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Robust Model for Network-constraint Hydro-thermal-wind Operation Planning Considering Uncertainties

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Abstract: Hydro-thermal-wind operation planning comprises a set of operational decisions made in an environment of uncertainty. The power demand and power generation are uncertain, mainly the hydro and wind power plants due to their dependence on natural factors. Robust optimization in planning under uncertainty has proven to be a very efficient technique, including the operation planning of electrical power systems. Thus, in this paper is proposed a computational model based on robust optimization to perform hydro-thermal-wind operation planning considering the uncertainties related to hydro and wind generation, besides the demand. The proposed model is verified through two test systems: one with two nodes and another one considering the southern Brazilian subsystem (33 nodes). Several study scenarios are simulated, indicating that the proposed formulation, when compared to the deterministic approach, presents a relatively low cost and a low risk of energy deficit.

Keywords: Robust optimization; Optimization under uncertainty; Hydro-thermal-wind operation planning.

NOMENCLATURE

$t = 1 \dots T$	Operation planning period;
$i = 1 \dots I$	Corresponding node;
$k = 1 \dots K$	Corresponding thermoelectric unit;
$w = 1 \dots W$	Corresponding wind power unit;
$r = 1 \dots R$	Corresponding reservoir;
$l = 1 \dots L$	Corresponding transmission line;
$m = 1 \dots M$	Corresponding hydroelectric unit upstream of reservoir r;
$d = 1 \dots D$	Corresponding demand;
$n = 1 \dots N$	Corresponding turbine-generator set of hydroelectric units;
Ω_i^W	Set of wind power units W connected to node i;
Ω_i^R	Set of hydroelectric units with reservoir R, connected to node i;
Ω_i^K	Set of thermoelectric units K connected to node i;
$\Omega_{i(s)}^L$	Set of transmission lines L connected to node i (s, indicates the power flow direction as leaving – sending energy – from node i);
$\Omega_{i(e)}^L$	Set of transmission lines L connected to node i (e, indicates the power flow direction as entering at node i);
Ω_r^M	Set of hydroelectric units upstream M of reservoir r;
Ψ	Set of optimization variables related to the maximization problem ($\widetilde{Pw}_{w,t}$, $\widetilde{Ph}_{r,t}$, $\widetilde{Pd}_{d,t}$);
Δ	Set of optimization variables related to the minimization problem ($V_{r,t}$, $Q_{r,t}$, $s_{r,t}$, $Pt_{k,t}$, $Ph_{r,t}$, $Pw_{w,t}$, $\theta_{i,t}$, $Def_{i,t}$);
$Pw_{w,t}$	Wind power related to unit w at period t (MWavg – MW average);
$\overline{Pw}_{w,t}$	Maximum wind power capacity of unit w at period t (MWavg);
$\widetilde{Pw}_{w,t}$	Maximum uncertain wind power related to unit w at period t (MWavg);
$Ph_{r,t}$	Hydropower unit production with reservoir r at period t (MWavg);
$\overline{Ph}_{rt}$	Maximum hydropower capacity related to the unit with reservoir r at period t (MWavg);
$\widetilde{Ph}_{rt}$	Maximum uncertain hydropower production related to the unit with reservoir r at period t (MWavg);
$V_{r,t}$	Volume related to reservoir r at period t (hm³);
$V_{r,t+1}$	Volume related to reservoir r at period $t+1$ (hm³);
$V_{r,t}^{avg}$	Average (avg) volume related to reservoir r at period t (hm³/month);
$V_{r,T}$	Volume of reservoir r at the last period of the operation planning T (hm³);
$\underline{V}_r$	Minimum volume allowed by reservoir r (hm³);
$\overline{V}_r$	Maximum volume capacity of reservoir r (hm³);
$Q_{r,t}$	Turbine flow related to reservoir r at period t (m³/s);
$\overline{Q}_r$	Maximum turbine flow capacity of reservoir r (m³/s);
$q_{n,r,t}$	Turbine flow of turbine-generator set n of reservoir r at period t (m³/s);
$Q_{m,t}$	Turbine flow of the unit immediately upstream m at period t (m³/s);
$s_{r,t}$	Spill related to reservoir r at period t (m³/s);
$\overline{s}_r$	Maximum spill capacity of reservoir r (m³/s);
$s_{m,t}$	Spill related to the unit immediately upstream m at period t (m³/s);
$af_{r,t}$	Water inflow related to reservoir r at period t (m³/s);
$af_{r,t}^{forecast}$	Forecast water inflow related to reservoir r at period t (m³/s);
$\widetilde{af}_{r,t}$	Maximum uncertain water inflow of reservoir r at period t (m³/s);
$phl_{n,r,t}$	Hydraulic losses of turbine-generator set n of reservoir r at period t (m);
$hl_{n,r,t}$	Net height of turbine-generator set n of reservoir r at period t (m);
$hb_{r,t}$	Gross height related to reservoir r at period t (m);
$hm(V^{avg})_{r,t}$	Upstream height related to reservoir r at period t (m);
$hj(Q)_{r,t}$	Downstream height related to reservoir r at period t (m);
$Pt_{k,t}$	Thermal power production related to unit k at period t (MWavg);
$\overline{Pt}_k$	Maximum thermal power production related to unit k (MWavg);
$\widetilde{Pd}_{d,t}$	Maximum uncertain demand related to demand d at period t (MWavg);
$\underline{Pd}_{d,t}$	Minimum demand related to demand d at period t (MWavg);
$\overline{Pd}_{d,t}$	Maximum demand related to demand d at period t (MWavg);
$Pd_{d,t}^{forecast}$	Forecast demand related to demand d at period t (MWavg);
β	Demand variability rate (%);
$Def_{i,t}$	Deficit of power production of each node i at period t (MWavg);
$Pl_{l,t}$	Power flow through transmission line l at period t (MWavg);
$Pl_{l,max}$	Maximum power flow allowed through transmission line l (MWavg);
b_l	Susceptance related to transmission line l (p.u);
$\theta_{s(l)}$	Node angle where the transmission line l is connected, indicating the direction of power flow as "leaving";
$\theta_{e(l)}$	Node angle where the transmission line l is connected, indicating the direction of power flow as "entering";
$\theta_{i,t}$	Node angle of node i at period t;
$C_{i,t}^{Def}$	Cost of energetic deficit related to not meeting the demand in the node i at period t (R$ 3500.00 for MWh);
c	Constant for the conversion of units (m³/s to hm³/month) equal to 2.628;
g	Gravity acceleration: 9.81 m/s²;
$\eta_{r,t}^{avg}$	Average yield of turbine-generator set related to reservoir r at period t. A constant value was assumed 0.85 (dimensionless);
ρ^{water}	Water density (approximately 1000 kg/m³);
$k_{n,r}$	Coefficient of hydraulic losses (s²/m⁵);
α	Constant to ensure no emptying of reservoirs ($0 \leq \alpha \leq 1$);
C_0	Thermoelectric fixed cost coefficient (R$)
C_1	Thermoelectric linear cost coefficient (R$/MW);
C_2	Thermoelectric quadratic cost coefficient (R$/MW²);
$a_0, \dots, a_4$	Quota polynomial coefficient upstream of the reservoir r;
$b_0, \dots, b_4$	Quota polynomial coefficient downstream of the reservoir r;
Γ^{Pd}	Budget of uncertainty related to demand;
Γ^{Pw}	Budget of uncertainty related to wind generation;
Γ^{Ph}	Budget of uncertainty related to hydro generation.

1. INTRODUCTION

The pursuit of renewable and low-polluting energy sources has transformed the electrical system around the world by introducing intermittent, non-dispatchable energy sources, usually located near the load, with highly stochastic generation. Despite the environmental benefits, wind energy is an intermittent energy source, which causes uncertainties in the electrical power to be generated. These characteristics require the study of new methods to plan the electrical power system operating properly. According to Conejo *et al.* (2016),

these uncertainties can be modeled by finite scenarios with individual probability for each scenario (stochastic programming – SP) or by sets of uncertainty of a defined size (robust optimization – RO). Two-stage SP is the most widely used technique for solving uncertain planning problems, however, it requires prior knowledge of probability distribution functions and can generate computationally unviable models due to the high number of scenarios. Thus, according Alem and Morabito (2015), RO arises as an alternative to SP because it requires only robust sets that can be easily formulated.

The operation planning of the electric power systems is a nonlinear and computationally complex problem. The literature presents different methods to find a solution to the energetic operation planning problems like Zhao and Zeng (2012), Lorca and Sun (2015), Moraes *et al.* (2017), Attarha, Amjady and Conejo (2018) and others. However, to the best of our knowledge, it was not identify any work developed that study the hydro-thermal-wind economic dispatch considering load, wind and hydropower uncertainties, and using RO to model the uncertain parameters and, furthermore, consider the nonlinear hydro production function, in the same problem.

On this context, this paper presents a formulation of hydro-thermal-wind dispatch under uncertainties in generation and demand. Thus, the contributions of this work include: (1) Model the uncertainties of wind power generation, hydropower generation and demand simultaneously; (2) Present a computational formulation of the multistage hydrothermal operation planning with high wind penetration, and also considering uncertainties; (3) Present a formulation for solving the hydro-thermal-wind dispatch problem using RO; (4) Evaluate the robustness of the proposed model and validate the proposed method using a real test system. This paper is organized as follows: Section 2 discusses the RO; Section 3 brings the mathematical formulation and solution approach; Section 4 presents simulations and their analysis; Section 5 presents the conclusions.

2. OPTIMIZATION UNDER UNCERTAINTY

Deterministic approach is the most traditional method used to model the problem of energetic operation planning, however, it does not allow the representation of the inherent uncertainties of the problem. RO is one of the techniques that allows the uncertainty representation, which aims to analyze the worst case, that is, look for the best possible solution, assuming that nature will behave in the worst possible way. Thus, this RO model minimizes (or maximizes) the objective under the worst case realization of uncertainty (Conejo *et al.* 2016).

Mathematically, the RO problem can be formulated as equations (1) to (4). In this case, the goal is to minimize the objective function ($f(x,u)$), which depends on the decision variables (x) and on the uncertain parameters (u). The minimization has to occur to the worst realization of the uncertain parameters (u), i.e, the maximization of u. Thus, after to find the worst realization of uncertainties ($max_{u\in\mathcal{U}}$) the decisions x (min_x) have to be found, to the final value of the objective function be minimal. Note, the uncertain parameter u is within the robust uncertain set ($\mathcal{U}$), and in this way, u resulted from maximization problem has to be within this robust set. Moreover, the equality and inequality constraints (equation (2) to (4)), must be respected – constraints are inflexible and no violation is allowed. Problem solving is integrated, making an interaction between the primal and dual problem. For readers interested in resolution using the dual decomposition method refers to Conejo *et al.* (2016). Other algorithms can be used to solve multistage problems, such as described in Lorca and Sun (2015), for example. Another strategy would be to use ready-made optimization tools, such as optimization toolbox available in MATLAB®, which has features such as the `fminimax` function, which is used in the present work and explained in more detail in section 3.

$$\max_{(u\in\mathcal{U})} \min_{(x)} f(x,u) \tag{1}$$

subject to:

$$h(x,u) = 0 \tag{2}$$

$$g(x,u) \leq 0 \tag{3}$$

$$x,u \geq 0 \tag{4}$$

3. PROBLEM FORMULATION

The proposed methodology is based on medium-term hydro-thermal-wind power dispatch (Finardi (2003); Takigawa (2010); Andriolo (2014); Silva (2014)), considering uncertainties of generation – hydro and wind power – and demand, using RO (Conejo *et al.* (2016); Bertsimas and Sim (2004)). Uncertainties regarding hydropower systems are due to future water inflows, while in wind power, those are due to the wind speed. Some assumptions are made in the proposed problem formulation: (1) To represent the intertemporal and spatial dependence of hydro generation, it is considered cascade plants, with or without storage; (2) The average yield is used instead of the production function in the hydropower production equation; (3) The transmission network is considered according to the linear model (DC – direct current).

3.1 Complete formulation

When RO is considered to deal with uncertainties, the deterministic problem usually employed becomes a two-level optimization problem. Thus, the proposed model is formulated in two levels ($max\ min$) – equation (5) – where the objective is to minimize the operational costs given the worst case realization of uncertainties – equation (6). The complete formulation is represented from equation (5) to (34). Equations (7) to (28) represent the first level formulation (operating costs minimization) and equations (29) to (34) represent the constraints of the second level (uncertainties maximization). In the sequence the detail explanations of the formulation is made.

$$\max_{\Psi} \min_{\Delta} f\left(Pt_{k,t}, Def_{i,t}\right) \tag{5}$$

where:

$$f\left(Pt_{k,t}, Def_{i,t}\right) = \sum_{t=1}^{T}\sum_{k=1}^{K}\{C_0 + C_1 Pt_{k,t} + C_2\left(Pt_{k,t}\right)^2\} + \sum_{t=1}^{T}\sum_{i=1}^{I}\{C_{i,t}^{Def} . Def_{i,t}\} \tag{6}$$

subject to:

$$\sum_{k\in\Omega_i^K} Pt_{k,t} + \sum_{r\in\Omega_i^R} Ph_{r,t} + \sum_{w\in\Omega_i^W} Pw_{w,t} + Def_{i,t} - \sum_{l\in\Omega_{i(s)}^L} Pl_{l,t} + \sum_{l\in\Omega_{i(e)}^L} Pl_{l,t} = \widetilde{Pd}_{d,t} \ \ \forall i, \forall t \tag{7}$$

$$Pl_{l,t} = b_l\left(\theta_{s(l)} - \theta_{e(l)}\right) \ \forall l, \forall t \tag{8}$$

$$V_{r,t+1} = V_{r,t} - c\left(Q_{r,t} + s_{r,t} - af_{r,t}\right) + \sum_{m\in\Omega_r^M} c\left(Q_{m,t} + s_{m,t}\right) \ \forall r, \forall t \quad (9)$$

$$Ph_{r,t} = 10^{-6}.g.\eta_{r,t}^{avg}.\rho^{water}.\sum_{n=1}^{N}(hl_{n,r,t}.q_{n,r,t}) \ \forall r, \forall t \quad (10)$$

$$hl_{n,r,t} = hb_{r,t} - phl_{n,r,t} \ \forall n, \forall r, \forall t \quad (11)$$

$$phl_{n,r,t} = k_{n,r}.q_{n,r,t}{}^2 \ \forall n, \forall r, \forall t \quad (12)$$

$$Q_{r,t} = \sum_{n=1}^{N} q_{n,r,t} \ \forall r, \forall t \quad (13)$$

$$hb_{r,t} = hm(V^{avg})_{r,t} - hj(Q)_{r,t} \ \forall r, \forall t \quad (14)$$

$$hm(V^{avg})_{r,t} = a_0 + a_1 V_{r,t}^{avg} + a_2(V_{r,t}^{avg})^2 + a_3(V_{r,t}^{avg})^3 + a_4(V_{r,t}^{avg})^4 \ \forall r, \forall t \quad (15)$$

$$hj(Q)_{r,t} = b_0 + b_1 Q_{r,t} + b_2(Q_{r,t})^2 + b_3(Q_{r,t})^3 + b_4(Q_{r,t})^4 \ \forall r, \forall t \quad (16)$$

$$V_{r,t}^{avg} = \left(V_{r,t} + V_{r,t+1}\right)/2 \ \forall r, \forall t \quad (17)$$

$$V_{r,T} \geq \alpha\left(\overline{V}_r\right) \ \forall r \quad (18)$$

$$\underline{V}_r \leq V_{r,t} \leq \overline{V}_r \ \forall r, \forall t \quad (19)$$

$$0 \leq Q_{r,t} \leq \overline{Q}_r \ \forall r, \forall t \quad (20)$$

$$0 \leq s_{r,t} \leq \overline{s}_r \ \forall r, \forall t \quad (21)$$

$$0 \leq af_{r,t} \leq \widetilde{af}_{r,t} \quad (22)$$

$$0 \leq Ph_{r,t} \leq \widetilde{Ph}_{r,t} \ \forall r, \forall t \quad (23)$$

$$0 \leq Pt_{k,t} \leq \overline{Pt}_k \ \forall k, \forall t \quad (24)$$

$$0 \leq Pw_{w,t} \leq \widetilde{Pw}_{w,t} \ \forall w, \forall t \quad (25)$$

$$-Pl_{l,max} \leq Pl_{lt} \leq Pl_{l,max} \ \forall l, \forall t \quad (26)$$

$$-\pi \leq \theta_{i,t} \leq \pi \ \forall i, \forall t \quad (27)$$

$$\theta_i = 0; \quad i = reference \quad (28)$$

subject to:

$$\widetilde{Ph}_{r,t} \in \left[0, \overline{Ph}_{r,t}\right] \ \forall r, \forall t \quad (29)$$

$$\frac{\sum_r(\overline{Ph}_{r,t} - \widetilde{Ph}_{r,t})}{\sum_r(\overline{Ph}_{r,t})} \leq \Gamma^{Ph} \ \forall t \quad (30)$$

$$\widetilde{Pw}_{w,t} \in \left[0, \overline{Pw}_{w,t}\right] \ \forall w, \forall t \quad (31)$$

$$\frac{\sum_w(\overline{Pw}_{w,t} - \widetilde{Pw}_{w,t})}{\sum_w \overline{Pw}_{w,t}} \leq \Gamma^{Pw} \ \forall t \quad (32)$$

$$\widetilde{Pd}_{d,t} \in \left[\underline{Pd}_{d,t}, \overline{Pd}_{dt}\right] \ \forall d, \forall t \quad (33)$$

$$\frac{\sum_d(\widetilde{Pd}_{d,t} - \underline{Pd}_{d,t})}{\sum_d(\overline{Pd}_{d,t} - \underline{Pd}_{d,t})} \leq \Gamma^{Pd} \ \forall t \quad (34)$$

3.2 Objective function and power balance

The proposed work aims to minimize the operational costs given the worst case realization of uncertainties. So the best operational decisions need to be made, which can face the worst behavior of the nature – equation (5). Once the costs regarding hydro and wind power system are considered almost null, only costs related to thermal plants and energy deficits are considered in this paper. Thus, the objective function is given by equation (6). Regarding the thermoelectric system the operational constraints also must be considered as equation (24). In relation to the power balance, the system demand must be reached. Therefore, the sum of all generation units has to be equal to the system demand as equation (7), where the deficit is also included to represent the demand that may not be achieved. It is important to notice that the demand of the system ($\widetilde{Pd}_{d,t}$) is obtained in the maximization problem, as will be explained in more details in section 3.6.

3.3 Hydroelectric system

In the operation of a reservoir, there is a conservation of water mass, which is represented by the equation (9) – water balance. The volume of a reservoir compromises the future system operation cost. Thus, equation (18) – volume goal – is considered to ensure that an appropriate volume is stored in reservoirs in the last planning period. The parameter α is a constant used to certify proper levels of water in the reservoirs. According to Andriolo (2014), α can be set according to the planning horizon, depending on whether it ends in wet or dry period. Moreover, related to the volume, it is necessary to constraint it in the other planning periods as equation (19). The hydropower production function consists of a set of highly nonlinear equations which are represented by equations (10) to (17). Equation (10) is the hydropower production function; equation (11) represents the net height of a turbine-generator set; (12) is the hydraulic losses of a turbine-generator set; (14) is the gross height of a reservoir; (15) represents the upstream height of a reservoir; (16) is the downstream height of a reservoir; and (17) is the average volume of a reservoir. Note the turbine flow of a reservoir, represented in equation (16), is obtained from equation (13).

It is also necessary to include the constraints related to turbine flow, spill of water, water inflow and hydropower production, which are given, respectively by equations (20), (21), (22) and (23). The maximum uncertain water inflow ($\widetilde{af}_{r,t}$), represented in equation (22), is obtained from the equation (35), which is included in the problem input data. In this study the forecast water inflow ($af_{r,t}^{forecast}$) used is, actually, the average of the historical data. Note the equation (35) aims to control the water inflow according to the uncertainty budget of hydroelectric production, once the uncertainty from hydropower comes from water inflow. The maximum hydro generation uncertain of a unit ($\widetilde{Ph}_{r,t}$), represented in equation (23), is obtained in the maximization problem, as will be explained below.

$$\widetilde{af}_{r,t} = (1 - \Gamma^{Ph}).af_{r,t}^{forecast} \ \forall r, \forall t \quad (35)$$

The uncertainty in hydropower was modeled as presented in Conejo *et al.* (2016). This uncertainty set is characterized by the equations (29) and (30). Constraint (29) is related to the physical limitation, imposing the lower and upper bound of $\widetilde{Ph}_{r,t}$. Equation (30) limits the variability of uncertain $\widetilde{Ph}_{r,t}$, through the so-called budget of uncertainty related to hydro generation (Γ^{Ph}). The uncertainty budget Γ^{Ph}can take values between 0 and 1. If Γ^{Ph} is chosen equal to 0, then $\widetilde{Ph}_{r,t} = \overline{Ph}_{r,t}$, i.e., uncertainty in the available capacity of generating units is not considered. On the other hand, if $\Gamma^{Ph}$is chosen equal to 1, then $\widetilde{Ph}_{r,t}$ can take any value within the interval $\left[0, \overline{Ph}_{r,t}\right]$, wherein the worst case realization of uncertainty, this value will be $\widetilde{Ph}_{r,t} = 0$. It means there is no hydro generation available in this scenario.

3.4 Wind power system

The wind power generation will be represented through available historical data. Regarding the uncertainties, they will also be considered similar to what was adopted for hydro generation according to equations (31) and (32). Moreover, it is necessary to consider the constraint related to the wind power generation as represented in equation (25). Thus, the maximum uncertain wind power production obtained in the maximization problem, will be used in the minimization problem as wind power production upper bound. In the present work, the uncertainty in wind generation is formulated based on the work of Conejo *et al.* (2016), where the uncertainty is modeled at the level of energy generation and not in the wind.

3.5 Transmission line

This work considers the DC transmission network model, as can be observed in the equation (8) – power flow over a transmission line. It is also necessary to consider the physical limitation of the lines and nodes, which are given, respectively, over equations (26) to (28). Thus, equation (26) constraints the power flow in the transmission line; equation (27) is responsible to constraint the angles of each node; and equation (28) defined the reference node of the system.

3.6 Demand

The demand uncertainty set was modeled according to Conejo *et al.* (2016). The demand has a range above and below expected values. This means that the worst realization of demand will be between a minimum (non-zero) and a maximum value as shown in equation (33). The maximum uncertain demand ($\widetilde{Pd}_{d,t}$) also depends on the demand uncertainty budget (Γ^{Pd}), as equation (34). Γ^{Pd} ranges from 0 to 1. When this value is zero, it means that $\widetilde{Pd}_{d,t} = \underline{Pd}_{dt}$. On the other hand, when Γ^{Pd} is equal to 1, it means that the maximum uncertainty size has been assumed, and, in this case, demand can take any value within the interval $[\underline{Pd}_{d,t}, \overline{Pd}_{dt}]$. The worst that can happen in this case is this value be $\widetilde{Pd}_{d,t} = \overline{Pd}_{dt}$. The demand resulted from the maximization problem, corresponds to the demand that has to be achieved in the power balance of equation (7). Moreover, to calculate the maximum and minimum demand, it is necessary to apply equations (36) and (37) that take into account the demand variability rate (β), which represents how much the demand values can deviate from the forecasted data.

$$\overline{Pd}_{d,t} = (1+\beta).Pd_{d,t}^{forecasted} \quad \forall d, \forall t \tag{36}$$

$$\underline{Pd}_{d,t} = (1-\beta).Pd_{d,t}^{forecasted} \quad \forall d, \forall t \tag{37}$$

Those values ($\overline{Pd}_{d,t}$ and $\underline{Pd}_{d,t}$) are included in the problem input data. In this paper the demand forecasted ($Pd_{d,t}^{forecasted}$) used is, actually, the historical data available.

3.7 Solution approach

The mathematical formulation presented was computationally implemented through MATLAB®. The function `fminimax`, available in the optimization toolbox, that allows to solve two-level problems, was employed to solve the proposed model. Readers interested in details on how this tool works can check the `fminimax` MATLAB® documentation available in Matlab (2011).

4. STUDY CASES – RESULTS AND DISCUSSIONS

To validate the formulation proposed, two test systems were used: 2 nodes test system and 33 nodes test system related to the southern Brazilian subsystem. To run the simulations in the 2 nodes test system a laptop computer with an Intel® Core™ i5 -6200U @ 2.30 GHz processor and 4.00 GB RAM was used. In the 33 nodes test system, due to the high computational cost associated to the problem, it was used a virtual computer available on Google Cloud platform with Quad Intel Xenon® Quad 2.2 GHz Hyper Threaded CPU with 30 GB RAM.

4.1 Data

Data used in the 2 nodes test system were adapted from Takigawa (2010). The structure of the system was elaborated by the authors themselves, in order to evaluate didactically the behavior of the system. As shown in Fig. 1 the system has a thermoelectric plant at node 2 and a hydroelectric and wind power plant at node 1. This system simulates a grid where there is a higher load center at node 2 and there is a line connecting node 1 to node 2.

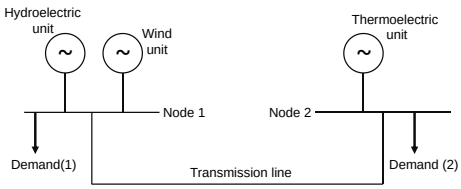

Fig. 1 Test system 1 – 2 nodes.

The 33 nodes test system was proposed by Alves (2007) and the data used is from Takigawa (2010) and Andriolo (2014). In order to validate the methodology proposed in this work, some modifications were done in the test system: (1) a 900 MW wind farm in Palmas – PR, Brazil, and a 700 MW thermoelectric plant in Araucária, Curitiba – PR, Brazil were included in the system; (2) using the data available in ONS (2018) each of the 9 load points were mirrored to find the demand for each node in the planning horizon; (3) the transmission lines capacity, which is adjacent to the added plants (1), were doubled to allow the power flow generated by these two new sources of energy included in the test system. Moreover, the system's hydropower plants are cascading as described in Andriolo (2014). For both test systems the planning horizon is equal to a year, discretized by months, forming 12 planning stages.

4.2 Test system 1 – 2 nodes

In addition to the information presented above, the following assumptions were made for simulations: (1) The initial reservoir volume was set to 73.8% of its maximum volume; (2) in the last planning period, the volume has to be greater than or equal to 73.8% of maximum; (3) maximum transmission line capacity is 750 MW; (4) β was assumed as 20%. Several scenarios considering these conditions and varying the uncertainty budgets were simulated and the total operational cost related to nine of them are presented in Table 1. However only three scenarios, considered the most relevant, highlighted in blue (Scenario I), green (Scenario II) and yellow (Scenario III) are discussed throughout this section.

In Scenario I all levels of uncertainty are considered equal to zero, which makes the model equivalent to the deterministic approach. In this case, there is no risk of unavailability in wind or water sources, and demand is minimal. Therefore, the operating cost (R$1e+5) is the lowest among the scenarios analyzed. The exchange of energy occurs from node 1 to node 2, since node 1 represents the largest generation subsystem with renewable sources, and node 2 represents a large load center. Moreover, given the system operation conditions there was no deficit in any month of the planning horizon, as presented in Table 2. Although the hydropower unit is

dispatching as much as possible (6281.271 MWavg for all periods), the amount is far from its maximum capacity (1500 MWavg per period), due to operating constraints corresponding to the volume goal. As a result, there is a dispatch from the thermoelectric plant to meet the demand during the 12 months. In the hydroelectric plant the turbine volume varies depending on the demand and availability of water inflow and volume constraints. In most months the turbined volumes are smaller due to the low water inflow, to the minimum volume constraint and mainly because water needs to be stored to satisfy the volume goal. This Scenario I did not show a volume of the spill, because low availability of stored water and low water inflow, make the reservoir does not reach its maximum limit in any month.

Table 1. Simulation results for 2 nodes test system – Total operational cost (R$), maximum transmission registered (MWavg) and respective period (t).

Γ^{Pd}	Parameter analyzed	Γ^{Ph} and Γ^{Pw}		
		0	**0.5**	**1**
0	Cost	1e+5	4.12e+6	3.07e+7
	Max. trans. [t]	750 [t8, t10, t12]	424.368 [t10]	-115.3 [t5]
0.5	Cost	4.28e+5	1.51e+7	3.05e+7
	Max. trans. [t]	750 [t4, t12]	572.527 [t10]	-138.761 [t10]
1	Cost	4.74e+5	1.51e+7	3.06e+7
	Max. trans. [t]	750 [t4 to t12]	572.527 [t10]	-136.18 [t10]

Table 2. Total generation in MWavg for each energy source (sum of 12 periods) from scenarios considered in Table 1.

Source of energy	Scenario I	Scenario II	Scenario III
Thermal	718.881	2008.64	8380.50
Hydro	6281.271	5841.79	0
Wind	8634.600	4317.30	0
Deficit	0	3963.59	9459.70

In Scenario II the uncertainty levels are relatively high, which means that half of hydro, wind and thermal generation will be unavailable. Thus, the available hydro and wind generation are not enough to meet the demand, which results in a power deficit in the system, as shown in Table 2. The operating cost for this scenario is R$1.51e+7, which is due to the dispatch of the thermoelectric plant and the fact that it is not possible to meet all the demand, causing a deficit, as already mentioned.

The Scenario III is characterized as a case of high uncertainty, where there is total unavailability of water and wind sources, and demand reaches its maximum value. Only part of the system demand is met, which implies on a power deficit as described also in Table 2. This is a purely thermoelectric operation and the reservoir level is kept constant throughout the planning horizon. The occurrence of this scenario is unlikely in the real world, but it was considered in this paper for analysis and verification purposes, mainly regarding the validation of the proposed mathematical and computational model.

Therefore, it was found that the operating cost of the system increases with increasing uncertainty in hydro and wind generation for the three cases analyzed. The power exchange between nodes 1 and 2 decreases with increasing generation uncertainty. In the extreme case (Scenario III), the power exchange is performed in the opposite direction, from node 2 to node 1. Thus, for this test system, the operating cost is directly proportional to the uncertainty and the exchange power flow is inversely proportional to the uncertainty.

4.3 Test system 2 – 33 nodes

In this test system it was simulated 9 scenarios, varying the uncertainty of generation and demand by 0, 0.25 and 0.75. The total operational costs obtained in the simulations can be seen in Table 3. The operating cost of the system is lower as lower is the degree of uncertainty and vice versa. Among the 9 simulated scenarios, just the results of the three scenarios highlighted in Table 3 are going to be discussed, once they were considered the most relevant. The scenario in blue (Scenario I), green (Scenario II) and yellow (Scenario III) show respectively, degrees of uncertainty equal to 0, 0.25 and 0.75.

Table 3. Simulation results for 33 nodes test system – Total operational cost (R$).

Γ^{Pd}	Γ^{Ph} and Γ^{Pw}		
	0	**0.25**	**0.75**
0	1.84e+6	5.04e+6	3.92e+7
0.25	7.71e+6	7.80e+6	1.42e+8
0.75	8e+6	5.86e+7	2.03e+8

In Scenario I demand and generation uncertainty are null. The operating cost (R$1.84e+6) is the lowest in this simulation because uncertainty is as low as possible. In Fig. 2 it is possible to note that the demand is mainly met by hydroelectric and wind sources, with a small participation of thermal generation in the months of greatest demand and/or lower water inflows. In this scenario, due to the high availability of lower cost generation sources, there is no energy deficit risk. Regarding the generation of energy from hydroelectric plants, it was found that the production of run-of-river plants is practically constant over the planning horizon, while in reservoir plants there are variations which are related to the water inflow of each period. In relation to the volume stored in each reservoir, most remain constant over the planning horizon and operating within their useful volume.

In Scenario II the uncertainty is relatively low, however, it is higher compared to Scenario I. The operating cost (R$7.80e+6) is higher than the operating cost of Scenario I (R$1.84e+6). There is a risk of deficit in the months of greatest demand (months 1, 2, 3 and 11). This energy deficit risk is verified due to the inability to fully meet the demand with the system's own generation. The Fig. 3 shows the hydro-thermal-wind dispatch in meeting demand during the planning horizon. It can be noted that the demand to be reached in Scenario II is higher than the one in Scenario I, as the increase in demand uncertainty causes uncertain demand to be greater than forecasted demand.

In Scenario III the operation cost (R$2.03e+8) is the highest among all simulated. This result is due to the high unavailability of hydro and wind generation, and great demand levels, which is a result of the uncertainty budget adopted. The Fig. 4 shows that under the conditions of Scenario III, system generation does not meet all demand, with a higher risk of deficit as more than half of the demand is not reached.

It was verified, therefore, that the results obtained in the 33 nodes test system follow the same logic of the 2 nodes test system: the operational cost of the system increases with the increase of uncertainties. The main difference in the results is related to the high computational time for 33 nodes test system simulation: in Scenario II, for example, where the budget of

uncertainty is relatively high for all parameters (0.50) the computational time for 2 nodes test system was 0.1594 min, while for the 33 nodes test system it was equal to 104.47 min. This is due to the complexity of this system (33 nodes), where the number of variables and constraints increase significantly (difference of 2448 variables compared to the 2 nodes test system).

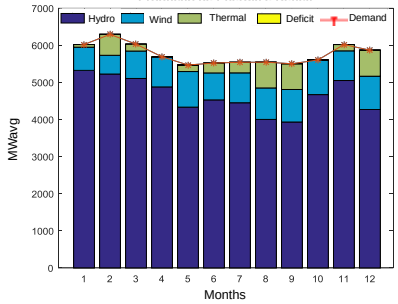

Fig. 2 Total power production and energy deficit from Scenario I.

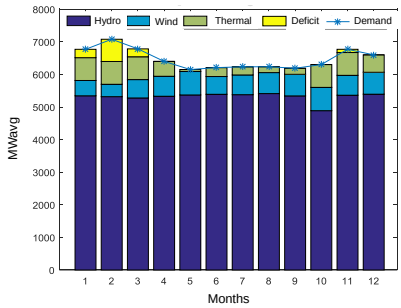

Fig. 3 Total power production and energy deficit from Scenario II.

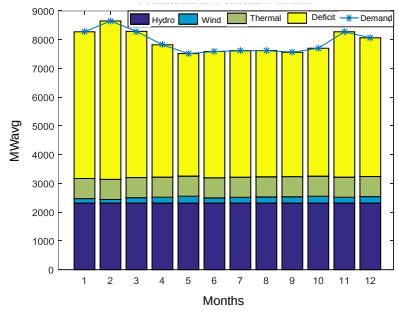

Fig. 4 Total power production and energy deficit from Scenario III.

5. CONCLUSIONS

The RO has been found to be a practical tool that allows to represent uncertain parameters by robust sets. The degree of uncertainty directly impacts the generation dispatch, deficit risk and consequently the operating cost of the system. In the 2 nodes test system, for example, in the worst case (Scenario III), the generation from deficit represents 53.02% of total production and 46.97% is from the thermal. While in the most optimistic case (Scenario I), the energy deficit is equal to 0% and the thermal represents just 4.60% of total generation. It shows the importance of different cases studies under uncertainty, otherwise the system will operate with low efficiency under bad scenarios. Future research proposes to use AC (alternating current) model for the transmission line and to use some decomposition strategy to solve the problem.

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REFERENCES

Alem, D. and Morabito, R. (2015). Planejamento da produção sob incerteza: programação estocástica versus otimização robusta. *Gestão & Produção*, 22(3), pp. 539–551.

Alves, W. F. (2007). *Proposição De Sistemas-Teste Para Análise Computacional de Sistemas de Potência*. Universidade Federal Fluminense, Niterói - RJ.

Andriolo, R. F. (2014). *Acoplamento Elétrico Energético no Planejamento da Operação em Médio Prazo com Restrição de Transmissão*. Universidade Federal do Paraná, Curitiba - PR.

Attarha, A., Amjady, N. and Conejo, A. J. (2018). Adaptive robust AC optimal power flow considering load and wind power uncertainties. *International Journal of Electrical Power and Energy Systems*, 96, pp. 132–142.

Bertsimas, D. and Sim, M. (2004). The Price of Robustness, *Operations Research*, 52(1), pp. 35–53.

Conejo, A. J., Baringo, L., Kazempour, S. J. and Siddiqui, A. S. (2016). *Investment in Electricity Generation and Transmission: Decision Making under Uncertainty*. 389 pages. Springer, Switzerland.

Finardi, E. C. (2003). *Alocação de Unidades Geradoras Hidrelétricas em Sistemas Hidrotérmicos Utilizando Relaxação Lagrangeana e Programação Quadrática Sequencial*. Universidade Federal de Santa Catarina, Florianópolis - SC.

Lorca, Á. and Sun, X. A. (2015). Adaptive Robust Optimization With Dynamic Uncertainty Sets for Multi-Period Economic Dispatch Under Significant Wind, *IEEE Transactions On Power Systems*, 30(4), pp. 1702–1713.

Matlab. (2011). Optimization Toolbox - User 's Guide.

Moraes, R. A., Fernandes, T. S. P., Arantes, A. G. B., Unsihuay-Vila, C. (2017). Short-Term Scheduling of Integrated Power and Spinning Reserve of a Wind-Hydrothermal Generation System with AC Network Security Constraints, *Journal Of Control, Automation And Electrical Systems*, 29(1), pp. 1–14.

ONS (2018) *Resultados da Operação: Histórico de Operação: Carga de Energia*. Available at: http://www.ons.org.br/Paginas/resultados-da-operacao/historico-da-operacao/carga_energia.aspx (Accessed: 3 October 2019).

Silva, S. R. (2014). *Planejamento Hidrotérmico Considerando a Penetração de Geração Eólica em Sistemas Elétricos de Potência*. Universidade Federal de Itajubá, Itajubá - MG.

Takigawa, F. Y. K. (2010). *Desenvolvimento de um Modelo Computacional para o Problema da Programação Diária da Operação de Sistemas Hidrotérmicos*. Universidade Federal de Santa Catarina, Florianópolis - SC.

Zhao, L. and Zeng, B. (2012). Robust unit commitment problem with demand response and wind energy. *IEEE Power and Energy Society General Meeting*, San Diego - CA.