On application of a zone IHMPC to an ESP-lifted oil well system


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Abstract: This paper presents an application of a stabilizing model predictive control (IHMPC) strategy with the underlying guarantee of feasibility to an oil production well system with Electric Submersible Pump (ESP) installation. The proposed controller is compared with a conventional finite-horizon MPC in which provides some unfeasible solutions in the presence of an unmeasured disturbance, due to the typical conflict among the ESP-lifted oil well system constraints. The results show IHMPC as a viable choice to improve ESP-lifted well production since it can incorporate the desired requirements and provide stabilizing control actions.

Keywords: Artificial lift, Electric submersible pump, Oil and gas, Model predictive control

1. INTRODUCTION

The electric submersible pump (ESP) is a widely used artificial lift method around the world when one seeks to produce a significant volume of oil and gas (Liang et al., 2015). The safe and stable operation of ESP-lifted oil wells is carried out by the so-called ESP operating envelope-like set (Takacs, 2009). This envelope comprises time-variant constraints (upthrust and downthrust) that rely on the dynamic of ESP flow rate and pump head.

To work around this, model predictive control (MPC) has played a prominent role over the last years, due to it can systematically deal with multivariable and constrained systems with associated economic targets. The research efforts on MPC based solutions for oil production wells with ESP installations are indeed in progress. However, the works investigated so far have focused on conventional MPC approaches, which are briefly highlighted as follows.

The primary work, proposed by Pavlov et al. (2014), presented an MPC formulation for tracking the ESP intake pressure, and the minimization of the ESP power consumption by a production choke opening target, incorporated explicitly in the objective function of the controller as well as the operational envelope constraints. Binder et al. (2014) used the same MPC controller formulation and ESP dynamic model to investigate the implementation of an embedded MPC on a controller logic programmable, but the ESP power minimization was regulated by ESP motor current target and downthrust and upthrust force limits were not included explicitly. Krishnamoorthy et al. (2016) designed an MPC controller with the same control objectives as in Pavlov et al. (2014) whereas the constraints were those same used by Binder et al. (2014). However, the linear model is obtained from the step-response approach on a high fidelity simulator of ESP-lifted well. Binder et al. (2019) explored some above-mentioned MPC formulations, Pavlov et al. (2014); Binder et al. (2014), by including measured disturbances in their formulations, such as reservoir pressure, in order to evaluate aspects of the control performance improvement. Delou et al. (2019) proposed an adaptive MPC control law in such a way that widens the ESP-lifted oil production operating range with step-response linear models. However, instead of tracking targets, the ESP power minimization was tracked by a conservative set-point, and the set of time-variant ESP operating envelope constraints was not incorporated into the control problem formulation.

Despite advances presented above, the MPC controllers investigated until now rely upon control laws which can easily yield unfeasible solutions due to the typical conflict among the ESP-lifted oil well system constraints. In the middle of theoretical MPC framework, stabilizing MPC strategies with the underlying guarantee of feasibility, experimentally tested in practice, e.g. Martin et al. (2019), can be an attractive way to face the challenging issues related to the oil production wells with ESP installations. In particular, to the best authors’ knowledge, there is no application of stabilizing MPC controllers devoted to ESP-lifted oil well systems, and this is the study object of the present work.

This paper concerns with the application of an infinite-horizon based stabilizing MPC (IHMPC) for oil production wells with ESP installed, hitherto unexplored yet. Moreover, another contribution of this paper lies at the tracking for maximizing the ESP oil production is properly designed within an implementable target zone scheme, including explicitly the associated downthrust and upthrust constraints, besides the optimizing target related to the production choke valve opening (keeping it as open as possible) and the set-point control of the ESP intake pressure. The control zone scheme used here softens, only when necessary, the typical conflict among the output...
constraints of the ESP-lifted oil well system by the use of the
slacked terminal constraints-type endpoint constraints, preserving
the stabilizing properties of the IHMPC control law and making it
implementable in practice as well.

This paper is organized as follows. Section 2 presents a
brief description of the ESP system, the conventional MPC
zone control and a control law with guaranteed feasibility.

The third section concerns with the control formulations
comparison in a scenario of mismatch simulation, set-
points tracking, unmeasured disturbance, and economic
target tracking. Finally, Section 4 offers some concluding

2. PROPOSED MPC SCHEMES FOR ESP

The ESP is a common artificial lift technology imple-
mented in wells where the oil does not flow naturally
(Krishnamoorthy et al., 2019). The operation of the ESP-
lifted oil well is given by two manipulated variables, the
pump rotational frequency \(f(t)\), and the production
choke opening \(z_c(t)\). Generally, there are several ESP
connected by the same manifold, which causes distur-
bances in the production due to the manifold pressure
\(p_m(t)\) variations.

As far as a dynamic simulation of the ESP-lifted oil well
is concerned, a system of differential-algebraic equations
(SDAE) proposed by Pavlov et al. (2014) is described as
follows \(^1\):

\[
\begin{align}
\dot{p}_{wh} &= 1.54 \times 10^8 (q_p - q_c) \\
\dot{p}_{bh} &= 0.8584 (p_r - p_{wh}) - 3.7 \times 10^8 q_p \\
q_p &= 5.02 \times 10^{-9} \left[ (p_{wh} - p_{wh} - 6.30 \times 10^8 q_p \right. \\
& \quad \left. + 9.32 \times 10^3 (H - 1 \times 10^3) \right] \\
q_c &= 2 \times 10^{-3} z_c \sqrt{p_{wh} - p_m} \\
p_m &= p_{bh} - 1.85 \times 10^5 q_p^{1.75} - 1.9 \times 10^6 \\
H &= 0.2664 f^2 + 133.09 f q_p - 1.41 \times 10^6 q_p^2
\end{align}
\]

where \(p_{wh}, p_{bh}, p_m, p_r\) are the wellhead, bottom hole,
take and reservoir pressures, respectively; \(q_p\) and \(q_c\) are
the average and production choke flow rates, respectively;
and \(H\) is the pump head. Figure 1 represents a simple
scheme of the artificial lift process used in this work.

In addition, the process lies within the so-called ESP
operational envelope. This envelope is limited by the
minimum and maximum rotational frequencies and two
curves indicated as upthrust and downthrust, resulting in a
region on which it is desirable to operate the pump to avoid
its mechanical degradation caused by unbalanced thrust
forces (cf. Figure 1). Besides that, the production flow rate
has to be kept in its desired reference and maximized. In
this way, the ESP control variables adopted in this work
will be represented by \(y \equiv [p_{wh}, H]^{\top}\) (controlled variables)
and \(\mathbf{u} \equiv [f, z_c]^{\top}\) (manipulated variables).

Based on this scenario, an automatic control must be able
to keep the ESP in safe and optimal operation, i.e. keeping
the process variables into an operational envelope, while
seeking the BEP (best operation point). As an alternative,

\(^1\) More details about equations and variables can be found in Delou
et al. (2019).

there are some solutions based on the PID control with
constraints being dealt with an ad hoc manner (Krish-
namoorthy et al., 2019). However, due to the capacity to
handle restrictions systematically, several researchers have
evaluated model predictive control (MPC) based solutions
for oil production wells with ESP installations over the last
years.

MPC controllers are known by a receding horizon op-
imization algorithm, whose decision variables – control
actions, are evaluated based on process predictions and
constraints. The linear state-space model-based standard
control law can be described as follows (Maciejowski,
2002):

**Problem 1.**

\[
\min_{\Delta \mathbf{u}_k} \sum_{j=1}^{p} \| y(k+j|k) - y_{sp} \|^2_{Q_y} + \sum_{j=0}^{m-1} \| \Delta \mathbf{u}(k+j|k) \|^2_R
\]

subject to:

\[
\begin{align}
\mathbf{u}_{min} &\leq \mathbf{u}(k-1) + \sum_{i=0}^{j} \Delta \mathbf{u} \quad |k + i| \leq \mathbf{u}_{max} \\
-\Delta \mathbf{u}_{max} &\leq \Delta \mathbf{u}(k+j|k) \leq \Delta \mathbf{u}_{max} \quad j = 0, \ldots, m - 1 \quad (2) \\
\Delta \mathbf{u}(k+j|k) &= 0, \quad v_j \geq m \\
x(k+j|k) &= A \cdot x(k+j-1|k) + B \cdot \Delta \mathbf{u}(k+j-1|k) \\
y(k+j|k) &= C \cdot x(k+j|k) \quad j = 1, \ldots, p \quad (3) \\
\end{align}
\]

where \(\Delta \mathbf{u}(k+j|k)\) are increments of manipulated vari-
ables and \(y(k+j|k)\) are predictions of controlled vari-
ables at time step \(k+j\) given the current state \(x(k)\);
\(y_{sp}\) are set-points of controlled variables; \(A, B\) and \(C\) are
space-state matrices of the system; \(\mathbf{u}_{max, min, \Delta \mathbf{u}_{max}}\).
\[ \Delta u_{\text{min}}, y_{\text{max}} \text{ and } y_{\text{min}} \text{ are the constraints of manipulated variables, increments of manipulated variables, and controlled variables, respectively; } \Delta u_k = [\Delta u(k|k)\top, \ldots, \Delta u(k+m-1|k)\top] \text{ is the vector of control actions; } p \text{ and } m \text{ are the prediction and control horizons, respectively; } Q_u \text{ and } R \text{ are weighting matrices of controlled and manipulated variables, respectively.} \]

As mentioned, the ESP operation is limited by an operational envelope (as seen in Figure 1) which can be built as a function of flow rate (denoted by \( q_p(t) \)) and pump head \( (H(t)) \). In this way, the MPC formulation presented in Problem 1 may be applied to a zone control scheme by setting constraints as downthrust and upthrust limits and zeroing the importance matrix element for the controlled variables, whose set-point tracking is not desired (Maciejowski, 2002); for this work it is applied to the pump head \( H \) \( (q_{in} = 0) \).

However, the control law presented by Problem 1 does not have neither the guarantee of stability nor the guarantee of global feasibility. For instance, at a given time step the optimization problem may be unfeasible, i.e. the controller is not able to evaluate a control action that lies within constraints.

To date, to the best authors’ knowledge, several studies have investigated variations of the conventional MPC approaches (Pavlov et al., 2014; Binder et al., 2014; Krishnamoorthy et al., 2016; Delou et al., 2019). So far, an open issue remains concerning applications of stabilizing MPC controllers to ESP-lifted oil well systems, which is one contribution of the present work. As an example of stabilizing MPC controllers, the Odloak family’s infinite horizon MPC (IHMP) controllers can be highlighted in Martins et al. (2014); Martins and Odloak (2016) to just name a few.

One of the main features of the Odloak family’s controllers is a modified cost function with guaranteed feasibility due to the addition of a suitable set of slack variables and an analytical expression of the step-response system based state-space model in the incremental form of inputs. With this approach, control actions can be always evaluated such that they drive the process to the desired reference value. In this scenario, González and Odloak (2009) proposed a control law applicable to ESP-lifted oil well-type open-loop stable processes and zone control. The IHMPC with zone control formulation is described as follows:

**Problem 2.**

\[
\min_{\Delta u_k, y_{sp}, \delta_y, \delta_u} \sum_{j=0}^{\infty} \|y(k+j|k) - y_{sp} - \delta_y\|_2^2 Q_y + \sum_{j=0}^{m-1} \|\Delta u(k+j|k)\|_2^2 R_u + \sum_{j=0}^{\infty} \|u(k+j|k) - u_{tg} - \delta_u\|_2^2 Q_u + \|\delta_y\|_2^2 S_y + \|\delta_u\|_2^2 S_u
\]

subject to (2) and:

\[
\begin{align*}
x(k+j|k) &= A \cdot x(k+j-1|k) + B \cdot \Delta u(k+j-1|k) \\
y(k+j|k) &= C \cdot x(k+j|k) \\
y_{\text{min}}(k+j|k) &\leq y_{\text{sp}} \leq y_{\text{max}}(k+j|k) \\
y(k+m|k) - y_{\text{sp}} - \delta_y &= 0 \\
u(k+m-1|k) - u_{tg} - \delta_u &= 0
\end{align*}
\]

where \( \Delta u_k, y_{sp}, \delta_y \text{ and } \delta_u \text{ are the decision variables of the optimization problem, in particular, } \delta_y \text{ and } \delta_u \text{ are the slack variables, } S_u \text{ and } S_y \text{ are weighting matrices of the slack variables; } u_{tg} \text{ are input targets and } Q_u \text{ is its respective weighting matrix. More details about } A, B \text{ and } C \text{ are described in González and Odloak (2009).} \]

In addition to slacks variables, from Problem 2, it can be seen that the zone constraints (4) (downthrust and upthrust limits) are imposed at the output set-point \( (y_{sp}) \). Therefore, the output constraints in Problem 2 are softer than (3), which also contributes to a fundamental issue on the optimization feasibility. Note that the set-point vector is an additional decision variable that is evaluated inside the limits to minimize the cost function. In this case, if the upper and lower bounds are equal, the zone control is reduced to set-point tracking control (González and Odloak, 2009), being such a scheme devoted here only to intake pressure \( p_{in} \).

Another interesting aspect of Problem 2 is that a steady-state economic target \( (u_{c}) \) is incorporated into the cost function. In this way, this controller strategy can drive the process close to the desired economic target while maintaining the process variables in their limits.

Based on discussed above, a comparison of these two control laws applied to the ESP-lifted oil well system will be provided in the next section.

### 3. SIMULATION RESULTS

Concerning the ESP control objectives, as mentioned before, it is necessary to keep the ESP inside the operational envelope (Figure 1). Also, it is important to keep the production flow rate \( (q_p(t)) \) in the desired reference. Thus, Pavlov et al. (2014) suggests, as a practical approach, to control the intake pressure, since affecting directly the production flow rate. In this way, the ESP control objectives adopted in this work can be summarized in:

1. Keeping the ESP intake pressure at the desired set-point;
2. Keeping ESP operation inside the operational envelope;
3. Maximizing the oil production volume.

Figure 2 provides the MPC implementation scheme of the plant-model mismatch simulation, where \( y \equiv [p_{in}, H]\top \) and \( u \equiv [f, z_c]\top \).

As regard control laws, MPC and IHMPC are applied to track and maintain the intake pressure \( (p_{in}(t) = y_1) \) reference while keeping the pump Head \( (H(t) = y_2) \) into the operational envelope by manipulating the rotational frequency \( (f(t) = u_1) \) and choke opening \( (z_c(t) = u_2) \).

The linearized model used for the MPC strategies was obtained at the equilibrium point \( p_{in,ss} = 6.0 \times 10^6 \text{ Pa}, H_{ss} = 592.12 \text{ m}, f_{ss} = 50 \text{ Hz}, z_{c,ss} = 50\% \text{ and } p_{in,ss} = 2 \times 10^6 \text{ Pa, and discretized for a sampling time equal to 1} \)

\[ 2. \] It is necessary a state estimator, since MPC formulations adopted in this work are based on state-space models with unmeasured states.

Thus, in this work, it was implemented the linear Kalman Filter, with the same covariance matrices.
For this reason, the conventional MPC was incapable of driving the ESP to the desired reference and maintaining the system within the operational envelope, as can be seen in Figure 4. The setting of controllers is summarized in Table 1.

Table 1. Setting of MPC controllers

<table>
<thead>
<tr>
<th>Parameters</th>
<th>MPC</th>
<th>IHMPCzone</th>
<th>IHMPCzone+target</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>60</td>
<td>( \infty )</td>
<td>( \infty )</td>
</tr>
<tr>
<td>( m )</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>( q_T )</td>
<td>[1, 0]</td>
<td>[1, 1]</td>
<td>[1, 1]</td>
</tr>
<tr>
<td>( r )</td>
<td>[1, 1]</td>
<td>[1, 1]</td>
<td>[1, 1]</td>
</tr>
<tr>
<td>( q_u )</td>
<td>-</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( S_u )</td>
<td>-</td>
<td>0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>( S_y )</td>
<td>-</td>
<td>( 1.1 \times 10^6 )</td>
<td>( 1.1 \times 10^6 )</td>
</tr>
<tr>
<td>( u_{min} )</td>
<td>[35Hz, 0%]</td>
<td>[35Hz, 0%]</td>
<td>[35Hz, 0%]</td>
</tr>
<tr>
<td>( u_{max} )</td>
<td>[65Hz, 100%]</td>
<td>[65Hz, 100%]</td>
<td>[65Hz, 100%]</td>
</tr>
<tr>
<td>( \Delta u_{max} )</td>
<td>[0.5Hz, 0.3%]</td>
<td>[0.5Hz, 0.3%]</td>
<td>[0.5Hz, 0.3%]</td>
</tr>
<tr>
<td>( y_{min}(t) )</td>
<td>[0, ( H_{1T}(t) )]</td>
<td>[( p_{in}(t), H_{1T}(t) )]</td>
<td>[( p_{in}(t), H_{1T}(t) )]</td>
</tr>
<tr>
<td>( y_{max}(t) )</td>
<td>[700, ( H_{1T}(t) )]</td>
<td>[( p_{in}(t), H_{1T}(t) )]</td>
<td>[( p_{in}(t), H_{1T}(t) )]</td>
</tr>
</tbody>
</table>

From Table 1 it is important to highlight some aspects:

- In the MPC, the element \( q_{u} \) is null so that Problem 1 can be applied as a zone control.
- \( q_u \) and \( S_u \) are null to simulate no target tracking in the IHMPC, denoted here as IHMPCzone.
- In the IHMPCzone and IHMPCzone+target, the limits for \( p_{in}(t) \) are equals to the desired reference so that it can be tracked.
- In all controllers, the limits of \( H(t) \) are defined by the downthrust (\( H_{DT}(t) \)) and upthrust (\( H_{UT}(t) \)) limits.

Regarding the performance objectives, the controllers are compared by the index ISE (integral of squared error), defined by

\[
\begin{align*}
\text{ISE}_y &= T_s \sum_{k=0}^{700} (y(k) - y_{Sp}(k))^2 \\
\text{ISE}_{\Delta u} &= T_s \sum_{k=0}^{700} (\Delta u(k))^2 \quad (6)
\end{align*}
\]

Moreover, the operating time outside the operating envelope (\( T_{out}(t) \)) is calculated as a performance index, and the oil production volume is evaluated as follows:

\[
V_T = T_s \sum_{k=0}^{700} q_c(k) \quad (7)
\]

The main difference between conventional MPC and IHMPC controllers are highlighted in Figure 3. The most interesting aspect of this figure is that the conventional MPC becomes unfeasible in the presence of an unmeasured disturbance (at time 100s and 350s).

For this reason, the conventional MPC was incapable of driving the ESP to the desired reference and maintaining the system within the operational envelope, as can be seen in Figure 4.

Figure 2. Implementation scheme of mismatch simulation.

second. To observe that the controllers start from the same point, but follow different routes and end in different stationary states.

Despite the stabilizing feature of the IHMPC formulations is guaranteed to nominal closed-loop, surprisingly, it is observed (Figure 8) that in a condition this case, the IHMPCzone+target simulation it was used a choke opening target throughout the simulation time, i.e., \( z_{c,tg}(t) = 1 \), \( t \in [0, 700] \) s.
even of plant-model mismatch, the objective functions of the IHMPC controllers maintain as a Lyapunov function (González and Odloak, 2009). This finding suggests that the linearized model was able to represent the nonlinear system in which the mismatch was not enough to provide instability.

In terms of manipulated variables dynamics, the MPC maintains the last control action until the system becomes viable again while the IHMPC continuously evaluates the control actions to reach the desired requirements, as shown in Figure 9.

From the point of view of performance indices, IHMPC with zone and target provides a slight improvement in the behavior of the control variables, decreasing the $ISE_{\Delta p}$ (Table 2), and maintaining the ESP inside the operational envelope for longer than the IHMPC zone performance, as can be seen in Figure 5 and Figure 6, and the $T_{out}$ index in Table 2. As a consequence, a greater control effort is required, (increasing $ISE_{\Delta u}$), as can be observed in Table 2.

Table 2. Comparison between normalized performance index of MPC and IHMPC

<table>
<thead>
<tr>
<th>Index</th>
<th>MPC</th>
<th>IHMPC zone</th>
<th>IHMPC zone+target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ISE_{\Delta p}$</td>
<td>10.14</td>
<td>2.43</td>
<td>2.14</td>
</tr>
<tr>
<td>$ISE_{\Delta u} \times 10^3$</td>
<td>6.4</td>
<td>7.6</td>
<td>9.4</td>
</tr>
<tr>
<td>$ISE_{\Delta z} \times 10^3$</td>
<td>3.7</td>
<td>12.4</td>
<td>13.7</td>
</tr>
<tr>
<td>$T_{out}$</td>
<td>452</td>
<td>270</td>
<td>230</td>
</tr>
<tr>
<td>$V_t/(m^3)$</td>
<td>6.37</td>
<td>6.81</td>
<td>6.86</td>
</tr>
</tbody>
</table>

From Figure 9, it can be observed that the IHMPC controllers behave similarly until instant 200s, while the MPC keeps MVs fixed (from 100s to 200s, and from 350s - Figure 9) due to the optimization infeasibility, resulting in less control effort and worse performance on intake pressure and pump head. From 100s to 200s, the conventional MPC goes infeasible but, it keeps choke opening higher than the IHMPCs, which provides greater oil production during this time (Figure 10). On the other hand, the system is kept out of the security region, which is undesirable (Figure 4).

In addition, Figure 9 and 10 show that from the instant ($t \geq 200$) the intake pressure reference decreases, IHMPCs compute control actions to reach the setpoint, which leads to greater oil production, while the conventional MPC goes infeasible again (from the instant 350s) keeping the choke opening in a lower value and providing a lower oil production volume.
production volume. Even with the input target activated ($q_u = [0, 1]$ and $S_u = [0, 100]$), IHMPC$_{zone+target}$ prioritizes the controlled variables, since the intake pressure reference increases. However, when intake pressure decreases ($t \geq 200$) the actions are calculated to reach the input target. In this way, IHMPC$_{zone+target}$ provides an increase in the choke opening associated with a soft decrease in the rotational frequency, which increases the oil production volume 0.7% more than IHMPC$_{zone}$ and 7.7% more than MPC (Figure 10 and Table 2).

4. CONCLUSION

The present study provides the first overall assessment of the an infinite-horizon based stabilizing MPC (IHMPC) application for an ESP-lifted oil well system.

The results indicate that IHMPC controllers can be applied to the ESP control problem without concern about the control law feasibility. The IHMPC formulations were capable of evaluating control action while the MPC one became unfeasible in the simulated scenario. Overall, the results indicate that the IHMPC can be a viable alternative to improve ESP-lifted oil well production since control formulation can easily incorporate the desired requirements.

Finally, the results shown here is subject to certain limitations. For instance, a fixed economic target and tuning parameters. Further investigation and experimentation concerning these issues are strongly recommended.

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