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An Alternative Proposal for Quantification of Technical Losses Using a Reduced Equivalent Circuit

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***Abstract*—Electrical grid technical losses affects the electricity tariff and the planning of new lines. Nonetheless, their assessment by means of time-series or real-time simulations is computationally burdensome for large grids. In order to achieve simplified equivalent circuits, reduction techniques have been developed. The only one that is applicable for unbalanced grids is the multiphase reduction, but it does not preserve the losses. This paper proposes an algorithm that allows indirect evaluation of total losses, thus extending the multiphase reduction technique. In the validation process, IEEE 34-bus and a Brazilian real feeder are evaluated. A daily time-series simulation, with hourly loadshapes, is performed in OpenDSS. Using this novel method, the losses errors obtained are lower than 2%. The proposed algorithm, therefore, may be applied to speed up the definition of electricity tariff and the planning process of unbalanced grids.**

***Index Terms*—Electrical Grid Reduction, Technical Losses, Distribution Grids.**

I. Introduction

The planning and operation of an electric grid determines the power quality delivered to customers. In this context, technical losses (TL) is a key aspect that needs to be evaluated. If they are too high, planning teams may decide to improve the grid infrastructure, adding more transmission lines or replacing some equipment with more efficient ones. Operation teams, for instance, need to compensate these losses with additional generation in order to assure system frequency stability.

Assessment of TL in the planning perspective is done by means of digital simulation. Given that the system changes dynamically during the day, a time series technique is usually performed. Nonetheless, this can be computationally burdensome, or even unfeasible, due to the grid extension and complexity, especially when several different scenarios need to be evaluated. In this context, reduction techniques can be applied to achieve simplified circuits, thus reducing the computational cost.

Reference [1] was the first, to the best of the authors' knowledge, published paper proposing and applying a reduction method. It focused on preserving the equivalence of the electric flux of the original and the reduced systems. The equivalence, however, is highly sensitive to operation changes. Moreover, generators eliminated in the reduction process have their contributions smeared over the remaining buses [2].

With the same objectives as Ward, that is to preserve the electric flux equivalence between the full system and the reduced one, references [3] and [2] achieved a less sensitive method with regard to operation changes. Despite this improvement, none of the methods assure equivalence of the voltage values between the original and the equivalent system. Additionally, their approach is based on the DC power flow, which is a rough approximation for distribution systems.

In [4], a distribution network reduction method that preserves both voltage and losses is proposed. The technique was applied in a Brazilian real feeder. As a result, the reduced system was approximately 3 times smaller than the original one, and the simulation time was reduced in 50%. Nonetheless, the technique is sensitive to the operating conditions.

In [5], a reduction method for real-time simulation of distribution grids is proposed. It eliminates all nodes that are not in a keep list, that are not topological nodes, nor have capacitors or regulators connected to it. Then equivalent lines are obtained by a Monte Carlo simulation in which the line length is randomly varied. The maximum voltage error obtained for the simulated feeders was 0.5%. Despite the low error, this method is computationally burdensome with up to 50000 Monte-Carlo loops. Moreover, technical losses are not addressed.

Reference [6] focused on the study of feeders with photovoltaic generation. The developed technique maintains the equivalence of the buses voltages and of the total power consumption. It also allows the user to select which buses should remain in the final circuit. Technical losses are not preserved though [7]. The same disadvantages pointed out in [4] applies here as well.

Reference [8] extended the approach presented in [6] to three-phase unbalanced systems, by means of the multiphase reduction technique. According to [8], this technique is the first one that is applicable to unbalanced grids. It is also independent of the operating condition in which the reduction was applied. For this purpose, a matrix mapping the loads in the original system to those in the reduced one is proposed. This allows the simulation of different scenarios without repeating the reduction process. Moreover, this technique does not need an initial solution from the original grid. TL, however, are not addressed and since this method is based on [6], it should have the same properties, that is preserving the buses voltages and the feeder total power, but not necessarily assuring losses equivalence.

Considering the aforementioned aspects, it can be observed that, despite reducing the computational effort, the existing techniques are not shown capable of providing accurate values of both voltages and losses for unbalanced grids.

This paper proposes a methodology for computing the TL of a large grid based on its equivalent circuit obtained applying the method proposed in [8]. It is worth mentioning that, although the losses of the original and the equivalent circuits are not necessarily the same, the multiphase reduction proposed in [8] provides tools that allow an indirect computation of the losses of the original system. It should be highlighted that this paper is the first one to provide an algorithm that allows evaluation of both voltages and losses for an unbalanced three-phase grid based on its reduced equivalent. OpenDSS is employed to solve the full and reduced systems. The proposed method is also compared with the openDSS loss calculation of the reduced system.

This paper is organized as follows. In the next section, the reduction method is provided, and the proposed algorithm for computing the losses is presented. Afterwards, the developed technique is applied in a daily simulations, with hourly load-shapes, of IEEE 34-bus and of a Brazilian feeder. In Section IV, conclusions are drawn.

II. Methodology

In this section, a description of the reduction technique developed in [8] is given, followed by an explanation of the proposed algorithm for computing the system TL.

A. Multiphase Distribution Feeder Reduction [8]

The multiphase distribution feeder reduction method is based on the bus elimination principle, and it assumes that all loads can be modelled as constant current. The buses that will be preserved in the reduction process are called critical buses (CBs). They can be selected by the user or automatically defined by the algorithm in order to preserve the system topology or to preserve some special equipment, such as shunt capacitors, voltage regulators, photovoltaic (PV) panels and transformers. The method can be divided into three main parts: i) topology detection; ii) off main tree reduction; and iii) main tree reduction.

In the first part, the grid is compared to a genealogical tree. Those buses that are adjacent to bus i constitute the set N_i. The one closest to the substation is called the parent P_i, and the remaining buses from N_i are called bus i children, composing the set C_i. If C_i is empty, then i is an end bus. The offspring

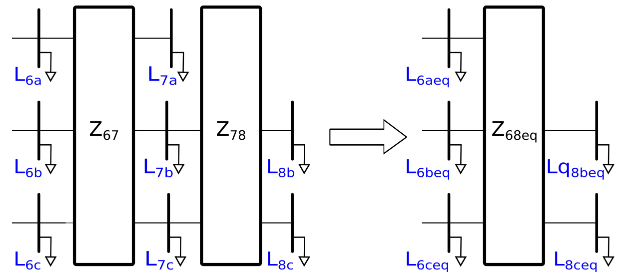

Fig. 1. Middle bus reduction example.

set Ω_i is composed from buses downstream bus i. Upstream buses are part of the the ancestor set Λ_i.

In the second part, buses outside the main tree are eliminated. The main tree is formed by the union of the farthest CB from the substation with its ancestors, that is those buses that are upstream. This step consists of recursively eliminating non-critical end buses and adding their load to their parent, until the end bus of that branch is a CB. It is worth mentioning that those transmission lines connected to the removed buses are also removed, and the remaining lines are kept unchanged.

At last, reduction is performed in the main tree. This is achieved by middle bus elimination, since in the previous step all non-critical end buses were removed. To explain this procedure, consider the following three buses shown in Figure 1.

After eliminating bus 7, the equivalent line is given by

$$\mathbf{Z_{68eq}} = \mathbf{Z_{67}^{red}} + \mathbf{Z_{78}^{red}} = \begin{bmatrix} z_{67bb} + z_{78bb} & z_{67bc} + z_{78bc} \\ z_{67bc} + z_{78bc} & z_{67cc} + z_{78cc} \end{bmatrix}, \tag{1}$$

in which $\mathbf{Z_{67}^{red}}$ and $\mathbf{Z_{78}^{red}}$ refers to the matrix impedance of the lines connecting the buses 6 and 7 and connecting the buses 7 and 8, without the elements regarding phase “a”. The subscript numbers denote the connected buses, and the subscript letters denote the phases affected. For instance, z_{67bc} represents the mutual impedance between phases “b” and “c” of the line connecting buses 6 and 7.

In order to map load changes in the original system to its equivalent a weighting matrix $\mathbf{W} \in \mathbb{C}^{(3n \times 3n)}$ is recursively computed, in which n is the number of buses in the system. Initially, if node i has a load, then the element of $\mathbf{W}$ in the ith column and in the ith row is equal to one, that is $w_{ii} = 1$. Otherwise, $w_{ii} = 0$. Off diagonals are equal to zero in the initialization of the reduction process.

After an end-bus elimination, the rows of $\mathbf{W}$ corresponding to the parent nodes of the removed bus (RB) are updated by adding to them the rows corresponding to the RB, which are then set to zero. For clarity purposes, we define the weighting sub matrix $\mathbf{W}_{ij}$ that contains the elements of $\mathbf{W}$ corresponding to the nodes of buses i and j. For instance, the weighting sub matrix of buses 1 and 1 is given by

$$\mathbf{W}_{11} = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix}, \tag{2}$$

in which w_{11} refers to the weight between phases a of the bus 1 of the original system and of the bus 1 in the reduced one. The other elements w_{ij} are defined likewise. If bus 8 is eliminated in the example shown in Figure 1, then

$$w_{(3*7-2)(3*8-2)} = 0, \tag{3}$$

because bus 8 does not have phase "a", and

$$w_{(3*7-1)(3*8-1)} = w_{(3*7)(3*8)} = 1, \tag{4}$$

given that the elements connected to phases "b" and "c"of bus 8 are mapped to phases "b" and "c"of bus 7. Moreover, since in the reduced system, bus 8 does not exist, the rows of $\mathbf{W}$ corresponding to this bus are set to zero.

For a middle bus reduction, the auxiliary matrices R_1 and R_2 are first computed and later applied to update $\mathbf{W}$. For the example of Figure 1, they are equal to

$$\mathbf{R_1} = \mathbf{Z_{68eq}}^{-1}\mathbf{Z}_{\mathbf{78}}^{\text{red}} \tag{5}$$

$$\mathbf{R_2} = \mathbf{Z_{68eq}}^{-1}\mathbf{Z}_{\mathbf{67}}^{\text{red}} \tag{6}$$

The equivalent load in this case is

$$\begin{bmatrix} L_{6beq} \\ L_{6ceq} \end{bmatrix} = \begin{bmatrix} L_{6b} \\ L_{6c} \end{bmatrix} + \mathbf{R_1} \begin{bmatrix} L_{7b} \\ L_{7c} \end{bmatrix} \tag{7}$$

$$\begin{bmatrix} L_{8beq} \\ L_{8ceq} \end{bmatrix} = \begin{bmatrix} L_{8b} \\ L_{8c} \end{bmatrix} + \mathbf{R_2} \begin{bmatrix} L_{7b} \\ L_{7c} \end{bmatrix} \tag{8}$$

$$L_{6aeq} = L_{6a} + L_{7a}. \tag{9}$$

The weighting sub matrices corresponding to nodes 6 ($\mathbf{W}_6$) and 8 ($\mathbf{W}_8$) are given by

$$\mathbf{W}_6 = \mathbf{W}_6 + R_1^{\dagger}\mathbf{W}_7 \tag{10}$$

$$\mathbf{W}_8 = \mathbf{W}_8 + R_2^{\dagger}\mathbf{W}_7, \tag{11}$$

in which $R_1^{\dagger}$ and $R_2^{\dagger}$ are

$$\mathbf{R_1}^{\dagger} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & R_1(1,1) & R_1(1,2) \\ 0 & R_1(2,1) & R_1(2,2) \end{bmatrix} \tag{12}$$

$$\mathbf{R_2}^{\dagger} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & R_2(1,1) & R_2(1,2) \\ 0 & R_2(2,1) & R_2(2,2) \end{bmatrix}. \tag{13}$$

This same procedure is recursively repeated for the entire main tree observing the existing nodes, until all non-critical buses have been eliminated.

At last, any load changes can be accounted for in the reduced system with usage of $\mathbf{W}$. In this sense, the equivalent loads are obtained after multiplying the original ones by $\mathbf{W}$.

B. Algorithm for Computing Losses

In order to develop the algorithm to compute the original system losses (P_{LO}), it is important to recall that the total active powers in the original (P_{TO}) and the reduced (P_{TR}) grids are equivalent. They are simply designated by $P_T = P_{TO} = P_{TR}$. This is the power provided by the substation to the feeder and should not be confused with the total active load consumption.

The reduced system does not have the same total active power load consumption (P_{CR}) as the original system (P_{CO}). The active power losses (P_L) are also not preserved. This means that

$$P_{CR} \neq P_{CO} \tag{14}$$

$$P_{LR} \neq P_{LO}, \tag{15}$$

in which the subscripts R and O are used to denote the reduced and the original system, respectively. This is expected because the losses of the removed lines are added to the equivalent loads in the reduction process. So, the active power losses of the original system (P_{LO}) will never be equal to (P_{LR}), which is obtained after running the power flow in the reduced grid, computing the losses of each element and adding them together. Nevertheless, P_{LO} can be indirectly computed by considering the following property

$$P_T = P_{LO} + P_{CO} \tag{16}$$

If P_{CO} can be determined from the reduced system, then P_{LO} can also be.

The vector containing the complex power of each bus load of the reduced grid $\overline{\mathbf{S}}_{\mathbf{CR}} \in \mathbb{C}^{(pn\times1)}$ can be related to the original loads vector $\overline{\mathbf{S}}_{\mathbf{CO}}$ via the $\mathbf{W}$ matrix as

$$\overline{\mathbf{S}}_{\mathbf{CO}} = \mathbf{W}^{-1} \times \overline{\mathbf{S}}_{\mathbf{CR}}, \tag{17}$$

in which $\mathbf{W}^{-1}$ is actually the pseudo inverse of $\mathbf{W}$. The total active power load consumption in the original system (P_{CO}) is given by

$$P_{CO} = \sum_i \text{Re}(\overline{S}_{CO}(i)), \tag{18}$$

in which i index all existing buses. Applying (18) in (16), the losses in the full circuit can be computed as

$$P_{LO} = P_T - \sum_i \text{Re}(\overline{S}_{CO}(i)). \tag{19}$$

Nonetheless, this applies only for a given load condition. In order to generalize the method, load changes ought to be considered.

In a time-series simulation, the loads active and reactive power daily changes are described in terms of a loadshape curve. In OpenDSS, the loadshape is a multiplier array $\mathbf{L} \in \mathbb{R}^{(1\times h)}$, in which h is the number of time changes to be simulated, in the case of this paper it means the number of simulated hours. Since loads with different loadshapes can be aggregated in an equivalent load during the reduction process, the equivalent loadshape of this equivalent load needs to be computed .

In order to obtain these equivalent loadshapes, each row of $\overline{S}_{CO}$ is multiplied by its corresponding loadshape array, resulting in the hourly complex power $\overline{\mathbf{S}}_{\mathbf{CO}}^{\mathbf{d}} \in \mathbb{C}^{(np\times h)}$. The upper index d stands for daily, since the simulation will be performed in the daily mode. The hourly power $\overline{\mathbf{S}}_{\mathbf{CR}}^{\mathbf{d}}$ is obtained from the weight matrix $\mathbf{W}$ as

$$\overline{\mathbf{S}}_{\mathbf{CR}}^{\mathbf{d}} = \mathbf{W} \times \overline{\mathbf{S}}_{\mathbf{CO}}^{\mathbf{d}}. \tag{20}$$

The equivalent loadshape $\mathbf{L_{CR}}$ is obtained by normalizing each row of $\overline{\mathbf{S}}^{\mathbf{d}}_{\mathbf{CR}}$ by its maximum active power. Mathematically,

$$\mathbf{L_{CR}}(i) = \frac{\overline{\mathbf{S}}^{\mathbf{d}}_{\mathbf{CR}}(i,:)}{\max\left(\overline{\mathbf{S}}^{\mathbf{d}}_{\mathbf{CR}}(i,:)\right)}. \tag{21}$$

The proposed technique is summarized in Algorithm 1.

Algorithm 1 Proposed Algorithm for Quantification of Technical Losses in daily time-series simulation

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Input: Original System Modeled
Output: Original System Technical Losses

Initialisation :

1: Obtain reduced equivalent circuit;
The equivalent loads and their loadshapes are obtained applying (20) and (21) with the loads nominal values;
2: **for** $i = 1$ to h (in which h is the total number of hours simulated) **do**
3: Run power-flow in the equivalent circuit for the hour i;
4: Compute both system total active power and loads complex powers, $P_{TR}(i)$ and $\overline{\mathbf{S}}_{\mathbf{CR}}(i)$, respectively;
5: Compute the original system losses applying (17) and (19;)
The values of $\overline{\mathbf{S}}_{\mathbf{CR}}(i)$ used in (17) are given by the power flow, rather than their nominal values obtained in step 1;
6: **end for**
7: Compute total losses for the whole day period;
This is achieved by adding up the values found for each hour i.

III. Results

In this section, the results obtained from the application of the proposed method in both the IEEE 34-bus test feeder and in a Brazilian feeder are presented. In order to first validate the implementation of the reduction technique, the voltage errors for different reduction percentages and for different hours of the day are evaluated. Then, the TL obtained from the proposed method are evaluated.

Seeking to validate the implementation of the reduction technique, the maximum voltage error between the original and the equivalent systems is presented and discussed. For this purpose, the number of buses is reduced from 0% up to approximately 70% in steps of 5%.

The reduction percentage is given by

$$RP = \frac{n_{\text{NCB}}}{n} \tag{22}$$

in which n_{NCB} is the number of non-critical buses eliminated and n is the total number of buses of the original system.

In each reduction step, a daily simulation is performed. The voltage error of each remaining bus, for each simulated hour and for each reduction percentage is computed. Then, the maximum error across the remaining buses of the Brazilian feeder is plotted against the percentage of reduction and the simulated hour, yielding the three dimensional graph shown in Figure 2.

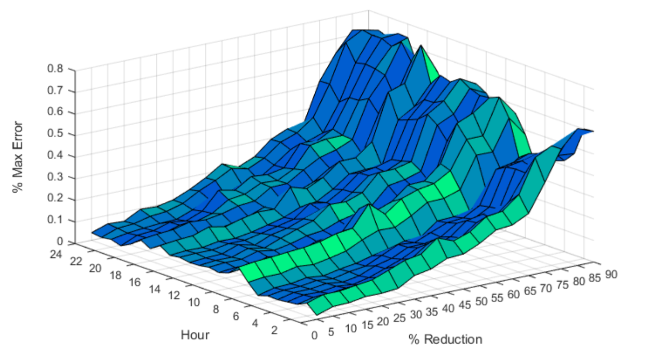

Fig. 2. Maximum voltage error in the Brazilian feeder for each hour and reduction percentage.

It can be observed in Figure 2 that the all values are less than 0.8%, similarly to those presented in [8], which were lower than 1.13% . They are not exactly the same because the feeder used is a different one. The results obtained with IEEE 34-bus test feeder are analogous, hence they are not shown here for concision purposes. Is is noteworthy that the error increases drastically for large reductions.

The total technical losses (TL) for a whole day period computed with the proposed method is presented next. The full system is simulated in OpenDSS and its results are used as reference. After simulating the reduced system, the OpenDSS computed losses are also compared to those obtained with the proposed indirect method.

The TL errors for the Brazilian feeder and for the IEEE 34-bus test feeder are shown in Figures 3 and 4, respectively.

In the Brazilian feeder, the error obtained with the proposed method was lower than 0.032% as shown in the upper plot of Figure 3. In the bottom plot of Figure 3, the error obtained with the OpenDSS loss computation of the reduced system is subtracted from the error obtained with the proposed method. It is noted that, for this feeder, the discrepancy between methods shows values below 7.25%, for an order of magnitude of 10^{-5}.

In the case of IEEE 34-bus test feeder, it is observed in the upper plot of Figure 4 that the error is approximately equal to 0.02% for reductions lower than 57%. The maximum error in the simulated scenarios was 0.12% for a reduction percentage of 70%. In this case, the order of magnitude of the discrepancy is 10^{-4} (In the bottom plot of Figure 4). Similarly to the Brazilian feeder, this proposed method gave losses errors similar to those obtained by the OpenDSS losses based reduced system power flow.

Based on the discussed results, the algorithm for computing the TL yields errors lower than 0.12% for reductions up to 70% of the system number of buses. Additionally, it is

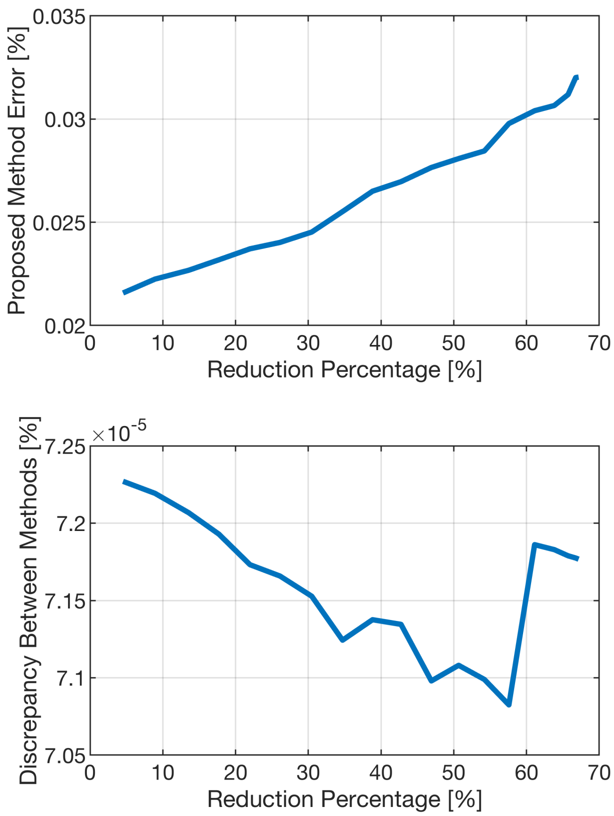

Fig. 3. TL error for the Brazilian feeder in a daily simulation.

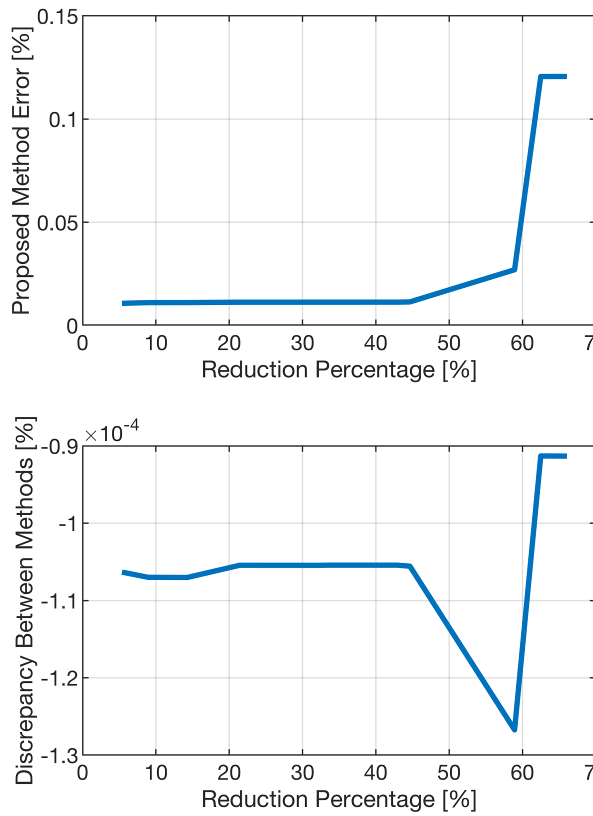

Fig. 4. TL error for the IEEE 34-bus test feeder in a daily simulation.

shown that the TL are preserved after the reduction technique for the two selected feeders. Nevertheless, this conclusion cannot be generalized since it is mathematically shown in [7] that the method philosophy does not assure TL equivalence. Unfortunately, neither [6]–[8] evaluated the TL errors in their investigations. The proposed method also presented errors below 0.12%, which may be due to the inversion process of the weighting matrix. Simulation of other feeders is still necessary to conclude if the general error introduced in the reduction process when calculating the TL with the the proposed indirect method is smaller than that obtained with OpenDSS computation.

IV. Conclusions

Computation of TL is a key aspect in planning, operation and tariff setting processes. However, simulation of large grids is computationally burdensome. Even though reduction techniques can be applied to obtain equivalent systems, none of them allow evaluation of both voltage and losses in unbalanced grids.

Considering the aforementioned aspects, this paper proposed a method for computing TL based on the multiphase reduction technique. As a result, both voltage and losses can be assessed in unbalanced grids.

In order to validate the reduction technique implementation, the voltage error for a Brazilian feeder was investigated. The maximum voltage error found was 0.8% for a 90% reduction in the Brazilian feeder, which is close to the errors presented by [8]. Then, both feeders had their TL assessed by simulation of their reduced equivalent in OpenDSS directly (with no further processing) and with further processing applying the proposed method. It was shown that in all cases, the errors introduced are smaller than 0.12%.

Considering the presented results, the proposed algorithm, as well as the multiphase reduction technique [8], are shown to be a feasible alternative for planning, operation and tariff definition processes. Evaluation of other feeders is still required to draw conclusions on the expected errors when applying these methods. Therefor, it is suggested for future works the evaluation of this algorithm in other systems and also in circuits with distributed generation.

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REFERENCES

[1] J. B. Ward, "Equivalent Circuits for Power-Flow Studies," *Transactions of the American Institute of Electrical Engineers*, vol. 68, no. 1, pp. 373–382, 1949. [Online]. Available: http://ieeexplore.ieee.org/document/5059947/

[2] D. Shi and D. J. Tylavsky, "A Novel Bus-Aggregation-Based Structure-Preserving Power System Equivalent," *IEEE Transactions on Power Systems*, vol. 30, no. 4, pp. 1977–1986, 2015.

[3] H. Oh, "A New Network Reduction Methodology for Power System Planning Studies," *IEEE Transactions on Power Systems*, vol. 25, no. 2, pp. 677–684, 2010.

[4] A. Coelho Neto, A. B. Rodrigues, R. B. Prada, and M. d. G. da Silva, "External equivalent for electric power distribution networks with radial topology," *IEEE Transactions on Power Systems*, vol. 23, no. 3, pp. 889–895, 2008.

[5] A. Nagarajan, A. Nelson, K. Prabakar, A. Hoke, M. Asano, R. Ueda, and S. Nepal, "Network reduction algorithm for developing distribution feeders for real-time simulators," in *2017 IEEE Power Energy Society General Meeting*, no. June, 2017, pp. 1–5. [Online]. Available: http://ieeexplore.ieee.org/document/8273938/

[6] M. J. Reno, K. Coogan, R. Broderick, and S. Grijalva, "Reduction of distribution feeders for simplified PV impact studies," *Conference Record of the IEEE Photovoltaic Specialists Conference*, pp. 2337–2342, 2013.

[7] M. J. Reno, R. J. Broderick, and S. Grijalva, "Formulating a simplified equivalent representation of distribution circuits for PV impact studies," Tech. Rep. April, 2013. [Online]. Available: https://www.researchgate.net/profile/Matthew_Reno/publication/264458156_Formulating_a_Simplified_Equivalent_Representation_of_Distribution_Circuits_for_PV_Impact_Studies/links/53e0294b0cf2a768e49f5bec.pdf

[8] Z. K. Pecenak, V. R. Disfani, M. J. Reno, and J. Kleissl, "Multiphase Distribution Feeder Reduction," *IEEE Transactions on Power Systems*, vol. 33, no. 2, pp. 1320–1328, 2018.