

# Web Environment for Diagnosis and Estimation of Power System Electromechanical Modes Based on Yule-Walker and Welch Methods<sup>\*</sup>

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**Abstract:** Real-time monitoring of the power grid has become increasingly viable due to the advances in phasor measurement units (PMUs) technology, allowing system operators to analyze a whole set of different events and dynamics present in the system. One particularly interesting dynamic observed via PMU data is electromechanical oscillations, which pose a real threat to power system stability. This paper presents the development of a web environment designed to estimate electromechanical modes present in the distribution grid based on real-time data from  $\mu$ PMU units. Those  $\mu$ PMUs are located at different campuses of the Universidade Federal do Paraná (UFPR). To achieve the mode estimation, the well-known Welch and Yule-Walker algorithms are used. A brief explanation about these methods is provided, as well as all the interface's features, such as the signal processing techniques and a set of configurable parameters by the end user whose influence on the overall result can then be assessed. To validate the usability of the web environment, a demonstration of both the Welch and Yule-Walker mode estimations using actual real-time  $\mu$ PMU data is displayed.

*Keywords:* Power System Mode Estimation; Phasor Measurement Units; Power System Stability; Power System Monitoring; Welch Method; Yule-Walker Method; Web Environment.

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## 1. INTRODUCTION

With the increase in electrical energy demands coming from industrial, commercial, and residential areas, power systems worldwide are becoming increasingly complex. To successfully fulfill those demands, the grid must integrate various energy generation sources in different locations and interconnect those sources to secure continuous energy delivery to the loads.

A problem that arises with this increasing power system complexity is oscillations related to a single generation source or between two generation sources. Depending on its frequency and damping ratio, oscillations in the power grid can reduce power flow thresholds in transmission lines and present a threat to system stability. Power flow changes in a system working near its stability limit may cause a chain of events resulting in oscillations in which amplitudes can quickly increase. A possible scenario would be the triggering of protective relays and tripping of transmission lines, consequently causing blackouts in some regions of the grid, showing just how drastic and insidious this nature of oscillatory instability can be (Rogers, 1996).

Power system oscillations can be captured by Wide Area Measurement Systems (WAMS), which are based on Phasor Measurement Units (PMU) and  $\mu$ PMU. Those measurement devices can be connected to one or more Phasor Data Concentrators (PDC). This setup provides visibility

of real-time system data, and consequently the grid's condition. Based on a WAMS network, PMUs are devices designed for measuring the main electrical grid signals, such as voltage, current, and phase angle. These measurements are referred to as synchrophasors and are complemented with precise time-stamping tags provided by the use of GPS technology, which allow operators to compare values sampled at the same time in different locations within the system (von Meier et al., 2017). Traditional PMUs are used to monitor data at transmission and substation levels, while  $\mu$ PMUs, due to their higher sampling rate, can be used to detect events present in distribution grids. Since PMUs can measure phase angles, the device's internal processor can use this information to obtain the grid's frequency (Vanfretti et al., 2013). This system frequency, paired with processing algorithms, allows us to estimate power system electromechanical oscillation modes.

Aside from PMUs, WAMS also contain components called PDCs. Those PDCs are responsible for collecting incoming PMU data and sort them based on their GPS time-stamps. They are also responsible for sharing this data with external programs in charge of the grid's control and protection. Moreover, this device also provides information about the communication status with connected PMUs and allows for data sampling configurations (de Oliveira, 2012). The application referred to in this paper gathers data coming from the  $\mu$ PMU/PDC network, which is maintained at the Electrical Eng. Department of UFPR. This network consists of 5  $\mu$ PMUs, four of those located in

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Curitiba/PR and one located in Palotina/PR, and 1 PDC. More details can be found in Oliveira et al. (2020).

In order to correctly estimate electromechanical modes in the power system, the implementation of processing algorithms for synchrophasor data is necessary. There are various methods to choose from to achieve this task. Within those methods, some of them can be classified as parametric and non-parametric methods (Thambirajah et al., 2010). Parametric methods aim to estimate a parametric model of the system's behaviour from the system's output or input/output measurements. The main electromechanical modes are calculated from the estimated model eigenvalues or poles. Methods that are labelled as parametric ones are the Yule-Walker (Trudnowski et al., 2008), the N4SID (Ding and Huang, 2015), the Vector Fitting (Schumacher et al., 2019) (Schumacher et al., 2018), among others. Meanwhile, non-parametric methods work exclusively with the system's output measurements to obtain the main electromechanical modes. The majority of these methods use spectral analysis techniques and have the disadvantage of not being able to inquire the damping ratio of oscillations, only its frequency. A method that is labelled as non-parametric is the Welch method (Welch, 1967).

Both parametric and non-parametric methods rely on some parameters design procedures (data downsampling, filtering, windowing, for example) and set-up parameters to work correctly. The design of those procedures and parameters may require some expertise to be applied.

This work presents a computational environment based on a web platform and the Yule-Walker and Welch methods. Both are implemented to aid decisions for electromechanical modes analysis from the UFPR  $\mu$ PMU/PDC network. This platform offers the possibility of accessing data from all 4  $\mu$ PMUs, presenting the user with the system's frequency condition. Configuration parameters such as time window and downsample rate can be selected by the user. Both methods can be selected and show the power system condition in its primary slow electromechanical mode. Therefore, their results can be easily compared in terms of the system's actual condition.

This paper is structured in 7 Sections. In Section 2, the electromechanical mode estimation problem based on  $\mu$ PMU/PMU data is reviewed. In Sections 3 and 4, the Welch and Yule-Walker methods are summarized, focusing on their tuning parameters. In Section 5, the web environment details are presented. Some results using actual system data are also presented in Section 5 to demonstrate the web environment applicability. In Section 6, the paper is concluded.

## 2. PROBLEM STATEMENT

Estimating power system electromechanical modes is a complex task, given the fact that it is mainly non-linear and time-varying. It may have many electromechanical modes that are close to each other, and the natural behaviour of the measurement signals are primarily stochastic due to random load switching (Trudnowski and Pierre, 2009).

In order to better understand how the power system behaves regarding oscillations and small-signal analysis, it is assumed that the system behaviour can be described as a set of differential equations as the following:

$$\begin{aligned} \dot{\underline{x}}(t) &= A\underline{x}(t) + B_L\underline{v}(t) \\ \underline{y}(t) &= C\underline{x}(t) + D_L\underline{v}(t) + \underline{\mu}(t) \end{aligned} \quad (1)$$

where  $t$  represents the time,  $t \in \mathbb{R}$ ,  $\underline{v}$  is the hypothetical random vector perturbing the system, generally caused by variations in system load and other system disturbances. Vector  $\underline{x}$  contains all the system states, including data about generators and transmission lines. Matrices  $A$ ,  $B_L$ ,  $C$  and  $D_L$  include the system's parameters, and vector  $\underline{\mu}$  is the assumed measurement noise. Lastly, vector  $\underline{y}$  is the measured system signals, which can be more than one (Trudnowski et al., 2008).

Assuming this constant intake of real-time frequency data coming from PMUs, the main objective is to estimate low-frequency electromechanical oscillations present in the power grid with the help of mathematical algorithms such as the Welch and Yule-Walker.

## 3. WELCH METHOD

The Welch method is classified within power system analysis as an algorithm that uses ambient data (Thambirajah et al., 2010) without the need of probing techniques. It is an example of a non-parametric spectral method.

Consider an incoming power system frequency signal on the basis of equation 1. This technique obtains the power density of frequency components present in this signal using the well-known Fourier transform. It computes an estimate of the power spectral density (PSD) by dividing the data into segments that can overlap between themselves. A periodogram for each segment is computed and finally averaged in order to obtain the resulting periodogram. One of the issues present in the periodogram method is the effect of offside lobe leakage that rises from the finite nature of the incoming data. To counter this problem, Welch proposes the use of windows (e.g. Hanning window) in each segment of overlapping data to smooth the signal edges (Welch, 1967).

In other words, to estimate a signal power spectral density, the Welch method averages a modified set of  $L$  periodograms, where each  $i$ th periodogram can be described as  $\hat{S}_{xx}^{(i)}(f)$ , where  $f$  is the frequency,  $f \in \mathbb{R}$  (Alkan and Yilmaz, 2007).

Then, the power density spectrum estimate is given by:

$$\hat{S}_{xx}^w(f) = \frac{1}{L} \sum_{i=0}^{L-1} \hat{S}_{xx}^{(i)}(f) \quad (2)$$

This method can be applied for one measurement, that is,  $y \in \mathbb{R}$ . One of the benefits of using the Welch method is the decrease of estimate variance due to the averaging of the modified periodograms instead of using one single periodogram for the entire data set. The stability of the estimate increases with the number of segments used, and

with data overlapping, more segments can be used despite the limited signal length.

A visual analysis of the peaks in the computed frequency-domain data allows us to estimate the main oscillating modes on the measured signal.

#### 4. YULE-WALKER METHOD

Another powerful method for power system analysis is the Yule-Walker method. It is classified as a parametric, non-recursive, and time-domain method. It's also based on ambient data, and it doesn't require probing signals (Thambirajah et al., 2010).

Consider an incoming power system frequency signal on the basis of equation 1. Since we are dealing with discrete-time signals, sampling time  $T$ , the frequency signal is  $y(k)$ ,  $k \in \mathbb{Z}$ , and the model is transformed to discrete-time. With this, the resulting system dynamics can be modeled as an auto-regressive moving-average (ARMA) model, which for a single signal measurement can be given as:

$$\hat{y}(k) = \sum_{j=1}^n a_j \hat{y}(k-j) + \sum_{l=1}^p \sum_{j=1}^{m_l} b_{lj} v_l(k-j) + \mu(k) \quad (3)$$

where  $n$  is the ARMA model order,  $p$  is the number of input terms (random loads), and  $m_l$  is the order of MA term for each input.  $a_j$  represents the AR parameters, while  $b_{lj}$  represents the MA parameters for input  $l$ .

In order to find the AR coefficients, the Yule-Walker method uses the auto-correlation values of  $y(k)$  (Trudnowski et al., 2008):

$$r(q) = \mathcal{E}\{y(k)y(k-q)\}, \quad (4)$$

where  $r(q)$  is the auto-correlation of  $y(k)$  and  $\mathcal{E}$  is the Expectation operator. In a certain finite window, with data extracted from the measurements, auto-correlation can be approximated by

$$r(q) = \frac{1}{N} \sum_{k=q+1}^N \{y(k)y(k-q)\}, \quad (5)$$

where  $N$  is the number of data points in the signal window considered to compute the model. From (Stoica and Moses, 1999), the output auto-correlation satisfies,

$$r(q) = - \sum_{j=1}^n a_j r(q-j), \quad q > m \quad (6)$$

where  $m = \max(m_l)$ . The AR coefficients can be estimated from the solution of a Least Square Problem based on:

$$\begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_N \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} = - \begin{bmatrix} r(m+1) \\ r(m+2) \\ \vdots \\ r(m+N) \end{bmatrix} \quad (7)$$

where:

$$R_i = [r(m+i-1) \ r(m+i-2) \ \cdots \ r(m+i-n)] \quad (8)$$

The  $n$  system modes can be computed from the poles of the estimated AR model using the following statement:

$$s_j = \ln(z_j)/T, \quad (9)$$

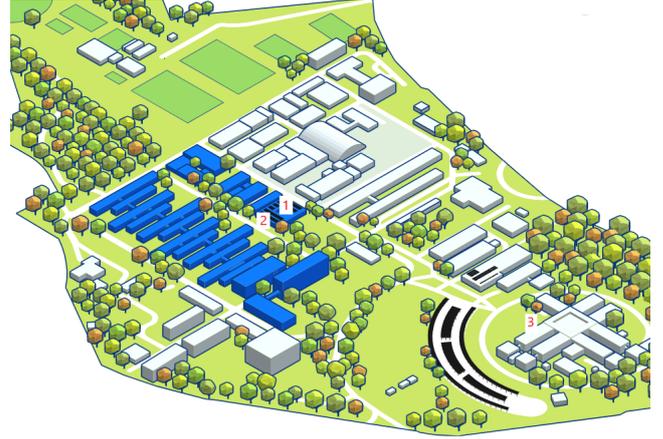


Figure 1. UFPR Polytech Campus and  $\mu$ PMU #1, #2 and #3 locations. Figure based on source: (P&D-PEE, 2021).

where  $j = 1, \dots, n$ .  $z_j$  are the roots of:

$$z^n + a_1 z^{n-1} + \dots + a_n = 0 \quad (10)$$

Then, the frequency (in Hz) and the damping ratio (in %) of the system estimated modes are calculated using:

$$f_j = \frac{\text{Im}(s_j)}{2\pi}$$

$$\xi_j = - \frac{\text{Re}(s_j)}{|s_j|} \times 100 \quad (11)$$

#### 5. WEB ENVIRONMENT

In this section, the development of a web platform for power system electromechanical modes analysis based on the grid's frequency signal is described. For that purpose, the following items will be discussed:

- (1) Power system frequency data acquisition via  $\mu$ PMUs;
- (2) Signal pre-processing;
- (3) Electromechanical mode estimation;
- (4) Graphical interface.

##### 5.1 Power system frequency data acquisition via $\mu$ PMUs

The UFPR's Polytech Center's power grid status is continuously monitored by 3  $\mu$ PMUs installed physically close to each other. The location of the measurement units present inside the university campus is represented by numbers in Figure 1.  $\mu$ PMU #1 is installed at the Electrical Eng. Department (DELT)/UFPR microgrid (more details in (Oliveira et al., 2020)), #2 is installed at DELT building substation, and #3 is placed near the UFPR photovoltaic plant (Oliveira et al., 2020) and will be operational soon. Aside from those 3 units, 2 others  $\mu$ PMUs are presented in different UFPR campuses. The #4 is located at the Agrárias Campus, in Curitiba, coordinates GPS -25.4124, -49.2489 the #5 is located at the Palotina Campus, west of Paraná, coordinates GPS -24.294612, -53.842245. Unit #6 is planned to be installed by COPEL in its own distribution grid.

Those  $\mu$ PMUs, model PSL PQube 3, can measure the grid's main parameters with a pretty high resolution.

According to the manufacturer, the device can measure frequency, plus voltage and current angle-magnitude pairs with a  $0.001^\circ$  resolution, with a sample rate of 120 Hz. Phasor data is streamed from the  $\mu$ PMU following the protocol IEEE Std. C37.118-2011.

Measurements are received and stored at a destination server, which works as the network's PDC. The server is located in a virtual machine that is hosted by a physical server managed by the DELT/UFPR (Oliveira et al., 2020). Within those virtual machines, it runs a Python script in charge of processing and storing incoming data to the local database. The PDC also hosts a web server application that provides visualization and analysis tools for the raw power grid data received (Pereira et al., 2019). The main PDC visualization software is called EmonCMS, and it allows users to access PDC data through an Application Programming Interface (API). The EmonCMS API provides frequency data coupled with a timestamp in a JavaScript Object Notation (JSON) format.

Based on the data stored at the UFPR-PDC, the paper's web environment for electromechanical modes analysis have been developed.

A Python program running on the server is responsible for interfacing with the PDC for pre-processing and calculations that makes a request to this API by sending a few configurable parameters. Those parameters include the API key, the desired PMU ID, the measurement's start and end timestamp, and the sampling frequency.

Within those parameters, the desired PMU, measurement window and sample time are selected by the user.

### 5.2 Signal pre-processing

After the frequency data is acquired by the API, this signal needs to be pre-processed before the electromechanical mode estimation algorithms (see Sections 3 and 4) can be applied. Firstly, this data is divided into 3 sections that are each processed individually and then grouped at the end. Each group is submitted to linear interpolation to recover data not sent by the  $\mu$ PMU.

A process of outlier removal is then applied. To achieve this task, for each section, a simple equation is used to identify if a point is an outlier:

$$|M| + \alpha\sigma > |y(k)| > |M| - \alpha\sigma \quad (12)$$

where  $y(k)$  is the measurement,  $M$  is the signal average for the current section,  $\sigma$  is the signal standard deviation, and  $\alpha$  is the threshold value to define an outlier regarding the standard deviation. For this application,  $\alpha$  has been chosen equal to 3.0, as in Leandro et al. (2015).

Then, a linear interpolation process is applied to fill the removed values during the outlier removal phase. The signal mean value is then subtracted to remove the linear trend.

Lastly, two digital 16th order Butterworth filters are applied. The first is a highpass filter with a cutoff frequency of 0.3 Hz to remove frequency components related to the systems control actions (Vanfretti et al., 2013). The last is a lowpass filter with a cutoff frequency of 7 Hz since all

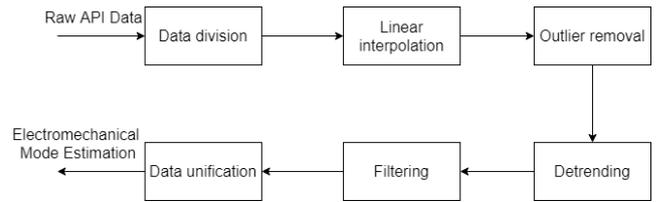


Figure 2. Data pre-processing routine

the electromechanical modes of interest are located below this threshold.

Figure 2 provides a graphical visualization of the data pre-processing routine that the script executes before moving to the next phase, estimating the electromechanical modes.

### 5.3 Electromechanical mode estimation

All of this data acquisition, pre-processing, processing and exportation is done by a Python program that runs every time the web application is accessed, refreshed or updated by user request.

The resulting signal from these phases is ready to be analysed using the methods discussed in Sections 3 and 4. As to electromechanical mode estimation, the Signal and StatsModels modules provide built-in Welch and Yule-Walker functions, respectively.

Regarding mode estimation using the Welch method, it takes as parameters the frequency time series and the signal sampling frequency. The length of each data segment and the number of points to overlap between segments also have to be provided. For this specific application, the length defined was 100 seconds with a 50% overlap, which is a good trade-off between accuracy while not using too much repeated data (Jones et al., 2001). Those two parameters are provided in terms of the numbers of samples.

Lastly, a Hanning Window is used to smooth points located at the edges of each segment. After the program calculations, the function returns two arrays, one containing the sample frequencies and the other containing the power spectral density of the input sampled frequency.

Estimations made using the Yule-Walker method work with the frequency measurement window data and autoregressive model order.

The end user can select the model order in the web environment. The default value set was 20. The function returns an array containing the AR coefficients and a floating point value representing the estimated residual standard deviation. The AR coefficients array is applied to equation 10, and the roots of the system characteristic equation are calculated. Continuous-time mode estimation is computed using equation 11.

### 5.4 Graphical interface

Data containing the results acquired by the Python script are transferred over to a website to be displayed to the end user. Two web applications were developed, one for each electromechanical mode estimation method discussed here.

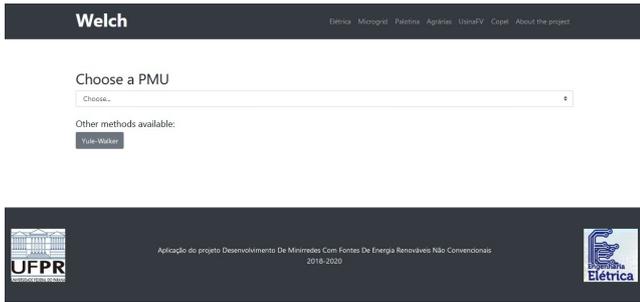


Figure 3. Web application initial page

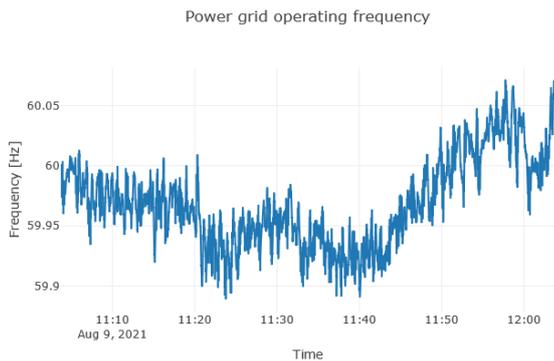


Figure 4. Frequency signal obtained from the web application

The initial page is shown in figure 3 and is identical for both applications. It displays the method's name on the page's upper-left position. At the upper-right position, it shows the option to select which PMU the user wants to analyse. Currently, there are 4 PMUs working with the real-time data available for analysis. 2 more units are planned to be integrated into the system.

After selecting a PMU, the user is redirected to a page containing information about which PMU is currently selected, the configurable parameters available for both methods and two graphs. The first graph contains the selected frequency window, which will be considered in the analysis. It is shown, as an example, in Figure 4. The second one shows the results of the electromechanical mode estimation. It is possible to interact with the graphs by zooming in and out the axis and regions of interest and also to download them as an image for future analysis and reports. The application refreshes every 5 minutes to always keep the most recent data available.

For both applications, the user can select two parameters that affect the algorithm estimation, the first being the desired time window, which ranges from 60 to 5 minutes before the current time. The second parameter is the downsample frequency, which can be selected from 15 to 20 Hz. These parameters are fed to the API, which returns the frequency data in the chosen format. Default values are 60 minutes time window and a sample rate of 15 Hz.

The Yule-Walker page also allows to select the model order, see equation 4. In this specific application, this parameter can range from 10 to 30, and the default value is 20.

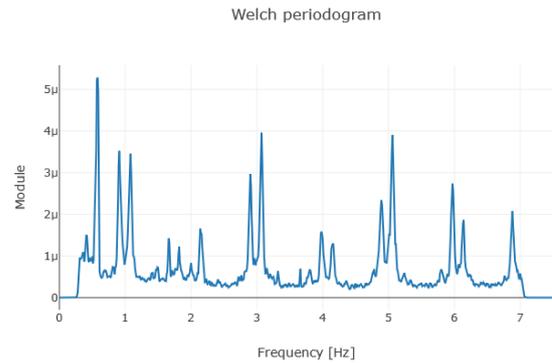


Figure 5. Electromechanical mode estimation obtained using the Welch method.

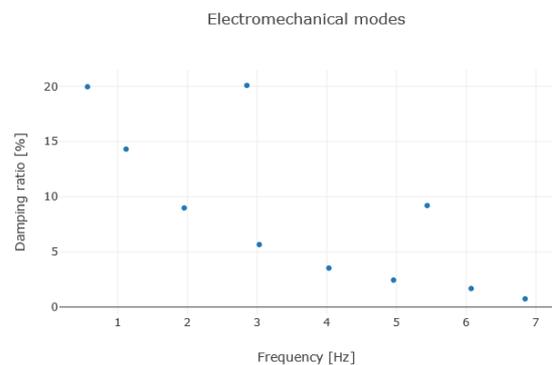


Figure 6. Electromechanical mode estimation obtained using the Yule-Walker method.

The Welch analysis result is a power spectrum density graph of the analysed frequency signal, being the main electromechanical modes estimated by the peak points in the curve. An example of this result is illustrated in figure 5, which contains a result on an ambient data using default values. The Welch web application can be accessed via a [link](https://sirius.eletrica.ufpr.br/welch/index.php) (<https://sirius.eletrica.ufpr.br/welch/index.php>).

The result presented by the Yule-Walker method is a scatter plot that indicates the mode's frequency in the  $x$  axis and the damping ratio in the  $y$  axis, and each point in this graph represents an oscillating electromechanical mode. An example is shown in figure 6, whose calculations were made using the default values. The Yule-Walker web application can be accessed via a [link](https://sirius.eletrica.ufpr.br/yulewalker/index.php) (<https://sirius.eletrica.ufpr.br/yulewalker/index.php>).

## 6. CONCLUSION

This article presented the development of a computational web environment that estimates the electromechanical modes of the power grid at that moment based on real-time data from the UFPR  $\mu$ PMU/PDC network. It utilizes the Welch and Yule-Walker algorithms and makes it possible to easily compare the results of both methods side-by-side, using the same time-domain data. The interface also allows the user to manually configure parameters that may influence the estimation results, such as the window time

and downsample rate. The platform is fully functional and can be accessed through the available mentioned links.

Future enhancements for this project include new estimation mode methods, an optimization of the Python script aiming for faster delivery of results, the integration of 2 more  $\mu$ PMUs and the ability to configure more estimation parameters, such as filter order, number of data divisions and window length and overlap for the Welch algorithm.

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