An Ill-conditioned System Study in a 11-bus Network and Characterization of the Problem considering the AC Power Flow in the MATPOWER Tool

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Abstract: This paper proposes a strategy to identify the parameter data for representing an 11-bus system for execution as a case in the MATPOWER software and discusses the ill-conditioned problem associated with it. The referred power system model is largely studied for evaluating iterative nonlinear method performances considering ill-conditioned systems. The case data are generated by converting the entry values of the system admittance matrix (matrix Y_{bus}) into a hypothetical network. The parameters of this network are identified in the form of longitudinal and transversal admittance in order to preserve the same precision of the given original matrix Y_{bus}. It is demonstrated that the power flow problem (PFP) solution diverges for a flat start guess when the standard Newton-Raphson (NR) method is used. However, by considering a very small reduction on the loading level the PFP converges. Even for this situation, the problem has elevated condition numbers along the iterations of the NR. Therefore, the paper focuses on the ill-conditioned study of the system, besides presenting case data which is appropriated to run in the MATLAB's format free code, MATPOWER.

Resumo: Este artigo propõe uma estratégia para identificar os dados dos parâmetros para representar um sistema mal-condicionado de 11 barras para execução como um caso no software MATPOWER. O referido modelo de sistema de potência é largamente estudado para avaliar desempenho de métodos não-lineares iterativos considerando sistemas mal-condicionados, mas em geral, seus dados são apresentados na forma de uma matriz admitância ao invés dos dados tradicionais de barras e de interconexões. Os dados para a rede elétrica são gerados pela conversão dos valores de entrada da matriz de admitância do sistema (matriz Y_{bus}) em uma rede elétrica hipotética. Os parâmetros desta rede são identificados na forma de admitância longitudinal e transversal a fim de preservar a mesma precisão da matriz original Y_{bus} fornecida. É demonstrado que a solução do problema de fluxo de potência (PFP) diverge para uma estimativa inicial *flat start* quando o método padrão de Newton-Raphson (NR) é usado. No entanto, considerando uma redução muito pequena no nível de carregamento, o PFP converge. Mesmo para esta situação, o problema tem números de condição elevados ao longo das iterações do NR. Portanto, o artigo se concentra no estudo mal-condicionado do sistema, além de apresentar dados de um caso apropriado para estudo do problema de fluxo de carga em programa com código livre, o MATPOWER.

Keywords: Ill-conditioned power system, Iwamoto's 11-bus system, MATPOWER, AC Power Flow, Newton-Raphson's method, condition number.

Palavras-chaves: Sistema de Potência Mal-Condicionados; Sistema de 11 barras de Iwamoto; MATPOWER; Fluxo de potência CA, Método de Newton-Raphson; Número de condição.

1. INTRODUCTION

Power Flow (PF) studies are an essential tool for analysis, planning and operation of power systems (Stevenson Jr and Grainger, 1994; Kundur, 2007). Due to its importance, PF techniques must be constantly updated in order to face the operational challenges of today's power system characteristics.

The classical techniques used to solve the Power Flow Problem (PFP) involve the Gauss-Seidel (GS), Newton-Raphson (NR) methods and their variants based on simplifications in the Jacobian matrix. Techniques such as the Fast Decoupled (FD) among others ones are included. The NR method can be considered the standard method used in industrial applications (Tostado-Véliz et al., 2018; Milano, 2009). The performance of a PF method is strongly linked to the condition of the system which depends on the following factors (Fan, 1989; Tripathy et al., 1982; Tostado-Véliz et al., 2018):

i. Large number of radial lines.
ii. Heavy loading conditions.
iii. Existence of negative line reactance.
iv. Lines with high resistance to reactance ratios.
v. Initial guess point outside of the region of attraction or far away from the solution.

These characteristics can cause instability and/or divergence for most standard PFP solver methods. In this case, the system is categorized as ill-conditioned (Fan, 1989; Milano, 2009). The solution of a PF for ill-conditioned cases is still a challenging for most of the available robust methodologies (Tostado-Véliz et al., 2020).

In Milano (2009), the PFP is classified into four categories as follows: 1) well-conditioned, when the PF solution exists and is reachable using a flat initial guess (i.e., all load voltage magnitudes equal to 1 pu and all bus voltage angles equal to 0) and a standard Newton–Raphson's method; 2) ill-conditioned, when the solution of the PFP does exist, but standard solution methods fail to get this solution starting from a flat initial guess; 3) bifurcation point, whose solution of the PF exists but it is either a saddle-node bifurcation or a limit-induced bifurcation; and 4) unsolvable, when the PF solution does not exist. In this paper, *we are interested in studying the second classification* and presenting figures on the mathematical characterization of the ill-conditioning of the problem.

When the initial guess is inside the region of attraction of the solution point, a numerical method is expected to converge. However, in some cases, it can occur divergence. In this case the initial guess is far away from the solution and robust numerical methods should be used before the NR method.

This paper presents an investigation on the well known ill-conditioned 11-bus test-system proposed in Iwamoto and Tamura (1981) and also studied in other works (Tripathy et al., 1982; Bonini-Neto et al., 2015; Fan, 1989). The main motivation for this study is justified by the fact that all of these previous works address the problem by considering a representation based on an admittance matrix. Then in this paper the main contribution is to propose a hypothetical network assuming connections based on the given Y_{bus} for the 11-bus system, as given in Tripathy et al. (1982). The data for this system is only available in the literature through its admittance matrix. The network data are converted to fictitious longitudinal and transversal parameters and implemented in the MATPOWER format cases (Zimmerman et al., 2011). Then, three operation point were investigated. It was studied the convergence process by monitoring the infinite norm of power mismatches and the condition number (Golub and Van Loan, 2013) of the Jacobian matrix of the PFP at each iteration. It is worth mentioning that the 11-bus system data was target of similar studies with respect to the identification of the network from the matrix Y_{bus}, such as in Bonini-Neto et al. (2015). However, the advantage of the data we provide in the MATPOWER case format is the fact that the software is used worldwide by power systems' researchers, educators, and professionals from academia, government, and industry (Zimmerman et al., 2011).

This paper is organized as follows: Section II discusses about the origin of the data relative to the Iwamoto's 11-bus system and presents a straightforward form to identify the branch and shunt data from the admittance matrix and other bus data. In Section III, simulations involving the synthesized network are presented and discussed the results in terms of condition number. Finally, Section IV highlights main conclusion on the study.

2. ORIGIN AND CONVERSION OF THE DATA

The ill-conditioned 11-bus system was proposed in Iwamoto and Tamura (1981), however without available data for reproducing their results. The authors in Tripathy et al. (1982) carried out investigations in this system based on the admittance matrix of the system as presented in Table 1. Therefore, network line or shunt connections were directly inserted in the entries $Y_{km} = G_{km} + jB_{km}$ of the system associated matrix Y_{bus}. The objective is to reproduce the table with the same decimal numbers presented in Tripathy et al. (1982). *The main purpose in this paper is to find a synthetic network with circuit elements and loads which allows to recover the matrix Y_{bus} exactly with the same entry values.* This way, we preserve the accuracy of the results found in the seminal papers that have studied the characteristics of the system.

Table 1. Y-bus matrix elements for the 11-bus system, $Y_{km} = G_{km} + jB_{km}$, with values in pu (Tripathy et al., 1982)

k	m	G_{km}	B_{km}	k	m	G_{km}	B_{km}
1	1	0.0	-14.939	5	5	2.581	-5.889
1	2	0.0	14.148	6	6	0.0	-55.556
2	2	12.051	-33.089	7	7	3.226	-4.304
2	3	0.0	6.494	7	8	-2.213	2.959
2	4	-12.051	13.197	8	8	2.893	-5.468
3	3	2.581	-10.282	8	9	-0.138	1.379
3	5	-2.581	3.789	8	10	-0.851	1.163
4	4	12.642	-74.081	9	9	0.104	-1.042
4	5	0.0	2.177	10	10	1.346	-6.110
4	6	0.0	56.689	10	11	-0.374	3.742
4	7	-0.592	0.786	11	11	0.283	-2.785

In order to synthesize line and shunt connection data for performing experiments in either production grade software or free style code software as MATPOWER

(Zimmerman et al., 2011), we generate a possible set of shunt (transversal) and branch (longitudinal) admittance to obtain Table 1 given data. Note that this is just one form to achieve the results, since many other options for devices could be synthesized. Table 2 provides branches elements connecting bus #k and #m in the admittance form, $y_{km} = g_{km} + jb_{km}$, while Table 3 gives the shunt elements connected at the bus #k, also in the form of admittance, $y_k = g_k + jb_k$. The shunt elements can be implemented as constant impedance load model in MATPOWER. The option to use data only in the form of admittance, y_k and y_{km}, is exactly to preserve the same values used for the entries of the original matrix Y_{bus}. The simple roundoff of a parameter value in the form of input data $z_{km} = 1/y_{km}$, as the format data used in MATPOWER, is sufficient to produce a rounding error. As we will see later through experiments, this type of error (either rounding or truncation) may causes serious numerical error in view of the convergence process of a numerical iterative method.

Table 2. Synthesized branch equivalent elements for the 11-bus system with data in pu

k	m	g_{km}	b_{km}	k	m	g_{km}	b_{km}
1	2	0.0	-14.148	2	3	0.0	-6.494
2	4	12.051	-13.197	3	5	2.581	-3.789
4	5	0.0	-2.177	4	6	0.0	-56.689
4	7	0.592	-0.786	7	8	2.213	-2.959
8	9	0.138	-1.379	8	10	0.851	-1.163
10	11	0.374	-3.742	-	-	-	-

Table 3. Synthesized shunt equivalent elements for the 11-bus system with data in pu

k	g_k	b_k	k	g_k	b_k
1	0.0	-0.791	2	0.0	0.750
3	0.0	0.001	4	-0.001	-1.232
5	0.0	0.077	6	0.0	1.133
7	0.421	-0.559	8	-0.309	0.033
9	-0.034	0.337	10	0.121	-1.205
11	-0.091	0.957	-	-	-

The generation and loading condition data for the 11-bus system are presented in Table 4. There is only one generation power connected at the bus #1, which is also the slack bus. Therefore, the information in the table refers to the voltage at the slack bus and the loads connected at each bus #k. The power values are given in the form of injected constant power in pu into the bus #k. Also, we have preserved exactly the same values as presented in Tripathy et al. (1982).

The schematic diagram for the 11-bus system is shown in Figure 1 (shunt admittance is omitted in the diagram). As can be seen, the system is predominantly radial, consisting of only 1 generation, 10 load buses (nonzero injections concentrated at buses #3, #5, #6, #9 and #11) and 11 interconnections. This radially characteristic (high R/X ratios) associated with the presence of only one generation, located at the end of the network to supply all loads, contributes to an ill-conditioned characteristic of the system (Fan, 1989).

3. TESTS AND RESULTS

This section aims to present experimental results obtained with the application of the traditional NR method to the PFP solution for the Iwamoto's 11-bus system. We investigate the characteristics of the system in terms of loading, operation point and numerical condition along of the iterative process of solution. The main focus is to demonstrate the ill-conditioning well reported in several works and provide a data bank for the MATPOWER cases.

Table 4. Generation and loading condition for the 11-bus system

Bus	$V(p.u.)$	$\theta(deg)$	$P(p.u.)$	$Q(p.u.)$
1	1.024	0.0	-	-
2			0.0	0.0
3			-0.128	-0.062
4			0.0	0.0
5			-0.165	-0.080
6			-0.090	-0.068
7			0.0	0.0
8			0.0	0.0
9			-0.026	-0.009
10			0.0	0.0
11			-0.158	-0.057

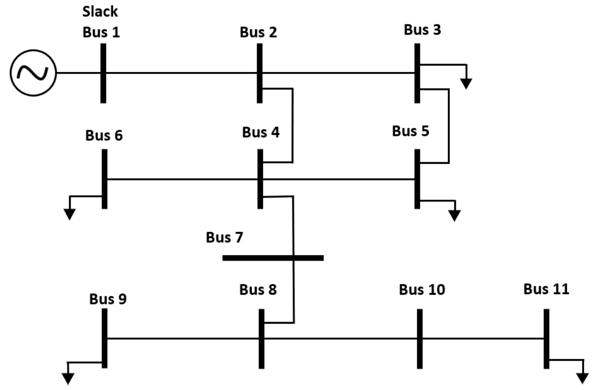

Figure 1. Schematic diagram of the 11-bus system

All adapted data used in the experiments are in Tables 2 to 4. All simulations were performed in the MATPOWER v6.0 (Zimmerman et al., 2011) and a case data labeled *case11.m* was produced. A convergence tolerance 10^{-8} pu for the power mismatch was adopted. A maximum number of 50 iterations for the convergence of the method has been established. The interconnection impedance input data for MATPOWER was calculated as $z_{km} = 1/y_{km}$ and this code was assigned replacing data of resistance and reactance for the branch $k-m$. We have preferred do not insert the calculated value of resistance and reactance directly in the field for this parameters at the MATPOWER case file just to avoid the introduction of truncation error in the synthesized data. Remember that we give the data for y_{km} as furnished in Table 2.

All PFP solutions were initialized by using flat start. The simulations were performed on a computer with the Intel® Core™ i5-9300H, 2.40 GHz; 8 GB of RAM; and 64-bit operating system.

3.1 NR convergence for relaxed tolerance and capacitive injection

As reported in Tripathy et al. (1982), the traditional NR method failed to obtain the PFP solution for the 11-

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bus system, when a convergence tolerance 10^{-4} MW or MVAR (or 10^{-6} pu) for MATPOWER is considered. We calculate a PFP solution for the tolerance 10^{-6} pu and indeed the convergence failed. Then the tolerance in this experiment was relaxed to 8×10^{-4} pu. For this situation, the infinite norm of the mismatches as a function of the iteration number is plotted in Fig. 2. Also, we verify that a single increment of the amount of reactive power injection at the bus #6 improves the convergence of the iterative process. From the original data, at bus #6 and nominal voltage, it was computed (identified) a synthetic capacitor injecting 1.133 pu of reactive power. Adding only 0.009 pu (less than 0.8%) at this bus, the NR applied to solve the PFP converges with 12 iterations. The norm along each iteration is also shown in Fig. 2. Note that the error 0.8 % might be even an intrinsic deviation in parameters associated to the device controlling the reactive power at bus #6.

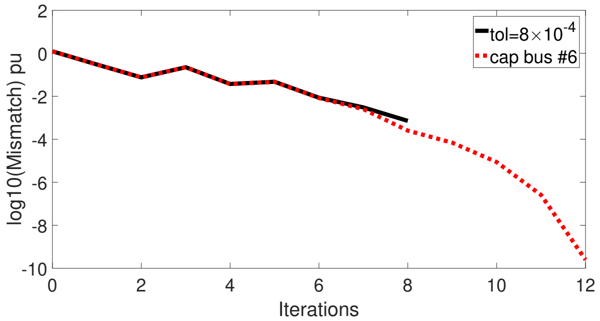

Figure 2. Error for the system considering the relaxed tolerance (black) and incremental capacitive injection at bus # 6 (red)

The resulting voltages and loads are exhibited in Tables 5 and 6.

We emphasize that the process for relaxed tolerance is divergent case the iterative process had proceeded beyond the 8^{th} iteration. Elevated values for the mismatches are verified (not shown in the plot). This result suggests the employment of a robust numerical method to deal with the problem. In Tripathy et al. (1982), an algorithm based on Brown's algorithm is used with this aim. Even in Iwamoto and Tamura (1981) a robust algorithm is proposed. However, by verifying the solution trend in Iwamoto and Tamura (1981), it presents stagnation and converging slowly for the interest solution with tolerance 10^{-6} pu. The study of robust methods for solving the problem is out of the scope of this paper and will be issue for other publication. Concerning the nodal voltages and power in Tables 5 and 6, the results indicates that despite the relaxed tolerance convergence, the loads has accuracy as the input data furnished in Table 4. And the voltage magnitude and phase have values very close of those ones found in the next simulations. Also, the simple injection of reactive at the bus #6 introduces differences on the voltage magnitudes and phase angles.

3.2 Influence of Loading Level

In the previous subsection, the original 11-bus PFP solution was obtained by the NR method only for a relaxed

Table 5. Solution obtained by the NR method for relaxed tolerance convergence

2*Bus	Voltage		Load	
	Mag.(pu)	Ang. (deg)	P (MW)	Q (MVAR)
1	1.024	0.000	-	-
2	1.056	-2.436	0.00	0.00
3	1.045	-4.133	12.80	6.20
4	1.030	-2.856	0.00	0.00
5	1.034	-4.877	16.50	8.00
6	1.050	-2.940	9.00	6.80
7	0.795	-12.564	0.00	0.00
8	0.888	-15.526	0.00	0.00
9	1.166	-16.506	2.60	0.90
10	0.787	-22.269	0.00	0.00
11	1.031	-25.193	15.80	5.70

Table 6. Solution obtained by the NR method when small value of power injection occurs at bus #6

2*Bus	Voltage		Load	
	Mag.(pu)	Ang. (deg)	P (MW)	Q (MVAR)
1	1.024	0.000	-	-
2	1.056	-2.443	0.00	0.00
3	1.046	-4.141	12.80	6.20
4	1.030	-2.878	0.00	0.00
5	1.034	-4.889	16.50	8.00
6	1.050	-2.962	9.00	6.80
7	0.791	-12.572	0.00	0.00
8	0.882	-15.555	0.00	0.00
9	1.158	-16.551	2.60	0.90
10	0.779	-22.388	0.00	0.00
11	1.020	-25.387	15.80	5.70

tolerance convergence. In this subsection, we demonstrate that reducing the loading level by only an infinitesimal amount, the NR method reaches the convergence for the given tolerance 10^{-8} pu. To assess the influence of loading, a scaling factor λ was used to decrease the active and reactive powers injected into the 11-bus system buses through $P_i^{sp} = \lambda P_i^{sp}$ and $Q_i^{sp} = \lambda Q_i^{sp}$. The parameter λ was progressively decreasing from 1 until reaching the threshold where convergence occurs considering precision in the fourth decimal place in λ. Then, the value immediately above λ was also considered for the simulations. We found the limit value for convergence as $\lambda = 0.9981$. Fig. 3 presents the evolution of the power equation mismatches for the NR method for $\lambda = 0.9981$ and $\lambda = 0.9982$. Note that the profile in the plot of Fig. 2 is similar with the plots in Fig. 3 until around iteration 8.

As can be seen from Fig. 3, a reduction of only 0.19% on the loading level in Table 4 is sufficient to lead to convergence of the PFP for the tolerance 10^{-8} pu and 13 iterations. On the other hand, when the reduction is 0.18% the convergence fails, leading to oscillations on the error plot around 10^{-4} pu.

3.3 Ill-conditioning

The condition number is frequently considered for measuring the degree of system ill-conditioning (Tripathy et al., 1982; Fan, 1989; Golub and Van Loan, 2013). In power systems it is commonly used the assumption of ill-conditioned system as presented in Milano (2009). By this assumption, a PFP is accepted as ill-conditioned when for a flat start guess the NR method diverges for the iterative problem so-

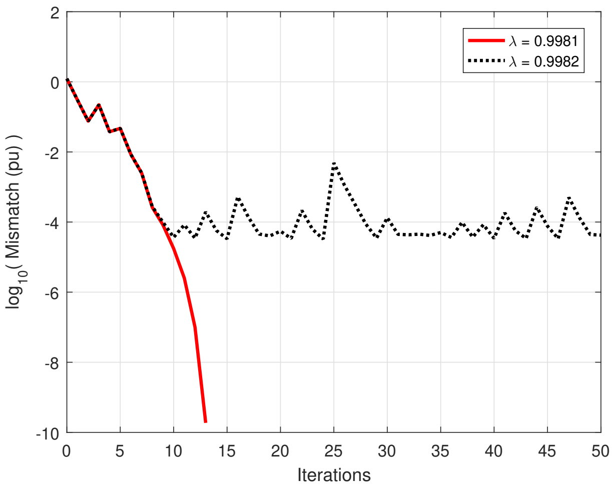

Figure 3. Mismatches for 11-bus system considering two load levels.

lution. Effectively, this is not the mathematical definition of condition number associated to a linear system, considering that the nonlinear system is iteratively resolved through linear system solutions.

It is more appropriate to apply the mathematical definition of condition number (see for instance Golub and Van Loan (2013) for details), either based on the computation of matrix norm or singular values. Let be the linear system $Jx = b$ related to an iteration of the PFP, where J is the associated Jacobian matrix for an iteration and b is the power mismatch vector. The condition number related to J is defined as $\kappa(J) = ||J|| \cdot ||J^{-1}||$, where $|| \cdot ||$ is a known norm (Golub and Van Loan, 2013). Then the condition number is norm dependent. In the limit case, when J is singular, $\kappa(J) = \infty$. Hence, an ill-conditioned system has a very high condition number. In the particular case of norm-2, $\kappa(J) = ||J||_2 \cdot ||J^{-1}||_2 = \frac{\sigma_{max}(J)}{\sigma_{min}(J)}$, where $\sigma_{max}(J)$ and $\sigma_{min}(J)$ are the maximum and minimum singular values of J, respectively.

Therefore, independently of the guess to be used to solve the linear system, the ill-conditioning of the system is associated with its condition number. We use the previous experiments based on the scaling factor attributed to the loads in Table 4 for $\lambda = 0.9981$ and $\lambda = 0.9982$ to demonstrate that despite the former nonlinear problem has flat start it converges. On the other hand the latter diverges and both presents high condition number along the iterative process, indicating that the errors increase tremendously for both systems.

The condition numbers for the Jacobian matrices J of each iteration of the PFP when solved by the NR method were computed by using the norm-2 approach. This process is straightforwardly applied inside the *newtonpf.m* code in MATPOWER. It is needed to compute only the maximum singular value of J and in the sequence, the minimum singular value. The MATLAB's native code *svds* was used as to implement the procedures as follows: $\sigma_{max} = svds(J, 1, 'largest')$; $\sigma_{min} = svds(J, 1, 'smallest')$; $\kappa = \sigma_{max}/\sigma_{min}$.

Figure 4 shows the condition number per iteration for the 11-bus system considering $\lambda = 0.9981$. As can be seen, although the system has a very high condition number in the seventh iteration, when compared to the values reported in Tripathy et al. (1982), around 1.9107×10^5, the NR method was able to obtain the PFP solution. Therefore, the result is indicative of very high condition number and so we can classify the profile of this scenario as ill-conditioned.

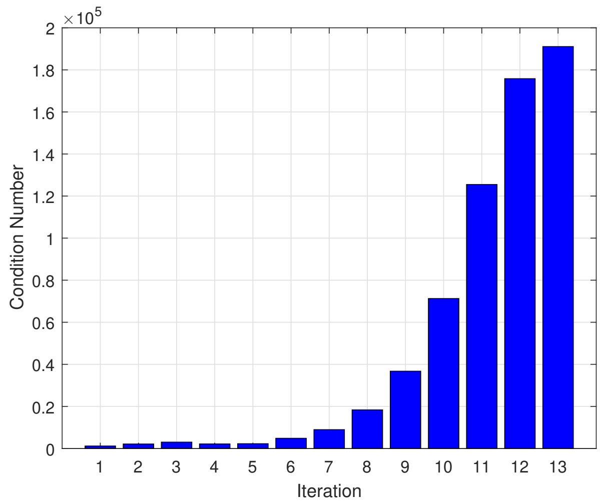

Figure 4. Condition number per iteration for the 11-bus system for the scenario $\lambda = 0.9981$.

The condition number per iteration for the second scenario ($\lambda = 0.9982$), where the load level was increased by just 0.0001 in relation to the previous one, is shown in Figure 5. As can be seen, the values are significantly high with orders of 10^6 and therefore justifying the difficulty of the NR method in obtaining the PFP solution for this scenario. The condition number in iteration 25, for example, is extremely high with a value of 3.08×10^6.

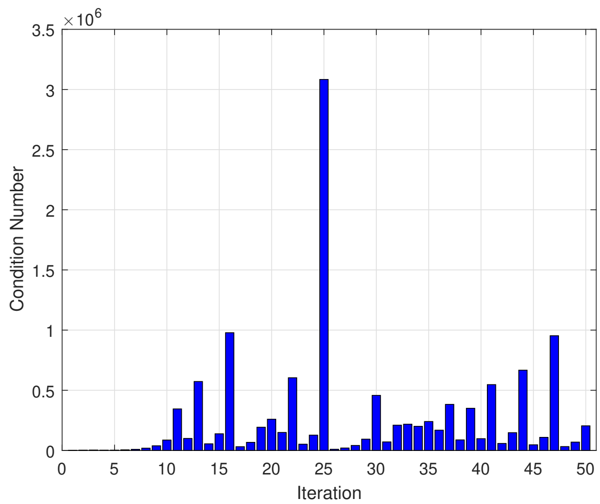

Figure 5. Condition number per iteration for the 11-bus system for the scenario $\lambda = 0.9982$.

3.4 Power Flow Solution

As presented in the Subsection 3.2, the NR method obtains the PFP solution in 13 iterations for the 11-bus system with $\lambda = 0.9981$ and convergence tolerance of 10^{-8} pu. The final solution for the PFP in this situation is given in the format of plots for the magnitude and phase in Figure 6. The numerical values for nodal voltages and load powers are exhibited in Table 7. Note that the loads are computed considering the loading factor $\lambda = 0.9981$ in relation to the loads (injected power in pu) shown in Table 4.

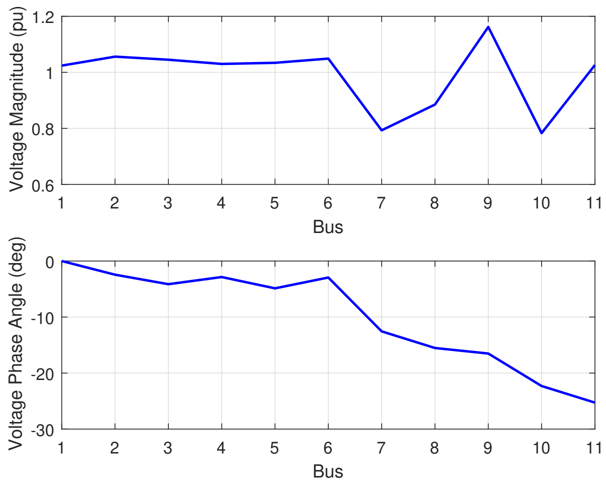

Figure 6. Voltage angles and magnitudes obtained by NR for the system assuming $\lambda = 0.9981$

Table 7. Solution obtained by the NR method for the scenario $\lambda = 0.9981$

2*Bus	Voltage		Load	
	Mag.(pu)	Ang. (deg)	P (MW)	Q (MVAR)
1	1.024	0.000	-	-
2	1.056	-2.437	0.00	0.00
3	1.045	-4.130	12.78	6.19
4	1.030	-2.854	0.00	0.00
5	1.034	-4.872	16.47	7.98
6	1.049	-2.939	8.98	6.79
7	0.793	-12.550	0.00	0.00
8	0.885	-15.521	0.00	0.00
9	1.162	-16.508	2.60	0.90
10	0.783	-22.307	0.00	0.00
11	1.026	-25.268	15.77	5.69

As shown in section II, the Iwamoto's 11-bus system corresponds to a radial network whose power generation occurs only in the bus #1. This power generation is located at one end of the network. Also, the network has loads far away from this power generation center (bus #11). Additionally, there are negative shunt resistance, large and small loads, large impedances at the interconnections etc. This results in a larger voltage phase angle at bus #11 with $-25.268°$. This configuration also causes low voltage magnitudes on connection buses #7, #8 and #10 (Fan, 1989), as shown in Figure 6.

4. CONCLUSIONS

This work presented a way to identify the parameter data for representing the Iwamoto's 11-bus system for execution as a case in the MATPOWER software. The referred power system model is largely studied for evaluating iterative nonlinear method performances considering ill-conditioned systems. The case data are generated by converting the entry values of a system admittance matrix assuming the same precision on the admittance matrix and identified network parameters, which are also synthesized in the form of admittance.

It is demonstrated that the power flow problem solution converges for a flat start guess and a very small reduction on the loading level. Even for this situation, the problem presents elevated condition numbers along the Newton-Raphson iterations of the nonlinear process. Therefore, characterizing and illustrating an ill-conditioned system. However, for a very reduced increment on the loading level of the convergent case, a divergence is verified for a prescribed convergence tolerance of 10^{-8} pu. On the other hand, for a relaxed convergence, say 8×10^{-4} pu, even the worst case reaches convergence through the NR method.

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