of the

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scheduling parameters but also the behavior of the nonlinear system are ignored during the sampling intervals. In the direct sampled-data approach, a time-varying input delay can be considered for the control of quasi-PLPV systems, modeled as continuous-time systems (Ramezanifar et al., 2012; Gomes da Silva Jr et al., 2018; Hooshmandi et al., 2018). This method does not require neither the quasi-PLPV dynamics nor the designed controller to be discretized. Furthermore, the scheduling parameters can vary over time, under the assumption that the bounds on the range of values and on the variation rates of the scheduling parameters exist and are assumed to be known.

Regarding the above discussion, the main contribution of this work is the proposition of new sufficient PLMI conditions to the synthesis of gain-scheduled controllers for sampled-data nonlinear systems, whose nonlinearities are bounded and have known variation rate. It is assumed that the nonlinear systems can be described in terms of quasi-PLPV realizations. The new sufficient PLMI conditions are derived after an expanded version of the Lyapunov-Krasovskii functional (LKF) adopted in Hooshmandi et al. (2018), and the reduced conservativeness of the proposed approach is attained by applying Wirtinger's Inequality to the resulting PLMI conditions (Seuret and Gouaisbaut, 2013). Furthermore, this paper also investigates the impact of the sampling time on the performance of the closed-loop sampled-data nonlinear systems. Such analysis can lead to the determination of a maximum allowable sampling period (MASP) for which both stability and performance are ensured.

This paper is organized as follows: Section 1 presents the state-of-the-art of the synthesis of gain-sheduling controllers for sampled-data nonlinear systems. Section 2 establishes a framework for the description of sampled-data nonlinear systems by means of quasi-PLPV models. In Section 3, the problem of synthesizing gain-scheduled sampled-data state-feedback controllers is addressed, while Section 4 discusses some computational aspects relevant for the definition of an SDP problem in terms of PLMI conditions. Furthermore, some numerical examples are exploited in this section to show the effectiveness of the proposed approach. Finally, Section 5 concludes this paper with final remarks and proposes possible future investigations.

Notation: $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ describe the real n-dimension euclidean space and the set of $n \times m$ real matrices, respectively. X^T is the transpose of X. X^H is the Hermitian operator defined as $X^H = X + X^T$. $X \succ 0$ ($X \prec 0$) implies that X is a positive (negative) definite matrix. The symbol $(*)$ denotes a symmetric term inside matrices. $diag(A,\ B)$ is a block-diagonal matrix composed by the blocks A and B. $\mathbf{I}_n$ is the identity matrix of size n. $\mathbf{0}_{n\times m}$ is a null matrix of dimension $n \times m$. $\mathbf{f} : \mathbb{R}^n \mapsto \mathbb{R}^m$ defines a real function $\mathbf{f}$ which maps n inputs to m outputs.

2. PROBLEM STATEMENT

Consider a nonlinear system described by the following set of equations:

$$\begin{aligned} \dot{x}(t) &= \mathbf{f}(x(t), u(t), w(t)), \\ y(t) &= \mathbf{h}(x(t), u(t), w(t)), \end{aligned} \tag{1}$$

in which $x(t) \in \mathbb{R}^{n_x}$ is the state, $w(t) \in \mathbb{R}^{n_w}$ is the disturbance, $u(t) \in \mathbb{R}^{n_u}$ is the control input, and $y(t) \in \mathbb{R}^{n_y}$ is the output. $\mathbf{f} : \mathbb{R}^{n_x+n_u+n_w} \mapsto \mathbb{R}^{n_x}$ and $\mathbf{h} : \mathbb{R}^{n_x+n_u+n_w} \mapsto \mathbb{R}^{n_y}$ are functions mapping $x(t)$, $u(t)$, and $w(t)$ to $\dot{x}(t)$ and to $y(t)$, respectively.

The description of nonlinear systems as quasi-PLPV models is made with the bounding-box method (Rotondo et al. (2013)). A polytopic, continuous, linear and quasi-PLPV realization of (1) can be then written as

$$\begin{aligned} \dot{x}(t) =& \mathbf{A}(\eta(t))x(t) + \mathbf{B}_1(\eta(t))w(t) + \mathbf{B}_2(\eta(t))u(t), \\ y(t) =& \mathbf{C}(\eta(t))x(t) + \mathbf{D}_1(\eta(t))w(t) + \mathbf{D}_2(\eta(t))u(t), \end{aligned} \tag{2}$$

where $\mathbf{A}(\eta(t))$, $\mathbf{B}_1(\eta(t))$, $\mathbf{B}_2(\eta(t))$, $\mathbf{C}(\eta(t))$, $\mathbf{D}_1(\eta(t))$, and $\mathbf{D}_2(\eta(t))$, with compatible dimensions, are matrices depending on the parameter vector $\eta(t) \in \mathbb{R}^N$. An affine representation of the system matrices is adopted, generically described as follows:

$$\mathbf{X}(\eta(t)) = \sum_{i=1}^{N} \eta_i(t)\mathbf{X}_i,\ \forall \eta(t) \in \Lambda_N, \tag{3}$$

with $\mathbf{X}_i$, for $i = 1, \ldots, N$, the vertices of $\mathbf{X}(\eta(t))$ and Λ_N an N-dimensional space described as

$$\Lambda_N = \left\{ \eta : \mathbb{R}^+ \to \mathbb{R}^N \mid \eta(t) \in \Delta_\eta,\ \dot{\eta}(t) \in \Delta_{\dot{\eta}} \right\}, \tag{4}$$

being Δ_η and $\Delta_{\dot{\eta}}$ compact admissible sets of the parameter and its derivative, which are defined by

$$\begin{aligned} \Delta_\eta &= \left\{ \eta \in \mathbb{R}^N : \underline{\eta}_i \le \eta_i(t) \le \overline{\eta}_i,\ i = 1, \ldots, N \right\}, \\ \Delta_{\dot{\eta}} &= \left\{ \dot{\eta} \in \mathbb{R}^N : |\dot{\eta}_i(t)| \le v_i,\ i = 1, \ldots, N \right\}, \end{aligned} \tag{5}$$

with $\underline{\eta}_i$ e $\overline{\eta}_i$, respectively, the lower and upper bounds of $\eta_i(t)$, and v_i the maximum absolute value for the variation rate of $\eta_i(t)$ (Hooshmandi et al., 2018).

Describing nonlinear systems as quasi-PLPV models, with the bounding-box method, requires the sets Δ_η and $\Delta_{\dot{\eta}}$ to be *a priori* known. In other words, the domain of discourse associated to the quasi-PLPV model (2) should be given. Such domain of discourse can be thus understood as a convex polyhedron whose vertices are either the bounds of the parameters η_i or the bounds of the time derivatives $\dot{\eta}_i$ (Rotondo et al., 2013).

3. CONTROLLER SYNTHESIS

This section proposes a gain-scheduled sampled-data state-feedback control law in the form

$$u(t) = u(t_n) = K(\eta(t_n))x(t_n),\ t \in [t_n, t_{n+1}) \tag{6}$$

to stabilize system (2) in closed-loop with an $\mathcal{L}_2$-gain guaranteed cost.

The control signal, held constant between two successive sampling instants, is equivalent to a time-delayed control input with respect to t. Thus, the control signal $u(t)$ can be understood as a delayed signal, whose delay $\tau(t)$ is given by

$$\tau(t) = t - t_n \le T_m,\ t \in [t_n, t_{n+1}). \tag{7}$$

The induced delay $\tau(t)$ is the time elapsed since the last sampling instant t_n and it cannot exceed the MASP T_m.

Assuming the structure of the gain-scheduled control law is known and given by (6), the quasi-PLPV dynamics (2) in closed loop are rewritten as

$$\begin{aligned}\dot{x}(t) =& \mathbf{A}(\eta(t))x(t) + \mathbf{B}_1(\eta(t))w(t)\\ &+ \mathbf{B}_2(\eta(t))\mathbf{K}(\eta(t_n))x(t_n),\\ y(t) =& \mathbf{C}(\eta(t))x(t) + \mathbf{D}_1(\eta(t))w(t)\\ &+ \mathbf{D}_2(\eta(t))\mathbf{K}(\eta(t_n))x(t_n).\end{aligned} \tag{8}$$

In order to obtain a uniform representation of (8), an expanded parameter vector $\rho(t) = \left[\eta^T(t_n)\ \delta^T(t)\right]^T$ is adopted, where $\delta(t) = \eta(t) - \eta(t_n)$ is the uncertainty between the real continuous parameters $\eta(t)$ and the sampled parameters $\eta(t_n)$.

The expanded parameter vector $\rho(t)$ is defined in the space Θ, so that

$$\Theta = \left\{\rho \in \mathbb{R}^{2N} : \rho(t) \in \Delta_\rho,\ \dot{\rho}(t) \in \Delta_{\dot{\rho}}\right\}, \tag{9}$$

in which Δ_ρ and $\Delta_{\dot{\rho}}$ are compact sets as formulated in (5). With the expanded parameter vector $\rho(t)$, the dynamics of the state vector $x(t)$ and the output vector $y(t)$ can be rewritten as follows:

$$\begin{aligned}\dot{x}(t) =& \mathbf{A}(\rho(t))x(t) + \mathbf{B}_1(\rho(t))w(t)\\ &+ \mathbf{B}_2(\rho(t))\mathbf{K}(\rho(t))x(t_n),\\ y(t) =& \mathbf{C}(\rho(t))x(t) + \mathbf{D}_1(\rho(t))w(t)\\ &+ \mathbf{D}_2(\rho(t))\mathbf{K}(\rho(t))x(t_n).\end{aligned} \tag{10}$$

Sufficient conditions for synthesizing gain-scheduled sampled-data controllers with guaranteed $\mathcal{L}_2$-gain cost are provided in Theorem 1, in which the system structure described in (10) is used. For the sake of simplicity, the time dependency of $\rho(t)$ is omitted. Theorem 1 is derived with the aid of Wirtinger's Inequality (Seuret and Gouaisbaut, 2013), described in the following lemma:

Lemma 1. (Wirtinger's Inequality). Given a constant symmetric positive-definite matrix R, the following inequality is verified for every function $\omega(u)$ continuously differentiable on the interval $[a,b] \to \mathbb{R}^n$:

$$\int_a^b \dot{\omega}^T(u)R\dot{\omega}(u)du \geq \frac{1}{b-a}\Omega_1^T R\Omega_1 + \frac{3}{b-a}\Omega_2^T R\Omega_2,$$

where $\Omega_1 = \omega(b) - \omega(a)$ and $\Omega_2 = \omega(b) + \omega(a) - \frac{2}{b-a}\int_a^b \omega(u)du$.

Theorem 1. Given scalars $T_m > 0$ and λ, if there exist symmetric positive-definite matrices $Q(\eta(t_n)) = Q(\rho) \in \mathbb{R}^{n_x\times n_x}$, $\Gamma_1(\eta(t_n)) = \Gamma_1(\rho)$, $\Lambda_1(\eta(t_n)) = \Lambda_1(\rho) \in \mathbb{R}^{2n_x\times 2n_x}$, $\Lambda_2(\eta(t_n)) = \Lambda_2(\rho) \in \mathbb{R}^{n_x\times n_x}$, matrices $Y(\eta(t_n)) = Y(\rho) \in \mathbb{R}^{n_u\times n_x}$, $\overline{N}_1(\rho)$, $\overline{N}_2(\rho) \in \mathbb{R}^{2n_x\times 3n_x+n_w}$, $L(\rho)$, $G(\rho)$, $\Upsilon(\rho) \in \mathbb{R}^{2n_x\times 2n_x}$, and a scalar $\gamma > 0$ satisfying the following PLMIs:

$$S_1^T Q(\rho)S_1 + T_m\lambda S_2^T\overline{Q}(\rho)L_0S_2 + \frac{T_m^2}{4}S_3^T\Lambda_2(\rho)S_3 \succ 0 \tag{11}$$

$$\begin{bmatrix}\Pi_1 + T_m\Pi_2 & * & * \\ T_mL_5 & -T_m\Lambda_1(\rho) & * \\ \Phi & \mathbf{0} & -\gamma\mathbf{I}\end{bmatrix} \prec 0 \tag{12}$$

$$\begin{bmatrix}\Pi_1 + T_m\Pi_3 & * & * & * \\ T_m\overline{N}_1(\rho) & -T_m\Gamma_1(\rho) & * & * \\ 3T_m\overline{N}_2(\rho) & \mathbf{0} & -3T_m\Gamma_1(\rho) & * \\ \Phi & \mathbf{0} & \mathbf{0} & -\gamma\mathbf{I}\end{bmatrix} \prec 0 \tag{13}$$

$$\begin{bmatrix}\Lambda_1(\rho) + \left(\Upsilon^T(\rho)\left(\overline{Q}(\rho) + L^T(\rho)\right)\right)^H & * \\ -L^T(\rho) + G^T(\rho)\Upsilon(\rho) & \Gamma_1(\rho) - G^H(\rho)\end{bmatrix} \prec 0 \tag{14}$$

for all $\rho \in \Theta$, in which

$$\begin{aligned}\Pi_1 =& \left(L_4^TL_1\right)^H - \lambda L_2^T\overline{Q}(\rho)L_0L_2 - \gamma L_8^TL_8\\ &+ \left(\overline{N}_1^T(\rho)M_1 + 3\overline{N}_2^T(\rho)M_2\right)^H\\ \Pi_2 =& \lambda\left(L_6^TL_0L_2\right)^H + \left(L_7^T\Lambda_2(\rho)L_3\right)^H + L_3^T\Lambda_2(\rho)L_3\\ \Pi_3 =& \left(\overline{N}_1^T(\rho)M_t\right)^H - L_3^T\Lambda_2(\rho)L_3\end{aligned}$$

$$M_1 = \begin{bmatrix}\mathbf{I} & -\mathbf{I} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix},\ M_2 = \begin{bmatrix}\mathbf{I} & \mathbf{I} & -2\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix}$$

$$M_t = \begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0}\end{bmatrix}$$

$$S_1 = [\mathbf{I}\ \mathbf{0}\ \mathbf{0}],\ S_2 = \begin{bmatrix}\mathbf{I} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{I} & \mathbf{0}\end{bmatrix},\ S_3 = [\mathbf{0}\ \mathbf{0}\ \mathbf{I}]$$

$$L_0 = [\mathbf{I}\ -\mathbf{I}]^T[\mathbf{I}\ -\mathbf{I}]$$

$$L_1 = [\mathbf{I}\ \mathbf{0}\ \mathbf{0}\ \mathbf{0}],\ L_2 = \begin{bmatrix}\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0}\end{bmatrix},\ L_3 = [\mathbf{0}\ \mathbf{0}\ \mathbf{I}\ \mathbf{0}]$$

$$L_4 = [\mathbf{A}(\rho)Q(\rho)\ \ \mathbf{B}_2(\rho)Y(\rho)\ \ \mathbf{0}\ \ \mathbf{B}_1(\rho)]$$

$$L_5 = \begin{bmatrix}\mathbf{A}(\rho)Q(\rho) & \mathbf{B}_2(\rho)Y(\rho) & \mathbf{0} & \mathbf{B}_1(\rho)\\ \mathbf{0} & Q(\rho) & \mathbf{0} & \mathbf{0}\end{bmatrix}$$

$$L_6 = \begin{bmatrix}\mathbf{A}(\rho)Q(\rho) & \mathbf{B}_2(\rho)Y(\rho) & \mathbf{0} & \mathbf{B}_1(\rho)\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix}$$

$$L_7 = [\mathbf{I}\ -\mathbf{I}\ -\mathbf{I}\ \mathbf{0}],\ L_8 = [\mathbf{0}\ \mathbf{0}\ \mathbf{0}\ \mathbf{I}]$$

$$\overline{Q}(\rho) = diag\left(Q(\rho), Q(\rho)\right)$$

$$\Phi = [\mathbf{C}(\rho)Q(\rho)\ \ \mathbf{D}_2(\rho)Y(\rho)\ \ \mathbf{0}\ \ \mathbf{D}_1(\rho)]$$

then, system (10) is asymptotically stable with aperiodic samplings lower than T_m and with a gain-scheduled sampled-data state-feedback controller given by $K(\rho) = Y(\rho)Q^{-1}(\rho)$. Furthermore, γ is an upper bound to the $\mathcal{L}_2$-gain of the closed-loop system.

Proof. Consider the following time-dependent LKF:

$$W(x,t) = V(x) + V_0(x,t) = V(x) + \sum_{i=1}^{3} V_i(x,t), \tag{15}$$

in which

$$V(x) = x^T(t)P(\rho)x(t)$$

$$V_1(x,t) = (t_{n+1} - t)\int_{t_n}^t \begin{bmatrix}\dot{x}(q)\\ x(t_n)\end{bmatrix}^T E(\rho)\begin{bmatrix}\dot{x}(q)\\ x(t_n)\end{bmatrix}dq$$

$$V_2(x,t) = (t_{n+1} - t)\begin{bmatrix}x(t)\\ x(t_n)\end{bmatrix}^T X(\rho)\begin{bmatrix}x(t)\\ x(t_n)\end{bmatrix}$$

$$V_3(x,t) = (t_{n+1} - t)(t - t_n)\nu^T(t)F(\rho)\nu(t)$$

with $\nu(t) = \frac{1}{\tau(t)}\int_{t_n}^t x(q)dq$, $P(\rho) = P(\eta(t_n))$, $F(\rho) = F(\eta(t_n))$, $E(\rho) = E(\eta(t_n))$ symmetric positive-definite matrices, and $X(\rho)$ defined as follows:

$$X(\rho) = \begin{bmatrix}X_1(\rho) & -X_1(\rho)\\ -X_1(\rho) & X_1(\rho)\end{bmatrix},$$

with $X_1(\rho)$ a symmetric matrix.

In order to ensure simultaneously closed-loop stability for quasi-PLPV models (10) and guaranteed $\mathcal{L}_2$-gain cost, the following conditions must be both respected for all $t \in [t_n,t_{n+1})$:

$$W(x,t) > 0 \tag{16}$$

$$\dot{W}(x,t) + \frac{1}{\gamma} y^T(t)y(t) - \gamma w^T(t)w(t) < 0 \tag{17}$$

Since $E(\rho) \succ 0$ by assumption, (16) is satisfied if the sum of the non-integral terms is positive. By defining $\overline{\xi}(t) = \left[x^T(t)\ x^T(t_n)\ \nu^T(t)\right]^T$ and combining the non-integral terms, one gets the PLMI

$$\overline{\xi}^T(t)\Bigg[S_1^T P(\rho) S_1 + (t-t_n) S_2^T X(\rho) S_2 + (t_{n+1}-t)(t-t_n) S_3^T F(\rho) S_3\Bigg]\overline{\xi}(t) > 0. \tag{18}$$

By assumption, $t - t_n \leq T_m$ and $t_{n+1} - t \leq T_m$ for all $t \in [t_n,t_{n+1})$. The maximum value of the quadratic relation $(t_{n+1}-t)(t-t_n)$ is $\frac{T_m^2}{4}$ and occurs when $t = \frac{t_{n+1}+t_n}{2}$. Considering the bounds of the time-dependent terms in (18), the feasibility of PLMI condition

$$S_1^T P(\rho) S_1 + T_m S_2^T X(\rho) S_2 + \frac{T_m^2}{4} S_3^T F(\rho) S_3 \succ 0 \tag{19}$$

guarantees that condition (18) is met.

Expanding (17), one has (20), with $\xi(t) = \left[\overline{\xi}^T(t)\ w^T(t)\right]^T$ and

$$S_4 = [\mathbf{A}(\rho)\ \mathbf{B}_2(\rho)\mathbf{K}(\rho)\ \mathbf{0}\ \mathbf{B}_1(\rho)]$$
$$S_5 = \begin{bmatrix} \mathbf{A}(\rho) & \mathbf{B}_2(\rho)\mathbf{K}(\rho) & \mathbf{0} & \mathbf{B}_1(\rho) \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$
$$S_6 = \begin{bmatrix} \mathbf{A}(\rho) & \mathbf{B}_2(\rho)\mathbf{K}(\rho) & \mathbf{0} & \mathbf{B}_1(\rho) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$
$$\phi = [\mathbf{C}(\rho)\ \mathbf{D}_2(\rho)\mathbf{K}(\rho)\ \mathbf{0}\ \mathbf{D}_1(\rho)]$$

An upper bound for the integral term in (20) can be found by means of Lemma 1, where $E(\rho)$ is held constant between two successive sampling instants:

$$-\int_{t_n}^{t} \begin{bmatrix} \dot{x}(q) \\ x(t_n) \end{bmatrix}^T E(\rho) \begin{bmatrix} \dot{x}(q) \\ x(t_n) \end{bmatrix} dq \leq \xi^T(t)\Bigg\{ -\frac{1}{\tau(t)}(\Omega_1^*)^T E(\rho)(\Omega_1^*) - \frac{3}{\tau(t)}(\Omega_2^*)^T E(\rho)(\Omega_2^*)\Bigg\}\xi(t) \tag{21}$$

in which $\Omega_1^* = M_1 + \tau(t)M_t$ and $\Omega_2^* = M_2$. Using the relation

$$(E(\rho)\Omega_i^* + \tau(t)N_i(\rho))^T E^{-1}(\rho)(E(\rho)\Omega_i^* + \tau(t)N_i(\rho)) \succeq 0,$$

for $i = 1, 2$, an upper bound for (21) is given by

$$-\int_{t_n}^{t} \begin{bmatrix} \dot{x}(q) \\ x(t_n) \end{bmatrix}^T E(\rho) \begin{bmatrix} \dot{x}(q) \\ x(t_n) \end{bmatrix} dq \leq \xi^T(t)\Bigg\{ \left[N_1^T(\rho)(\Omega_1^*)\right]^H + \tau(t) N_1^T(\rho) E^{-1}(\rho) N_1(\rho) + \left[3N_2^T(\rho)(\Omega_2^*)\right]^H + 3\tau(t) N_2^T(\rho) E^{-1}(\rho) N_2(\rho)\Bigg\}\xi(t) \tag{22}$$

Considering (22) and in order to linearize the product of variables in (20), choose

$$P(\rho) = Q^{-1}(\rho),\ X_1(\rho) = \lambda Q^{-1}(\rho),$$
$$E(\rho) = \Lambda_1^{-1}(\rho),\ F(\rho) = Q^{-1}(\rho)\Lambda_2(\rho)Q^{-1}(\rho),$$

and apply the congruence transformation

$$\tilde{\Omega} = diag\big(Q(\rho),\ Q(\rho),\ Q(\rho),\ \mathbf{I}_{n_w}\big)$$

to both sides of (20), yielding, after some manipulations, (23).

Since matrices Ω_1^* and Ω_2^* are composed of constant submatrices and they are independent with respect to the disturbances $w(t)$, one has the following equivalence

$$\Omega_i^*\tilde{\Omega} \equiv \overline{Q}(\rho)\Omega_i^*,$$

for $i = 1, 2$. Hence, the changes of variables

$$\overline{N}_i(\rho) = \overline{Q}(\rho) N_i(\rho)\tilde{\Omega},$$

for $i = 1, 2$, is adopted.

As performed in Hooshmandi et al. (2018), the linearization of condition (23) can be achieved by imposing the relaxation

$$\Gamma_1^{-1}(\rho) \succeq \overline{Q}^{-1}(\rho)\Lambda_1(\rho)\overline{Q}^{-1}(\rho). \tag{24}$$

Substituting (24) in (23), the resulting terms can then be grouped based on their dependence on time:

$$\left[\Pi_1 + \frac{1}{\gamma}\Phi^T\Phi\right] + (t_{n+1}-t)\left[\Pi_2 + L_5^T\Lambda_1^{-1}(\rho)L_5\right] + (t-t_n)\left[\Pi_3 + \overline{N}_1^T\Gamma_1^{-1}(\rho)\overline{N}_1 + 3\overline{N}_2^T\Gamma_1^{-1}(\rho)\overline{N}_2\right] \prec 0 \tag{25}$$

Provided that (25) is affine with respect to t, it is sufficient to ensure that (17) holds for both $t = t_n$ and $t = t_{n+1}$. By applying Schur complements, these two conditions are guaranteed by means of the PLMI conditions (12) and (13), respectively.

Notice also that inequality (24) is non convex due to the product of three decision variables. The linearization of such term is borrowed from Hooshmandi et al. (2018), and the PLMI presented in (14) ensures that (24) holds for all $t \in [t_n,t_{n+1})$, considering that matrix $\Upsilon(\rho)$ is known when computing the control gain $K(\rho)$.

With the proposed changes of variables, the PLMIs conditions ensuring (16) must be accordingly adapted. Applying the congruence transformation $diag\big(Q(\rho),\ Q(\rho),\ Q(\rho)\big)$ to both sides of (19), expanding and collecting terms yields (11), which is a sufficient condition to ensure (16). This completes the proof.

Remark 1. The adopted LKF is a looped functional, since $V_0(x,t_n) = V_0(x,t_{n+1}) = 0$. The main interest behind looped functionals lies in guaranteeing that $W(x,t) = V(x)$ at any jump instant. As a result, expansive jumps are allowed within a sample interval, as long as the storage function accordingly decreases. If a monotonically decreasing $V(x)$ is considered, then system stability is ensured. For further details, please refer to (Seuret and Gouaisbaut, 2013).

4. NUMERICAL SIMULATIONS

In this section, the effectiveness of the proposed approach is presented by means of numerical simulations. The

$$\xi^T(t)\Bigg\{\big(S_4^T P(\rho)L_1\big)^H + (t_{n+1}-t)S_5^T E(\rho)S_5 + (t_{n+1}-t)\Bigg[\big(S_6^T X(\rho)L_2\big)^H + L_2^T\dot{X}(\rho)L_2\Bigg] - L_2^T X(\rho)L_2$$

$$+(t_{n+1}-t)\Bigg[(L_7^T F(\rho)L_3)^H + L_3^T F(\rho)L_3\Bigg] - (t-t_n)L_3^T F(\rho)L_3 + \frac{1}{\gamma}\phi^T\phi - \gamma L_8^T L_8\Bigg\}\xi(t) - \int_{t_n}^{t}\begin{bmatrix}\dot{x}(q)\\ x(t_n)\end{bmatrix}^T E(\rho)\begin{bmatrix}\dot{x}(q)\\ x(t_n)\end{bmatrix}dq < 0 \quad (20)$$

$$\big(L_4^T L_1\big)^H + (t_{n+1}-t)L_5^T\Lambda_1^{-1}(\rho)L_5 + (t_{n+1}-t)\Bigg[\lambda\big(L_6^T L_0 L_2\big)^H\Bigg] - \lambda L_2^T\overline{Q}(\rho)L_0L_2 + (t_{n+1}-t)\Bigg[(L_7^T\Lambda_2(\rho)L_3)^H + L_3^T\Lambda_2(\rho)L_3\Bigg]$$

$$-(t-t_n)L_3^T\Lambda_2(\rho)L_3 + \frac{1}{\gamma}\Phi^T\Phi - \gamma L_8^T L_8 + \Big[\overline{N}_1^T(\rho)(\Omega_1^*)\Big]^H + \tau(t)\overline{N}_1^T(\rho)\overline{Q}^{-1}(\rho)\Lambda_1(\rho)\overline{Q}^{-1}(\rho)\overline{N}_1(\rho)$$

$$+\Big[3\overline{N}_2^T(\rho)(\Omega_2^*)\Big]^H + 3\tau(t)\overline{N}_2^T(\rho)\overline{Q}^{-1}(\rho)\Lambda_1(\rho)\overline{Q}^{-1}(\rho)\overline{N}_2(\rho) \prec 0 \quad (23)$$

---

adopted simulation environment is described, as well as any constraint required for the solvability of the optimization problem issued in Theorem 1. For the sake of simplicity, first order (affine) polynomials are admitted for every decision variable. The presented example is evaluated in light of Theorem 1 as a way of assessing the closed-loop stability and $\mathcal{L}_2$-gain performance when sampled-data controllers (6) are applied. Different MASPs T_m and different bounds on the parameters variation rate are considered.

4.1 Computational aspects

The solution of the optimization problems derived in this paper in terms of PLMIs considers the usage of SDP. The computational packages SeDuMi (Sturm, 1999), for the solution of convex optimization problems, and YALMIP (Lofberg, 2004) and ROLMIP (Agulhari et al., 2019), for the description of the PLMI conditions, were exploited. All programming was performed in Matlab.

For computing the control law (6), both the states $x(t)$ and the parameters $\eta(t)$ are assumed to be available at each sampling instant t_n.

For comparison purposes, other techniques from the literature were also implemented under the same framework.

The solution for the conditions reported in Theorem 1 is attained by an iterative procedure, developed in Algorithm 4.1. For that, an initial feasible solution for $\Upsilon(\rho)$ has to determined. As described by Hooshmandi et al. (2018), the inequality

$$\Lambda_1(\rho) + \varepsilon^2\Gamma_1 - 2\varepsilon\overline{Q}(\rho) \prec 0, \quad (26)$$

can replace PLMI (14), in the first iteration, for obtaining an initial solution $\Upsilon_0 = -\Gamma_1^{-1}\overline{Q}(\rho)$. Note that ε is some given positive scalar and that Γ_1 is a parameter-independent matrix during initialization only.

4.2 Results

Example 1. (Hooshmandi et al. (2018)). Consider a quasi-PLPV realization of an inverted pendulum, whose states are the angle $x_1(t)$ of the pendulum with respect to the vertical axis and the angular velocity $x_2(t)$. The system matrices are given by

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Algorithm 1 Iterative procedure for synthesizing gain-scheduled $\mathcal{L}_2$-gain sampled-data controllers.

Initialization:
1) Adopt a value for maximum allowable sampling period T_m.
2) Set $\varepsilon = 1$, $\lambda_0 = 1$, $\lambda = 1$, $\gamma_0 = 10$, $\epsilon = 0.01$, $k_{max} = 20$, and $k = 1$.
3) Given ε and λ, minimize γ under PLMI conditions (11), (12), (13), and (26) for obtaining $Q(\rho)$ and Γ_1.
4) Set $\overline{Q}(\rho) = \text{diag}(Q(\rho), Q(\rho))$, $\Upsilon_0 = -\Gamma_1^{-1}\overline{Q}(\rho)$, and $\gamma_1 = \gamma$.
Iterative procedure:
While $k < k_{max}$ or $|\gamma_k - \gamma_{k-1}| > \epsilon$ **do**
5) Given $\Upsilon_{k-1}(\rho)$ and λ_{k-1}, minimize γ under PLMI conditions (11), (12), (13) and (14) for determining $Q(\rho)$, $\Lambda_1(\rho)$, $\Lambda_2(\rho)$, $\Gamma_1(\rho)$, $G(\rho)$, $L(\rho)$, $Y(\rho)$, $\overline{N}_1(\rho)$, and $\overline{N}_2(\rho)$.
6) Set $Q_{k-1}(\rho) = Q(\rho)$, $\Lambda_{1,k-1}(\rho) = \Lambda_1(\rho)$, $\Lambda_{2,k-1}(\rho) = \Lambda_2(\rho)$, $\Gamma_{1,k-1}(\rho) = \Gamma_1(\rho)$, $G_{k-1}(\rho) = G(\rho)$, $L_{k-1}(\rho) = L(\rho)$, $Y_{k-1}(\rho) = Y(\rho)$, $\overline{N}_{1,k-1}(\rho) = \overline{N}_1(\rho)$, and $\overline{N}_{2,k-1}(\rho) = \overline{N}_2(\rho)$.
7) Given $Q_{k-1}(\rho)$, $\Lambda_{1,k-1}(\rho)$, $\Lambda_{2,k-1}(\rho)$, $\Gamma_{1,k-1}(\rho)$, $G_{k-1}(\rho)$, $L_{k-1}(\rho)$, $Y_{k-1}(\rho)$, $\overline{N}_{1,k-1}(\rho)$, and $\overline{N}_{2,k-1}(\rho)$, minimize γ under PLMI conditions (11), (12), (13) and (14) to obtain $\Upsilon(\rho)$ and λ.
8) Set $\Upsilon_k(\rho) = \Upsilon(\rho)$, $\gamma_k = \gamma$, and $\lambda_k = \lambda$.
9) Set $k = k + 1$.
End

$$A(\eta) = \begin{bmatrix}0 & 1\\ 12.63 - 4.66\eta(t) & 0\end{bmatrix}, \; B_1(\eta) = \begin{bmatrix}0\\1\end{bmatrix}$$
$$B_2(\eta) = \begin{bmatrix}0\\ -0.077 - 0.098\eta(t)\end{bmatrix}$$
$$C(\eta) = [1 \;\; 0], \; D_1(\eta) = 0, \; D_2(\eta) = 0.006 + 0.002\eta(t)$$

with

$$\eta(t) = \left(1 - \frac{1}{1 + \exp(-7[x_1(t) - \pi/4])}\right) \times \left(\frac{1}{1 + \exp(-7[x_1(t) + \pi/4])}\right), \; 0 \le \eta(t) \le 1.$$

Algorithm 4.1 is executed in order to synthesize gain-scheduled controllers to the inverted pendulum system. The attained results are compared with Hooshmandi et al. (2018) in terms of upper bounds to the $\mathcal{L}_2$-gain of the closed-loop system for several MASPs, T_m, and for several bounds for the variation of $\dot{\eta}(t)$. The results are shown in Tables 1 and 2, respectively.

Table 1. Upper bounds for the $\mathcal{L}_2$-gain for different MASPs T_m and $|\dot{\eta}(t)| \leq 0.1$.

T_m (s)	Hooshmandi et al. (2018)	Proposed approach
0.01	0.159	0.052
0.05	0.161	0.072
0.1	0.166	0.093
0.15	0.433	0.115
0.2	Infeasible	0.166
0.239	Infeasible	0.175

Table 2. Upper bounds for the $\mathcal{L}_2$-gain for different variation rates $|\dot{\eta}(t)|$ and $T_m = 0.1$ s.

Bounds on $\lvert\dot{\eta}(t)\rvert$	Hooshmandi et al. (2018)	Proposed approach
0.3	0.171	0.097
0.5	0.177	0.099
0.8	0.193	0.105

The reported results show that the designed controller leads to smaller (improved) $\mathcal{L}_2$-gain upper bounds both for larger T_m and for larger variation bounds on $|\dot{\eta}(t)|$.

Assuming an initial condition $x(0) = [1.2\ 2]^T$, a disturbance input $w(t) = 3\sin(2\pi t)$, $T_m = 0.15$ s and $|\dot{\eta}(t)| \leq 0.1$, the quasi-PLPV representation of the inverted pendulum is simulated. The control law

$$K(\eta(t_n)) = [480.16\ 120.88] - [260.27\ 70.27]\,\eta(t_n) \quad (27)$$

is considered when simulating the closed-loop system for the approach presented in (Hooshmandi et al., 2018). Furthermore, the controller designed with the proposed approach is given as $K(\eta(t_n)) = Y(\eta(t_n))Q^{-1}(\eta(t_n))$, with

$$Y(\eta(t_n)) = [30.4797\ 85.9713] - [11.7697\ 22.8722]\,\eta(t_n)$$

$$Q(\eta(t_n)) = \begin{bmatrix} 0.3642 & -0.8595 \\ -0.8595 & 3.6956 \end{bmatrix} + \begin{bmatrix} 0.0432 & 0.1699 \\ 0.1699 & 0.4567 \end{bmatrix} \eta(t_n)$$

Figures 1, 2 and 3 depict, respectively, a comparison of the state $x(t)$ responses, of the controlled output $y(t)$ response, and of the control signal $u(t)$. The sampling periods are shown in Figure 4, where each stem indicates when the sampling occurred and its amplitude represents the time to be elapsed to the next sampling. The figures illustrate that the proposed control law ensures that the sampled-data controlled system is stable, despite the amplitude of the disturbance signal. Furthermore, the induced $\mathcal{L}_2$-gain norm for the closed-loop system with the sampled-data controller (6) designed is $\gamma^\star = 0.1015$, which is below the reported upper bound in Table 1. Contrast that with the induced $\mathcal{L}_2$-gain obtained by the method of Hooshmandi et al. (2018) with the control law (27), which is $\gamma^\star = 0.2361$.

Example 2. (Gomes da Silva Jr et al. (2018)). Consider the following PLPV system:

$$A(\eta) = \begin{bmatrix} 0 & 1 \\ 0.1 & 0.4 + 0.6\eta(t) \end{bmatrix}, \ B_1(\eta) = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \ B_2(\eta) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C(\eta) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \ D_1(\eta) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \ D_2(\eta) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

with

$$\eta(t) = \sin(\alpha t), \ |\eta(t)| \leq 1, \ |\dot{\eta}(t)| \leq \alpha,$$

in which α is a given bound for the variation rate of the parameter $\eta(t)$.

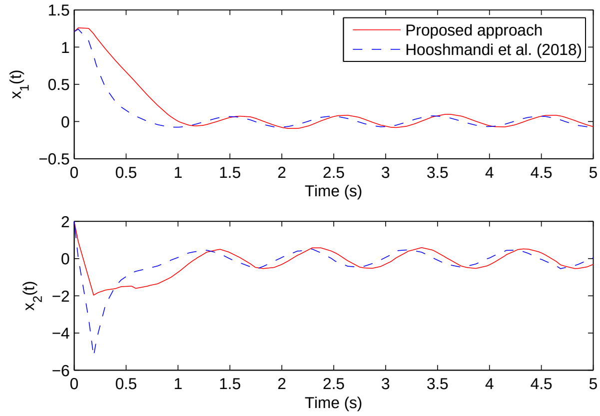

Figure 1. Comparison of the states response for $x(0) = [1.2\ 2]^T$, $w(t) = 3\sin(2\pi t)$, $T_m = 0.15$ s, and $|\dot{\eta}(t)| \leq 0.1$ for Example 1.

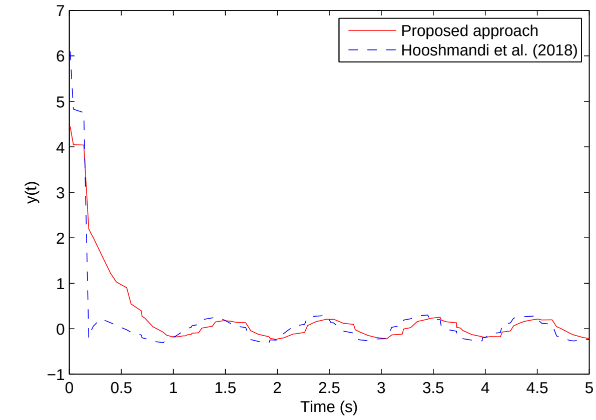

Figure 2. Comparison of the output response for $x(0) = [1.2\ 2]^T$, $w(t) = 3\sin(2\pi t)$, $T_m = 0.15$ s, and $|\dot{\eta}(t)| \leq 0.1$ for Example 1.

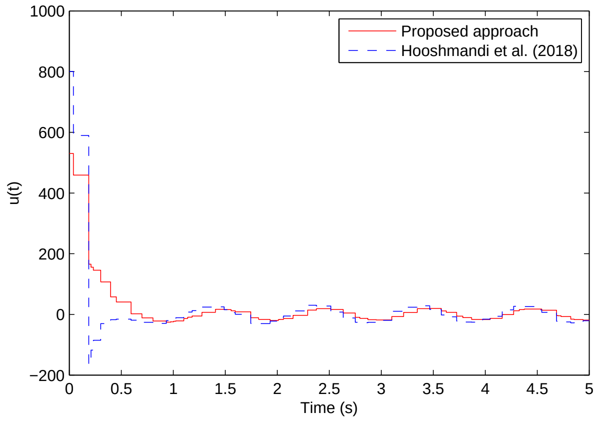

Figure 3. Comparison of the control signal for $x(0) = [1.2\ 2]^T$, $w(t) = 3\sin(2\pi t)$, $T_m = 0.15$ s, and $|\dot{\eta}(t)| \leq 0.1$ for Example 1.

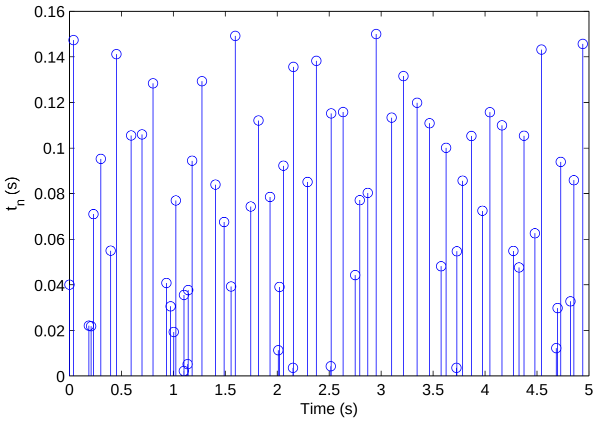

Figure 4. Aperiodic sampling time with MASP $T_m = 0.15$ s for Example 1.

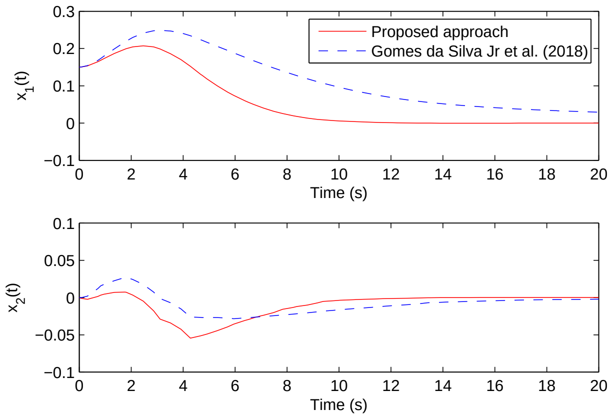

Figure 5. Comparison of the states response for $x(0) = [0.15\ 0]^T$, $w(t) = e^{-t}\sin(t)$, $T_m = 1.349$ s, and $\alpha = 0.2$ for Example 2.

Gain-scheduled sampled-data controllers for the analyzed PLPV system are synthesized through the procedure outlined in Algorithm 4.1. The achieved results are compared with the approach of Gomes da Silva Jr et al. (2018), regarding two different scenarios: firstly, for a fixed upper bound to the $\mathcal{L}_2$-gain, the maximum allowable sampling period T_m is estimated; secondly, for a chosen maximum allowable sampling period T_m, the upper bounds to the $\mathcal{L}_2$-gain are evaluated.

For the first scenario, an upper bound of $\gamma = 15$ and a maximum variation rate $\alpha = 0.2$ for the parameter were adopted. In this setup, the MASP attained by the proposed method is $T_m = 1.659$ s, whereas the method of Gomes da Silva Jr et al. (2018) provided $T_m = 1.349$ s. In the second scenario, the same variation rate $\alpha = 0.2$ is considered and a MASP $T_m = 1.349$ s is used. In this case, the method of Gomes da Silva Jr et al. (2018) yields an upper bound for the $\mathcal{L}_2$-gain of $\gamma = 6.3007$, while the proposed approach provides $\gamma = 1.2000$, which is more than five times smaller. Contrasting the results shows that the proposed conditions are less conservative than others available in the literature.

Choosing an initial condition $x(0) = [0.15\ 0]^T$, a disturbance input $w(t) = e^{-t}\sin(t)$, $T_m = 1.349$s and $\alpha = 0.2$, the PLPV model is simulated. The simulation of the closed-loop system in the approach presented in (Gomes da Silva Jr et al., 2018) employs the sampled-data controller

$$K(\eta(t_n)) = -[0.1508\ \ 0.7422] + [0.0033\ \ -0.5522]\,\eta(t_n). \tag{28}$$

The control law designed with the proposed approach is given as $K(\eta(t_n)) = Y(\eta(t_n))Q^{-1}(\eta(t_n))$, where

$$Y(\eta(t_n)) = -[0.6456\ \ 0.0659] + [0.3976\ \ -0.1519]\,\eta(t_n)$$

$$Q(\eta(t_n)) = \begin{bmatrix} 3.4173 & -0.3375 \\ -0.3375 & 0.2618 \end{bmatrix} + \begin{bmatrix} -0.7029 & 0.0937 \\ 0.0937 & -0.0579 \end{bmatrix} \eta(t_n)$$

Figures 5, 6 and 7 show a comparison of the state $x(t)$ responses, of the controlled output $y(t)$ responses, and of the control signal $u(t)$, respectively. The aperiodic sampling instants are outlined in Figure 8. As in the previous example, the figures demonstrate that the proposed sampled-data controller quickly brings the states to the equilibrium point, even in the presence of external disturbances. The induced $\mathcal{L}_2$-gain estimated by the method of Gomes da Silva Jr et al. (2018) with the control law (28)

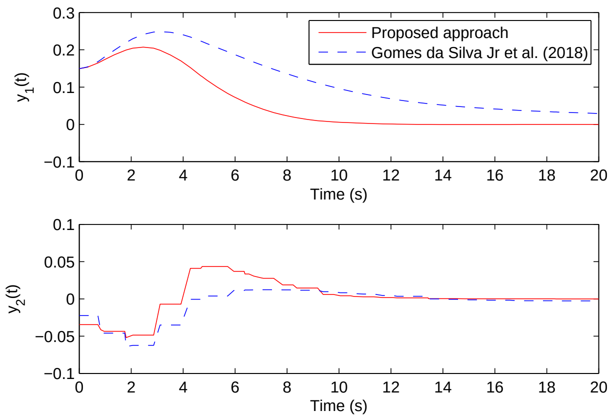

Figure 6. Comparison of the outputs response for $x(0) = [0.15\ 0]^T$, $w(t) = e^{-t}\sin(t)$, $T_m = 1.349$ s, and $\alpha = 0.2$ for Example 2.

is $\gamma^\star = 1.6827$. Such gain is larger than the induced $\mathcal{L}_2$-gain norm for the closed-loop system with the synthesized controller (6), which is computed as $\gamma^\star = 1.1362$. Note also that the proposed approach provided a tighter upper-bound than the method of Gomes da Silva Jr et al. (2018).

5. CONCLUSION

This paper addressed the gain-scheduled sampled-data controller synthesis for nonlinear systems, whose nonlinearities are bounded and have bounded rates of variation. Under these assumptions and with the bounding-box method, nonlinear systems were recast as quasi-PLPV models. Gain-scheduling controllers were designed for the minimization of the $\mathcal{L}_2$-gain norm assuming several maximum allowable sampling periods T_m. The methodology was based on the definition of a Lyapunov-Krasovskii functional and the use of Wirtinger's Inequality to provide LMIs with reduced conservativeness. When compared to previous works, the numerical examples show that the proposed approach ensures the stability for the nonlinear systems, represented in terms of quasi-PLPV models, with larger T_m and improved $\mathcal{L}_2$-gain performance.

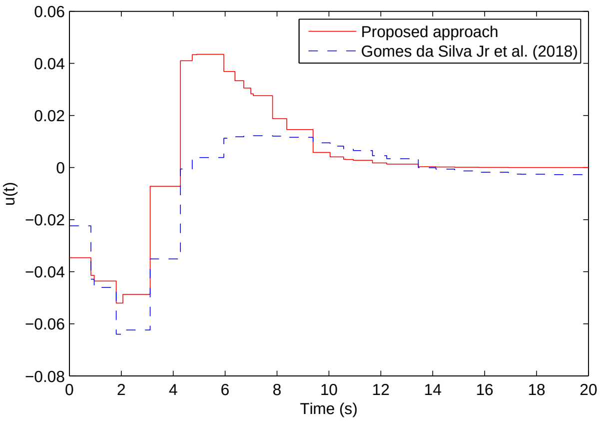

Figure 7. Comparison of the control signal for $x(0) = [0.15\ 0]^T$, $w(t) = e^{-t}\sin(t)$, $T_m = 1.349$ s, and $\alpha = 0.2$ for Example 2.

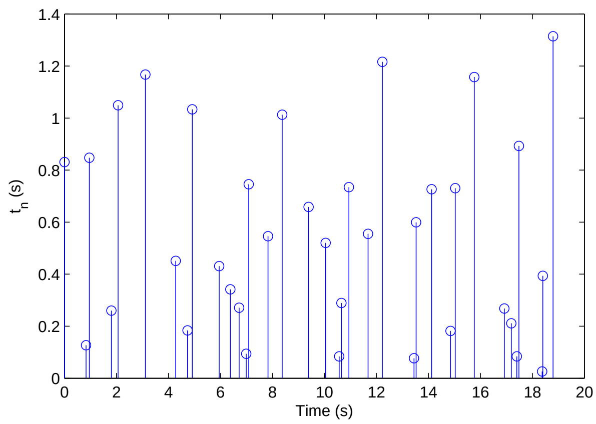

Figure 8. Aperiodic sampling time with MASP $T_m = 1.349$ s for Example 2.

As possible future works, the framework presented in this paper can be extended to include in the PLMI derivation process (i) slack variables by using Finsler's Lemma, or (ii) null terms, as proposed in (Seuret and Gouaisbaut, 2013). The authors also suggest (iii) the application of the synthesized gain-scheduling controllers in real-world systems.

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