

# Evaluation of Cross-Differential Protection Applied to Double-Circuit Transmission Lines Under Inter-Circuit Faults

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**Abstract:** This paper deals with the performance evaluation of cross-differential protection applied to double-circuit transmission lines which share the same tower, considering the occurrence of inter-circuit faults. Aiming to do so, sensitivity and transient analysis were carried out on a 300 km double-circuit transmission line with 500 kV of rating voltage, modeled on Alternative Transients Program (ATP). By the analysis of the obtained results, the cross-differential protection function is able to detect faults between two different phases of the two existent circuits and has a high instantaneous coverage for such type of fault.

*Keywords:* Double-Circuit Lines, Cross-Differential Protection, Inter-Circuit Faults, Alpha Plane, Power Systems, ATP.

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## 1. INTRODUCTION

The social and economic expanding demand is one of the main factors related to the need of power systems development, responsible for providing a continuous service of electrical energy. According to this, double-circuit transmission lines have been extensively used due to its economic and environmental advantages when compared with single-circuit lines. Such lines are increasing in use due to its high-power transfer capacity, along with sharing the same transmission towers and right-of-way, therefore improving transmission capability and reliability (Bo et al., 2003)(Wang et al., 2005c)(Apostolov et al., 2007)(Sanaye-Pasand and Jafarian, 2011). In a context where is intended to gather efficiency with cost-benefit, these lines are useful.

However, such lines are strongly affected by mutual coupling effects. The negative and positive sequence coupling are whimsy, and can be disregarded, differently from the zero sequence coupling (Apostolov et al., 2007). As the distance elements are affected by the zero sequence currents, this mutual coupling brings challenging issues to this protection element, since it affects the positive sequence impedance computation (Sanaye-Pasand and Jafarian, 2011).

Also, it is worthy to mention that due to conductors layout, inter-circuit faults occurrence in multi-circuit lines is sufficient high to be studied by protection engineers when implementing a protection scheme (Agrasar et al., 1997)(Spoor and Zhu, 2005). Although the earthed cross-country fault is the most common inter-circuit fault, the unearthed inter-circuit one also provide some issues due to zero-sequence currents present in each circuit and

underreach of distance protection. In general, inter-circuit faults have a high chance of incidence due to conductor galloping, bushfire activity or broken conductors at specific circuit (Spoor and Zhu, 2005). Due to this, the algorithms must be sensitive to those topology of faults, to avoid critical damages to the power system.

A frequently used protection scheme for this arrangement is the longitudinal differential protection for each circuit associated with distance function. However, this configuration requires a reliable communication channel, which results in the increase of costs and protection complexity, besides introducing a new point of failure. On that way, a protection scheme with similar performance that does not need any communication between the relays may bring advantages to protection schemes (Bo et al., 2003).

Therefore, the cross-differential protection has been reported to enhance the protection scheme, covering some aspects that the longitudinal differential and the distance elements cannot accomplish. This function is immune to the zero-sequence mutual coupling effect, and as a one-terminal based function, it is independent of communication channels, which entails in cost savings and security enhancement (Sanaye-Pasand and Jafarian, 2011). This protection element is based on the relation between currents measured of each circuit from the transmission line. A fault is detected when the difference between the currents amplitudes exceed a specified threshold (Wang et al., 2005b).

Accordingly, this paper aims to evaluate the cross-differential protection among inter-circuit faults. Using the alpha plane representation, the protection behavior is presented. To do so, the instantaneous coverage for inter-circuits faults is illustrated by means of sensitivity analyses for fault resistance and location variation. A transient

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analysis is also presented, in order to visualize the alpha plane trajectories.

## 2. CROSS-DIFFERENTIAL PROTECTION ALGORITHM

The cross-differential protection is a one-terminal based function. Its essential principle is established on the amplitude comparison of each circuit's current, considering a double-circuit transmission line (Wang et al., 2005a).

The conventional principle of cross-differential protection, presented in Figure 1, is based on the comparison of the currents phasors magnitudes of the two circuits, according to (1) and (2), where  $\bar{I}_{circuit,1}$  and  $\bar{I}_{circuit,2}$  are the currents phasors of circuit 1 and 2, and  $I_{op}$  is a operating threshold. When one of mentioned equations is satisfied, a fault on respective circuit is detected (Wang et al., 2005b).

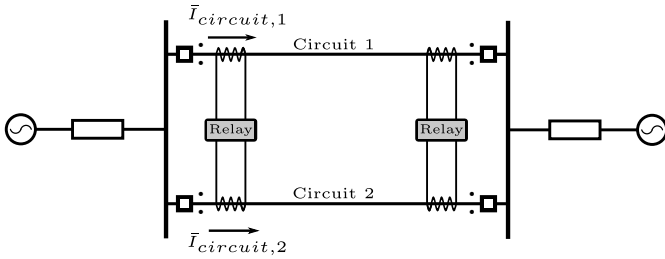


Figure 1. Cross-differential representation.

$$|\bar{I}_{circuit,1}| - |\bar{I}_{circuit,2}| \geq I_{op} \rightarrow Trip (Circuit1) \quad (1)$$

$$|\bar{I}_{circuit,2}| - |\bar{I}_{circuit,1}| \geq I_{op} \rightarrow Trip (Circuit2) \quad (2)$$

During no fault or external fault conditions, electric quantities are analogous, being different on internal faults, in such a way that the similarity is broken and then the fault can be detected (Wang et al., 2005b). However, the operating values must be set to high values due to unbalance current during external fault, differential current of healthy phase for phase-earth fault in successive operation status and greatest load current during single line operation (Wang et al., 2005b).

Therefore, the percentage cross-differential protection can be applied to enhance the protection scheme. This technique consists on the comparison between the reference and parallel circuit currents phasors,  $\bar{I}_1$  and  $\bar{I}_2$ . Two parameters are used to develop this function, the operating current ( $I_{op}$ ) and the restraining current ( $I_{res}$ ), presented at (3) (Wang et al., 2005a)(Wang et al., 2005b)(Roberts et al., 2001).

$$I_{op} = |\bar{I}_1| - |\bar{I}_2| \quad I_{res} = |\bar{I}_1| + |\bar{I}_2| \quad (3)$$

The condition of operation is given by (4), where K is a protection sensitivity parameter.

$$I_{op} \geq KI_{res} \quad I_{op} \geq I_{pickup} \quad (4)$$

The use of the pickup element is necessary to provide the protection security against false differential currents

generated, in part, by current transformers inaccuracies (Roberts et al., 2001).

### 2.1 Alpha Plane Representation

The alpha plane formulation, previously presented by Warrington (1962), is a geometrical description of the ratio of phase or sequence currents phasors through the complex plane. It has been a convenient technique to evaluate current differential characteristics, and is known that percentage differential elements can be represented into alpha plane, in such a way that both restraining and operating zones can be found by the protection specifications (Benmouyal, 2005).

The alpha plane representation applied to the cross-differential protection is given by the ratio of the reference circuit current phasor ( $\bar{I}_1$ ) and the parallel circuit current phasor ( $\bar{I}_2$ ). This formulation is presented in (5) (Neves and Silva, 2018).

$$M = \frac{\bar{I}_1}{\bar{I}_2} \quad (5)$$

It is possible to map the operating zone on alpha plane, using the first condition presented in (4) and implementing similar procedure presented in Neves (2019).

$$|\bar{I}_1| - |\bar{I}_2| \geq K(|\bar{I}_1| + |\bar{I}_2|) \quad (6)$$

Dividing (6) by  $|\bar{I}_2|$ , turns in (7).

$$\left| \frac{\bar{I}_1}{\bar{I}_2} \right| - 1 \geq K \left( \left| \frac{\bar{I}_1}{\bar{I}_2} \right| + 1 \right) \quad (7)$$

Manipulating (7), a circular restraining characteristic is obtained, with adjustments described in (8). This zone is illustrated on Figure 2.

$$Center = (0,0) \quad Radius = \frac{1+K}{1-K} \quad (8)$$

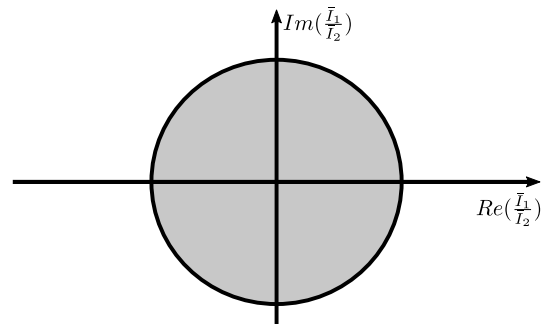


Figure 2. Restraining characteristic.

In order to represent the second condition presented in (4) on alpha plane, the coefficients are blocked on coordinates (1,0) when magnitude of  $I_{op}$  is lower than  $I_{pickup}$ . In such a way, even the reference or the parallel elements coefficients are blocked on that specific coordinates for a given terminal, in order to improve the cross-differential protection security.

## 2.2 Superimposed Currents Method

The cross-differential function can be used to enhance the protection scheme of the double-circuit line, however this element is sensitive to load conditions and difference between source capacities at each terminal. The presence of weak sources may provide similar currents magnitudes during fault condition, in such a way that the cross-differential protection could lose sensitivity (Wang et al., 2005a). To overcome these shortcomings, the superimposed currents method is applied. To do so, (3) becomes (9) and (5) becomes (10), with currents  $\Delta \bar{I}_1$  and  $\Delta \bar{I}_2$  defined in (11) (Wang et al., 2005a)(Neves and Silva, 2018).

$$I_{op} = |\Delta \bar{I}_1| - |\Delta \bar{I}_2| \quad I_{res} = |\Delta \bar{I}_1| + |\Delta \bar{I}_2| \quad (9)$$

$$M = \frac{\Delta \bar{I}_1}{\Delta \bar{I}_2} \quad (10)$$

$$\Delta \bar{I}_1 = \bar{I}_1 - \bar{I}_{1,pre-fault} \quad \Delta \bar{I}_2 = \bar{I}_2 - \bar{I}_{2,pre-fault} \quad (11)$$

where  $\bar{I}_{1,pre-fault}$  and  $\bar{I}_{2,pre-fault}$  are the pre-fault currents measured at the reference and parallel circuits, respectively.

## 2.3 Operating Modes

The cross-differential protection performance is based on two common operating modes, nominated as instantaneous and successive modes. The instantaneous operating mode, which usually occurs for close-in end and middle line faults, occurs in situations that the current configuration is capable of making the relay installed in a terminal of the line to detect the fault independently from the remote detection and subsequent circuit-breakers opening. On the other hand, the successive operating mode, commonly represented by remote end faults, occurs in situations that the relay only trips after the circuit-breakers opening at the remote terminal (Wang et al., 2005a)(Borges and Silva, 2014). This operating mode lasts more than the instantaneous mode, and is highly dependent of the circuit-breakers opening time, which usually lasts at least one-and-a-half cycle (Schweitzer et al., 2015).

## 3. METHODOLOGY

In order to evaluate the cross-differential protection under inter-circuit faults, a set of simulations were performed on a system modeled on Alternative Transients Program (ATP). The inter-circuit fault model used to evaluate the protection performance is presented as follows.

### 3.1 Inter-Circuit Fault Model

In order to model inter-circuit faults on double-circuit lines, two topologies of such faults were used, as presented in Figure 3. This representation simulates common faults that would occur, involving the transmission towers or only the phase cables (Saha et al., 2015). This representation could represent conductor galloping or even broken conductors. The fault resistance parameters adopted on simulations are related to those presented in Figure 3.

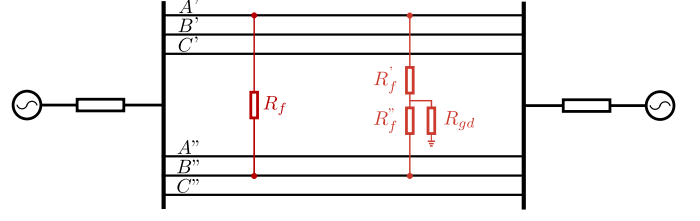


Figure 3. Fault model applied.

### 3.2 ATP Model

The system used to evaluate the cross-differential protection in ATP software is presented in Fig. 4. This system consists on a double-circuit transmission line with 500 kV of rating voltage and 300 km of extension, adjacent to two *Thevenin* equivalents. The line parameters were obtained following the geometry of the Danúbio Tower described in Saliba et al. (2013), which represents the geometry of a existing double-circuit transmission line of national interconnected system from Brazil. The line model used to represent the system consists on Clarke distributed parameters with two individually transposed 3-phase lines with mutual coupling (Leuven, 1987). The current transformer consisted on class C800 2000-5A, according to ANSI C57.13, with 0.75  $\Omega$  of secondary resistance (IEEE, 2004).

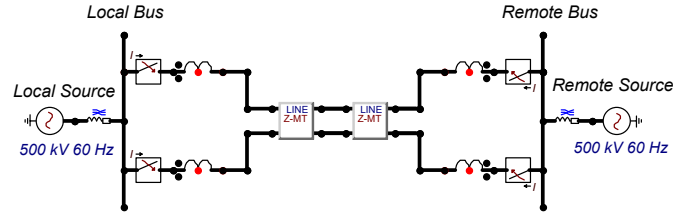


Figure 4. System modeled on ATPDraw software

## 4. PERFORMANCE EVALUATION

In order to evaluate the cross-differential protection performance under inter-circuit faults, sensitivity and transient analyses were performed. In this paper, four analyses were done. They are described as follows:

- Sensitivity analysis for location variation of a solid A'B'' inter-circuit fault;
- Sensitivity analysis for location variation of a solid A'B'' to ground inter-circuit fault;
- Sensitivity analysis for fault resistance variation of a A'B'' to ground fault applied on 75% of the line;
- Transient Analysis of solid A'B'' fault applied on 65% of the line evolutive to a solid A'B'' to ground fault;

The sensitivity analyses were obtained considering the sinusoidal steady-state phasor solution provided by ATP. This kind of analysis considers the steady-state fault quantities. The transient analysis consisted on the sampling of the currents signals filtered by a third order Butterworth filter with 180 Hz of cut-off frequency. The sampling procedure considered 16 samples per cycle of sampling rate and the phasor estimation were performed through modified cosine filter algorithm (Hart et al., 2000).

#### 4.1 Case 1 - Effect of location variation for a solid A'B'' inter-circuit fault

The first case presents the cross-differential protection performance upon the location variation for a solid A'B'' fault, according to Figure 3, applied between 0% to 100% of line length, considering steps of 0.1%.

According to alpha plane response, presented in Figure 5, it is noticed that the local element of circuit 1, presented in Figure 5a, detected the fault for locations between 0% and 95.7%, whereas the remote element of such circuit, presented in Figure 5b detected the fault for locations between 5.7% and 100%. Therefore, phase A fault was detected instantaneously in circuit 1 between 5.7% and 95.7%, in such a way that the instantaneous coverage for such fault was 90% on such circuit.

Considering now the alpha plane response for circuit 2 elements, it is noticed that the local element of such circuit, presented in Figure 5c, detected the fault for locations between 0% and 95.7%, whereas the remote element, presented in Figure 5d detected the fault for locations between 5.7% and 100%. Therefore, phase B fault was detected instantaneously in circuit 2 for faults between 5.7% and 95.7%, which results in a instantaneous coverage of 90% for such fault. Comparing with the fault detection of circuit 1, it is worthy to mention that the elements of both circuits detected the faults applied at the same locations.

According to the results aforementioned, then it can be concluded that the A'B'' fault was detected for locations between 5.7% and 95.7%, with 90% of instantaneous coverage for such fault.

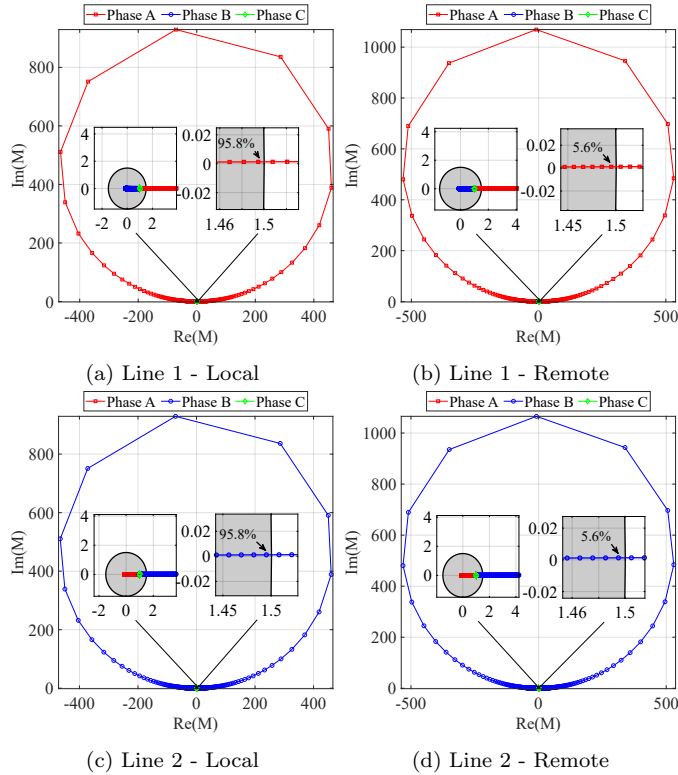


Figure 5. Case 1

#### 4.2 Case 2 - Effect of location variation for a solid A'B'' to ground inter-circuit fault

The second case presents now the cross-differential protection performance considering the location variation for a solid A'B'' to ground fault. The fault was also applied from 0% to 100% of line length, with steps of 0.1%.

Through the alpha plane response, presented in Figure 6, it is observed that the local element of circuit 1, illustrated in Figure 6a, identified the fault between 0% and 95.7%, while the remote element of that circuit, presented in Figure 6b identified the fault between 5.7% and 100%. Hence, phase A fault was instantaneously identified in circuit 1 between 5.7% and 95.7%, with a 90% instantaneous coverage.

According the alpha plane response for circuit 2 elements, it is observed that the local element of that circuit, illustrated in Figure 6c, identified the fault between 0% and 95.7%, while the remote element identified the fault between 5.7% and 100%. Hence, phase B fault was instantaneously identified in circuit 2 between 5.7% and 95.7%, with a 90% instantaneous coverage.

Therefore, the A'B'' to ground fault was identified between 5.7% and 95.7%, with 90% of instantaneous coverage for such fault, the same coverage from the previous case.

However, by the analysis of the alpha plane coefficients, it was seen that for cases 1 and 2, the coefficients tended to have higher magnitude for faults applied near to the half of the line. Furthermore, it was noticed that this behavior was caused mainly due to the reduction of healthy circuit fault contribution, as a result of the reduced values of superimposed currents, which is influenced by fault contribution at each side of the system.

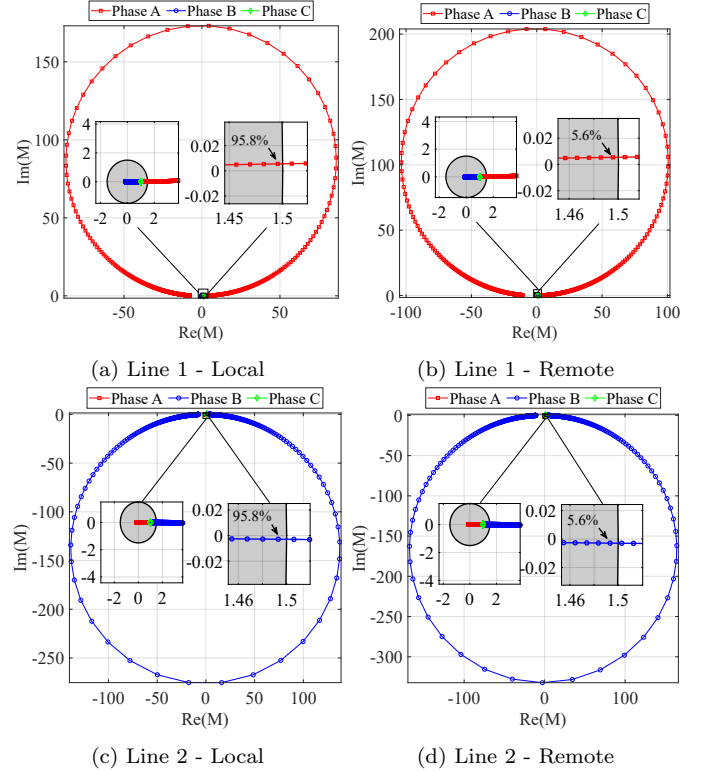


Figure 6. Case 2

#### 4.3 Case 3 - Effect of fault resistance variation for a A'B'' to ground fault applied on 75% of the line

The third case presents the cross-differential protection performance considering the fault resistance variation for an A'B'' to ground fault applied on 75% of line length. Aiming to do so, the three fault resistances, presented in Fig. 2, were varied independently, from 0  $\Omega$  to 1000  $\Omega$ , maintaining the remaining as 0  $\Omega$ .

According to alpha plane response, presented in Figure 7, it's noteworthy to point out that the circuit 1 elements, presented in Figures 7a and 7b, detected the the fault on phase A for all resistance fault values simulated, whereas the circuit 2 elements, presented in Figures 6c and 6d, detected phase B faults for all values simulated.

A noticeable aspect to be considered is that, through the analysis of the results presented in Figure 7, it is possible to identify that the elements of each circuit are more affected by the variation of resistances directly connected to each one, in such a way that the circuit 1 elements are more affected by the variation of  $R_f'$ , whereas circuit 2 elements are more affected by the variation of  $R_f$ .

As expected, healthy phase elements did not detect the fault, since their coefficients remained at restraining zone, however, the alpha plane coefficients for faulty phases on the healthy circuits approximated to the point (0,0), since the superimposed currents on the faulty circuit are higher in magnitude. Hence, it is noticeable that the cross-differential protection was slightly affected for fault resistance variation. This behavior is evident due to the use of superimposed currents method, which usually enhances the protection sensitivity against fault resistance.

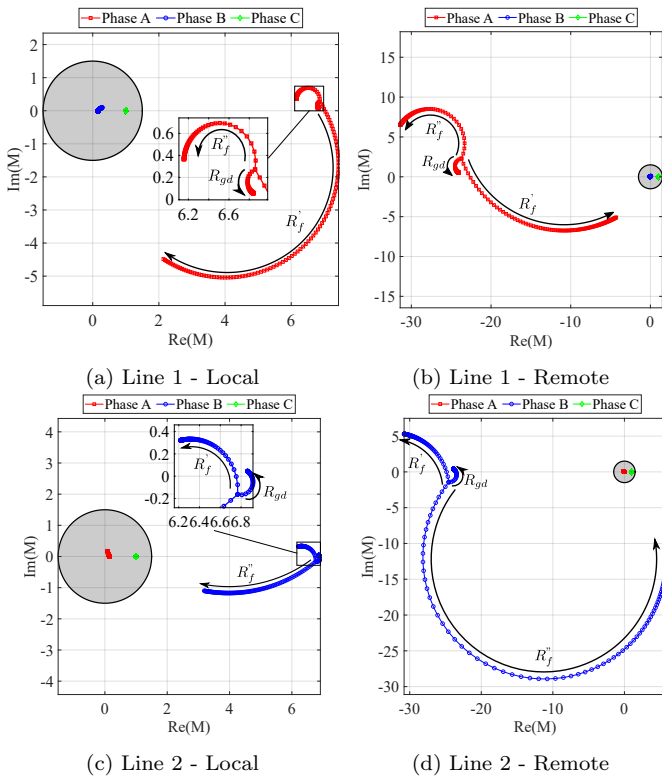


Figure 7. Case 3

#### 4.4 Case 4 - Transient Analysis of solid A'B'' fault applied on 65% of the line evolutive to a solid A'B'' to ground fault

The fourth case presents a transient analysis of the cross-differential protection. In order to visualize the alpha plane behavior, the system was submitted to a solid A'B'' fault applied on 65% of line length, which evolved after 50 ms into a solid A'B'' to ground fault applied at the same location.

According to alpha plane response, presented in Figure 8, it is notable that all elements detected the fault correctly, in such a way that the protection could operate on the instantaneous operating mode for the first fault applied. After the appliance of the solid A'B'' to ground fault, the coefficients move from the region 1, correspondent to solid A'B'' fault steady-state coefficients, to the region 2, correspondent to solid A'B'' to ground fault steady-state coefficients.

Considering the real scenario, where the circuit-breakers would extinguish fault arcing current between on-and-a-half cycle and three cycles after the trip (Schweitzer et al., 2015), the behavior of cross-differential protection under the A'B'' to ground fault appliance would not be identified in that case. However, it is convenient to demonstrate this response in order to analyze the alpha plane trajectories for such fault. Indeed, the protection could perform a secure element for such fault applied on the considered line, in such a way that this topology of protection function could be used to enhance the protection scheme.

Therefore, through this case, it was possible to visualize the alpha plane trajectories under an evolutive inter-circuit fault.

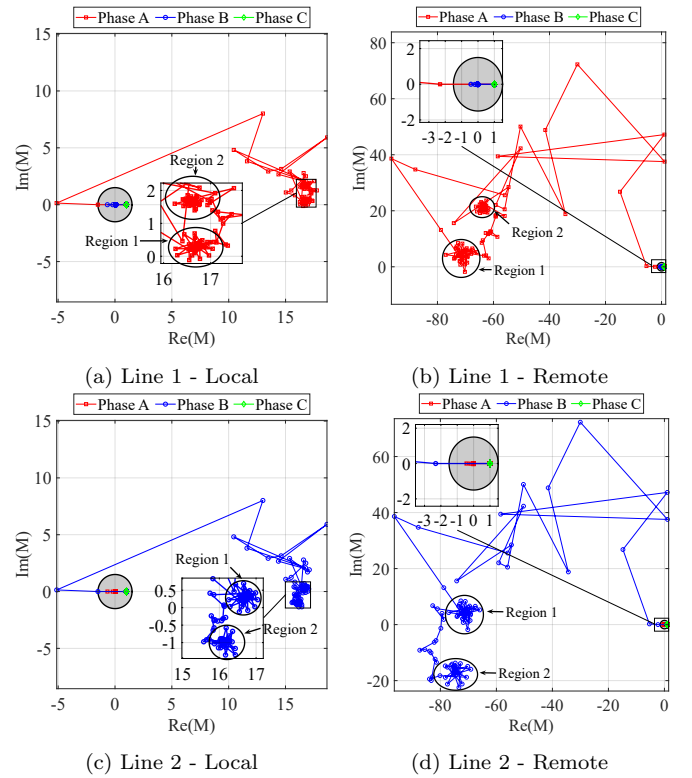


Figure 8. Case 4

## 5. CONCLUSIONS

This paper presented the performance evaluation of cross-differential protection applied to double-circuit transmission lines under inter-circuit faults. In order to do so, simulations of a 500 kV system with 300 km of extension were performed in ATP software. Sensitivity analyses of location and fault resistance variations were performed, considering the fault steady-state phasors. Also, a transient analysis was performed, in order to visualize the alpha plane trajectories of such faults.

According to the results of the location variation analysis, presented in 4.1 and 4.2, it is possible to visualize the protection behavior upon this kind of variation, and it is also possible to identify the instantaneous coverage for the inter-circuit faults applied, which maintained in 90% of the line length. Considering distance elements, which are usually designed for covering about 80% of each circuit length on first zone, therefore protecting each circuit with 60% of simultaneous first zone (Anderson, 1998), the cross-differential protection could provide a higher coverage.

Through results of the fault resistance variation analysis, presented in 4.3, it was possible to see the protection sensitivity related to fault resistance. In the simulated case, the cross-differential protection identified the fault for all fault resistances considered, thanks to the use of the superimposed currents method. As distance elements are subject to underreaching caused by fault resistance, according to Benmouyal et al. (2017), the cross-differential protection could be useful to enhance the protection scheme.

The last case, presented in 4.4, presented a transient analysis of a evolutive inter-circuit fault. Through the alpha plane response, it was seen how the protection behaved when submitted to an evolutive inter-circuit fault.

Hence, the cross-differential protection provided a great performance for the inter-circuit faults applied. Moreover, as a one-terminal based function immune to zero-sequence mutual coupling effect, it comes up as a great solution to reinforce the protection schemes of double-circuit lines.

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