Offline FMPC Applied to the 3SSC Boost Converter

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Abstract: This article develops a novel application for fuzzy model predictive control (FMPC),
applying this control law to a three state-switching cell (3SSC) boost converter. The converter
is modeled using augmented state space equations through a polytopic approach, associated
with fuzzy membership functions in order to reject disturbances caused by the change in the
operating point. Furthermore, the FMPC combines Model Predictive Control (MPC), Takagi-
Sugeno (T-S) and Parallel-Distributed Compensation (PDC) fuzzy methodologies. In addition,
an offline approach for the FMPC is presented, whose gains are obtained via stability ellipsoids
and stored in a lookup-table. The obtained results highlight the performance of the proposed
method in comparison with the benchmark controller, through analysis of time responses,
stability ellipsoids and performance index $J_{\infty}$.

Keywords: Fuzzy Control, Model Predictive Control, 3SSC Boost Converter, Linear Matrix
Inequalities, Stability Ellipsoid, Offline FMPC.

1. INTRODUCTION

The advances on methods and computation capacity of microprocessors allowed the development and application
of powerful and sophisticated control strategies (Vazquez et al., 2016), among these strategies stand out the model
predictive control (MPC) and the fuzzy control. These control techniques feature useful advantages and can be
applied in several fields; recent applications can be seen in Sari et al. (2019), Fateh and Fateh (2020) and Bartsch
et al. (2019).

According to Wang (2009), the MPC has been widely
used in industrial and academic applications, due to its
attractive features. The MPC consists of different control
methods that share common aspects. Some examples of
these strategies are the Generalized Predictive Control
(GPC), the Robust Model Predictive Control (RMPC) and
the Dynamic Matrix control (DMC). The main property
of MPC is the prediction ability in a process model.
Moreover, MPC strategies execute a control law that
minimizes a certain cost function over a prediction horizon
(Camacho and Bordons, 2007).

Among MPC’s methods, the procedure developed by
Kothare et al. (1996) highlights. This controller found
many applications due to its capacity to guarantee the
stability and performance, even when subject to system
constraints, model uncertainties, multivariable process,
disturbances and time delay. Evidences of the success of
this approach can be found in Maccari Jr et al. (2019),
Rego and Costa (2020), and Capron and Odloak (2013).
Nonetheless, this control strategy presents difficulties in
controlling complex nonlinear systems as confirmed in
Khairy et al. (2010) and Yu-Geng et al. (2013).

Oppositely, fuzzy controllers are well suitable for dealing
with nonlinear models. According to Kovacic and Bogdan
(2006), this aspect explains the increase in applications
using this approach. The Takagi-Sugeno (T-S) methodology
offers a way to model systems using fuzzy theory, and
is popular because of its proven feature as an universal
approximator (Takagi and Sugeno, 1985). In this context,
Lu (2018), Treesatayapun (2019) and Abbasi and Jalali
(2020) illustrate promise results using fuzzy techniques.

Many researchers found a good solution to the disadvan-
tages of MPC through the association of fuzzy control with
MPC, as pointed by Espinosa et al. (1999). Recent develop-
ments for the Fuzzy Model Predictive Control (FMPC)
can be explore in Wu et al. (2015), Méndez et al. (2016),
Teng et al. (2017), and Wang et al. (2018).

Considering the FMPC positive background, this study
proposes a novel application of the FMPC applied to the
three-state switching cell (3SSC) boost converter. The
power electronics field is an attractive application for
advanced control techniques. According to Costa (2017),
this interest is based on the need to use robust control
methods to assure the stability of these systems, even in
the presence of disturbance, change of the operating point,
and constraints to the process.

Thus, this article proposes the following contributions: (1)
a system model using the augmented state space equations
associated with fuzzy membership functions and a poly-
topic structure; (2) applying the FMPC to the converter
through the T-S and Parallel-Distributed Compensation (PDC) fuzzy methods, aiming at a stable system using state-feedback gains; (3) an offline formulation of the FMPC using the stability ellipsoids stored in a lookup-table; (4) a comparison between the FMPC and the classic Linear Quadratic Integral (LQI), described in Luong and Tso (2014), under the same design conditions.

The results obtained in this study evidences the effectiveness of the proposed FMPC applied to output voltage control of the 3SSC Boost converter. The presented paper adopts the output time response considering the disturbances originated from the input voltage and load. Also, the lookup table is displayed using the impulsive response data of $x_{set}$ provided by Costa (2017), though it adapted to the proposed control technique.

The paper is divided as follows: Section 2 introduces the boost converter model. Next, Section 3 presents the proposed control strategy. Section 4 shows the collected results and reports the controller performance. Finally, Section 5 discusses the main conclusions of the article.

## 2. BOOST CONVERTER MODEL

The proposed FMPC is applied to a DC-DC step-up converter. The model used is a 3SSC boost converter presented in Costa (2017). Figure 1 illustrates the topology of the converter. Besides, Table 1 provides the specifications of the circuit structure.

![Figure 1. 3SSC Boost Converter (Costa, 2017).](image)

### Table 1. 3SSC boost converter electrical parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Voltage ($V_i$)</td>
<td>26–36 [V]</td>
</tr>
<tr>
<td>Output Voltage ($V_o$)</td>
<td>48 [V]</td>
</tr>
<tr>
<td>Duty Cycle ($D_{cycle}$)</td>
<td>0.25–0.46</td>
</tr>
<tr>
<td>Sample time ($T_s$)</td>
<td>1 [ms]</td>
</tr>
<tr>
<td>Inductor filter ($L$)</td>
<td>35 [μH]</td>
</tr>
<tr>
<td>Output capacitor ($C_o$)</td>
<td>4000 [μF]</td>
</tr>
<tr>
<td>Capacitor intrinsic resistance ($R_{ca}$)</td>
<td>26.7 [mΩ]</td>
</tr>
<tr>
<td>Load resistance ($R_o$)</td>
<td>2.3–6.1 [mΩ]</td>
</tr>
<tr>
<td>Output power ($P_o$)</td>
<td>380–1000 [W]</td>
</tr>
</tbody>
</table>

The continuous system model is represented by the medium state-space equations given in (1).

$$
\begin{align*}
\dot{x} &= A_t x(t) + B_t u(t) \\
y(t) &= C_t x(t) + D_t u(t)
\end{align*}
$$

where the state variable is $x(t) = [i_L, V_c, V_i]^T$, $i_L$ is the inductor current and $V_c$ the capacitor voltage. The output voltage is given by $y(t) = V_o(t)$ and $u(t)$ represents the control signal. The state-space matrices $A_t$, $B_t$, $C_t$ and $D_t$ are expressed in (2), (3), (4) and (5), respectively.

$$
\begin{align*}
A_t &= \begin{bmatrix}
\frac{R_L + (1 - D_{cycle}) (R_o || R_o)}{L} & \frac{(1 - D_{cycle}) R_o}{L (R_o + R_o)} \\
\frac{1}{C_o (R_o + R_o)} & \frac{L (R_o + R_o)}{C_o (R_o + R_o)}
\end{bmatrix} \\
B_t &= \begin{bmatrix}
\frac{R_o (1 - D_{cycle}) R_o + R_o}{L (R_o + R_o)} \\
\frac{-1}{C_o (R_o + R_o)}
\end{bmatrix} \left( \frac{V_i}{R'} \right) \\
C_t &= \begin{bmatrix}
(1 - D_{cycle}) (R_o || R_o) \\
\frac{R_o}{R_o || R_o}
\end{bmatrix} \\
D_t &= -V_i \frac{R_o || R_o}{R'}
\end{align*}
$$

The term $R' = (1 - D_{cycle})^2 R_o + D_{cycle} (1 - D_{cycle}) (R_o || R_o)$. To evaluate the controller performance under disturbances, a variation in the operation point is applied to the converter. According to Costa (2017), the uncertainties of the 3SSC converter occur due to the change in the input voltage ($V_i$) and the output power ($P_o$). Thus, the 3SSC boost converter has two uncertain parameters, the load resistance ($R_o$), which changes depending on the output power ($P_o$), and the input voltage ($V_i$), which is a function of the duty cycle ($D_{cycle}$). These parameters are expressed in (6) and (7), respectively.

$$
\begin{align*}
R_o &= f(P_o) = \frac{V_i^2}{P_o}, P_o \in [380,1000][W] \\
D_{cycle} &= f(V_i) = 1 - \frac{V_i}{V_o}, V_i \in [26,36][V]
\end{align*}
$$

Joining (1), (6), and (7), the state space model can be represented as showed in (8).

$$
\begin{align*}
\dot{x} &= A_t (P_o, V_i) x(t) + B_t (P_o, V_i) u(t) \\
y(t) &= C_t (P_o, V_i) x(t) + D_t (P_o, V_i) u(t)
\end{align*}
$$

The model presented in (8) is a continuous Linear Time Variant (LTV) system. Discretizing (8) in the sample time displayed at Table 1, the system becomes (9).

$$
\begin{align*}
x(k + 1) &= A (P_o, V_i) x(k) + B (P_o, V_i) u(k) \\
y(k) &= C (P_o, V_i) x(k) + D (P_o, V_i) u(k)
\end{align*}
$$

where the discretization method follows the one adopted by Costa (2017).

Using the polytopic structure in (9) and the variations in time showed in Figure 2, the vertices of the discretized system are given by (10)-(13).

$$
\begin{align*}
f&(36V,1000W) \\
A_1 &= \begin{bmatrix}
-0.2838 & -7.7479 \\
0.0634 & -0.1136
\end{bmatrix} \quad B_1 = \begin{bmatrix}
580.4784 \\
65.2796
\end{bmatrix} \\
C_1 &= \begin{bmatrix}
0.0198 & 0.9885
\end{bmatrix} \quad D_1 = -0.7304
\end{align*}
$$

$$
\begin{align*}
f&(26V,1000W) \\
A_2 &= \begin{bmatrix}
0.0958 & -8.4508 \\
0.0691 & 0.2660
\end{bmatrix} \quad B_2 = \begin{bmatrix}
851.9917 \\
53.4467
\end{bmatrix} \\
C_2 &= \begin{bmatrix}
0.0143 & 0.9885
\end{bmatrix} \quad D_2 = -1.0054
\end{align*}
$$
3. CONTROL STRATEGY

The proposed control strategy combines the MPC with Fuzzy Control, the theoretical aspects of these techniques are introduced in this section. Subsection 3.1 formulates the augmented state space model applied to the 3SSC Converter. Subsequently, subsection 3.2 discusses the T-S fuzzy model for the system. Furthermore, subsection 3.3 introduces the FMPC control law and subsection 3.4 defines the implemented offline algorithm. Lastly, the block diagram for the control system is illustrated in subsection 3.5.

3.1 Augmented state space model

Following what was presented in Costa (2017), a integral control with two degree-of-freedom is added to the proposed scheme. Such control is adjusted by the variables $g$ and $h$, which are adopted as $g = 1$ and $h = 10$ in order to guarantee the best performance.

The augmented model for the 3SSC Boost Converter is represented by (14).

$$x(k + 1) = \hat{A}_j x(k) + \hat{B}_j u(k)$$

$$y(k) = \left[ \begin{array}{c} C_j \end{array} \right] \begin{bmatrix} 0 \\ \hat{D}_j \end{bmatrix} x(k)$$

With,

$$\hat{A}_j = \begin{bmatrix} A_j & 0 \\ -hC_j & g \end{bmatrix}, \quad \hat{B}_j = \begin{bmatrix} B \\ -hD_j \end{bmatrix}$$

$j = 1, \ldots, N$, com $N$ representando a quantidade de vértices do sistema.

3.2 Takagi-Sugeno Fuzzy model

The augmented model presented in (14) is used to implement the FMPC proposed by Yeh et al. (2006). In order to perform this technique the converter model must be represented using the T-S fuzzy inference system. Wang (1997) defines a T-S fuzzy model as a representation of a nonlinear system through locals linear input-output relationship.

Thus, a discrete T-S fuzzy system is characterized through IF-THEN rules for its $i$-th subsystems, as shown in (16).

$$\text{Rule } i: \begin{cases} \text{If } Z_i(k) = \mu_{i1} \ldots \text{ and } Z_p(k) = \mu_{ip} \\ \text{Then } x(k + 1) = \hat{A}_i x(k) + \hat{B}_i u(k) \end{cases}$$

where $Z_i(k)$, $Z_2(k), \ldots, Z_p(k)$ are the premise variables, $\mu_{ij}$ are the memberships degrees, $\hat{A}_i \in \mathbb{R}^{nxn}$ and $\hat{B}_i \in \mathbb{R}^{nxm}$ are the augmented state matrices, with $i = 1, 2, \ldots, r$, being $r$ the number of rules.

The global output is expressed by (17).

$$x(k + 1) = \sum_{i=1}^{r} h_i(Z(k)) (\hat{A}_i x(k) + \hat{B}_i u(k))$$

Where $h_i(Z(k))$ represents the weight of each rule and is given by (18).

$$h_i(Z(k)) = \frac{w_i(Z(k))}{\sum_{i=1}^{r} w_i(Z(k))}$$

With,

$$w_i(Z(k)) = \prod_{l=1}^{p} M_{il}(Z(k))$$

Moreover, the PDC is used to obtain the control law, as given in (20).

$$u(k) = \left( \sum_{i=1}^{r} h_i(Z(k)) F_i \right) x(k)$$

Where, $F_i$ represents the gain associated with each fuzzy rules.

To represent the 3SSC boost converter through fuzzy logic a two-rules T-S fuzzy system is implemented. Using the the duty cycle as input variable, which is a function of the input voltage, as expressed by (7). The fuzzy layout of this variable is done through trapezoidal membership functions, illustrated in Figure 3.

3.3 Fuzzy Model Predictive Control

Following Yeh et al. (2006), the FMPC control law is implemented using the Linear Matrix Inequalities (LMI) approach. This method allows to deal with uncertainties and add constraints to the system.

This strategy consists of calculating, at each sample time, a state-feedback control law that minimizes the cost function given in (21).

$$\min_{x(k)} \max_{u(k)} J_{\infty}(k),$$

where

$$J_{\infty}(k) = \sum_{i=0}^{\infty} [\hat{X}(k + i) + U(k + i)],$$

Figure 2. Variation in the operating point of the 3SSC converter.
Where, \( x(k) \) is the augmented state vector, \( u(k) \) is the input signal and \( y(k) \) is the output signal. \( W = W^T \geq 0 \) and \( R = R^T > 0 \) are symmetric weighting matrices and \( \Omega \) is the convex hull of plant model \([A(k + i) B(k + i)] \in \Omega, i \geq 0\).

Therefore, FMPC can be described as the LMI problem in (24), considering the augmented system with its model uncertain designed by polytopes.

\[
\min_{\gamma, Q, Y_i, \gamma} \quad x(k|k)^T Q x(k|k) \geq 0 \tag{25}
\]

subject to the constraints given in (25)-(27).

\[
\begin{bmatrix}
Q \\
W^T Q \\
R^T Y_j \\
S \\
\sqrt{2} W^T Q \\
\sqrt{2} R^T Y_j
\end{bmatrix} \begin{bmatrix}
\alpha \\
\gamma I \\
\gamma I \\
\gamma I \\
\gamma I
\end{bmatrix} \geq 0 \tag{26}
\]

Where, \( S = A_j Q + B_j Y_j + A_i Q + B_i Y_j \).

Moreover, the constraints \( \|x(k)\|_2 \leq u_{max} \) and \( \|y(k)\|_2 \leq y_{max} \) are assured if the constraints given in (28) and (29), respectively, are followed.

\[
\begin{bmatrix}
Q \\
Y_j u_{max}^* \\
C \frac{S}{2} y_{max}^* I
\end{bmatrix} \geq 0 \tag{28}
\]

\[
\begin{bmatrix}
Q \\
S \gamma I \\
\frac{S}{2} \gamma I
\end{bmatrix} \geq 0 \tag{29}
\]

The feedback gains for the FMPC can be obtained by (30).

\[
F_i = Y_i Q^{-1} \tag{30}
\]

3.4 Offline algorithm

Considering the FMPC control law given in subsection 3.3 and the converter fuzzy model in subsection 3.2, the proposed offline FMPC algorithm method is obtained as follows:

For an offline system, given an initial condition \( x_2 \), a sequence of minimizers \( (\gamma, Q, Y_i, Y_j) \) is calculated following (21)-(29). Take \( k = 1 \)

- Compute the minimizers \( (\gamma, Q, Y_i, Y_j) \) with the additional constraints \( Q > 0 \) and \( Q = Q^T \) and keep \( Q, Y_i \) and \( F_i \) in a lookup table;
- If \( i < N \), choose the state \( (x_2)_{k+1} \) satisfying \( \|x_{k+1}\|_{Q^{-1}} \leq 1 \). Take \( k := k+1 \) and go to step one.

Lookup table: given the initial condition \( \|x(0)\|_{Q^{-1}} \leq 1 \) take the state \( x(k) \) for the respective time \( k \). Plot the search around \( Q^{-1} \) in the lookup table to find the biggest \( j \) (or the smallest ellipsoid).

Apply the control law (20).

3.5 Block Diagram

Based on the exposed in the aforementioned subsections, Figure 4 displays the block diagram for the proposed augmented FMPC applied to the 3SSC boost converter.

4. RESULTS ANALYSIS

This section discusses the obtained results for the FMPC applied to the discretized 3SSC boost converter. The controller performance is analyzed through time response, stability ellipsoids, and performance metrics. Furthermore, a comparison between FMPC and LQI control is presented.

Figure 5 shows the output response for the FMPC in comparison with the classic LQI control law. It can be seen that both controllers are able to follow the reference tracking even after changing the operation point, presenting oscillations only at these moments. However, the FMPC has a faster and stabler performance, presenting less oscillations and lower overshoots and undershoots.

The control signals are illustrated in Figure 6, showing once again a better performance of the proposed controller. Since the LQI control law presented a slower and more oscillatory outcome. Moreover, the input constraint \( |u(k)| < 1 \), \( k \geq 0 \) is satisfied.
The stability performance of the controller is assessed using invariant ellipsoids. Following Costa (2017), for the 3SSC boost converter, the worst condition is the impulse response. Thus, if the system’s impulse response remains within the ellipsoid boundaries and this response tends to zero on steady state, then the system is stable.

The first step in this kind of analysis is to obtain the impulse response considering the nominal operating point. Thus, considering that the converter nominal condition is given in (10), a set of twenty voltage points was obtained to design the ellipsoid, given by $x_{2,ni} = [48.0000 - 3.6942 - 23.6553 11.3360 7.8756 - 9.0638 - 0.5195 4.9506 - 1.6958 - 1.9171 1.6496 0.3478 - 1.0017 0.2161 0.4384 - 0.2874 - 0.1152 0.1962 - 0.0177 - 0.0957 0.0473]$. 

Figure 7 presents the ellipsoids for the matrices $Q_i$, with $i = 1, 2, ..., 20$, considering the set $x_{2,ni}$. Analyzing Figure 7, the closed-loop stability can be verified, since the size of the ellipsoids decreases as $i$ approaches 20.

Furthermore, Figure 8 displays the projection for the impulse response for each N ellipsoid. According to Costa (2017), the closed loop system is stable to any value of N; thus, choosing the ellipsoid N=20 the trajectory of the impulse response for the controller is illustrated in Figure 9. It is possible to notice that the impulse response is restricted inside the limits of the ellipsoid. In addition, the impulse response converge to the origin, thus it can be concluded that the proposed controller guarantees the system stability.

Figure 5. Output response $y(k) = V_o(k)$.

Figure 6. Control signal $u(k)$.

Figure 7. Stability Ellipsoids.

Figure 8. The closed-loop impulse response bounded by ellipsoids in $x_{set} \times i_L \times V_c$.

Figure 9. Impulse response for N=20.
The controller was also evaluated and compared using the cost function given in (14) as a performance index. The result obtained for the FMPC is $J_{∞} = 9.9627 \times 10^6$ while for the LQI $J_{∞} = 1.0062 \times 10^6$, reinforcing the superiority of the FMPC in comparison with LQI.

5. CONCLUSION

The designed procedure demonstrate the effectiveness of the FMPC, taking into consideration the time response with variations of the input voltage and the load, and also the $J_{∞}$ performance index. In addition, the FMPC exhibited satisfactory results considering the ellipsoid stability analysis for the offline representation. Therefore, compared to the LQI under the same design conditions, the offline FMPC controller proved its viability application for 3SSC boost converters.

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