AN EFFICIENT SENSITIVITY ANALYSIS BASED METHOD FOR CALCULATING LOAD MARGINS TO VOLTAGE COLLAPSE

Adriana L. Tognete
adriana@densis.fee.unicamp.br

Carlos A. Castro
ccastro@dsee.fee.unicamp.br

UNICAMP/FEEC/DSEE
C.P. 6101
13081-970 Campinas SP

ABSTRACT – In this paper an efficient method for calculating load margins to voltage collapse is proposed. The method consists of computing gradual load increases based on sensitivity analysis. It is well suited for large systems, since heuristics are added to the basic algorithm in order to explore their particular characteristics. Those heuristics result in significant computational time savings. Also a special power flow method with step size optimization is used when either (a) systems become ill-conditioned (near collapsing) or (b) no real solution is found after some load increase. In case no real solution is found, the results provided by the special power flow can be used in order to define control actions (load shedding) to pull the system back into the feasible operating region. Simulations have shown that the method is very efficient, accurate and robust.

1 INTRODUCTION

Voltage stability has now been considered as a very important aspect of power system planning and operation analysis. It has been receiving special attention in the last decade. Due to many reasons, including economical issues, modern power systems tend to operate close to their limits. Voltage collapse has been proving to be a limiting factor to the operation of those systems, and became one of the main topics among researchers. Many contributions have been given with the goal of increasing the knowledge about the phenomenon of voltage collapse. A large number of research works trying to explain the voltage collapse phenomenon and its mechanisms have been published. Also analysis of real incidents as well as the importance of representing systems components have been discussed. Basically those works approach the problem from either static or dynamic standpoints. These approaches are related to the model adopted for the system.

Dynamic approaches present more accurate results. Basically they can be based either on non-linear analysis, being bifurcation theory (Chiang *et alii*, 1990) one of the most important ones, or small signal analysis (Rajagopalan *et alii*, 1992). Many papers focused on the importance of an appropriate load modelling (Sekine and Ohtsuki, 1990). Static approaches are computationally more efficient. Moreover, they are well suited for studies of the determination of stability limits. They can be very important tools in the detection and prevention of voltage collapse, especially in real time operation environments (as in modern energy management systems), where time constraints are very strict. Voltage stability studies based on static approaches use the power flow equations or some modified version of them. Static approaches can be based either on sensitivity studies (Gao *et alii*, 1991) or in power flow solutions (Sekine *et alii*, 1989). They are appropriate for the computation of:

- proximity indices, which are scalars that quantify the distance from the current operating point to a critical point where voltage collapse occurs, even though they may not have any physical meaning. Several different indices have been proposed in the literature. In Löf *et alii* (1992) a proximity index based on the minimum singular value of the Jacobian matrix was presented. The method of (Löf *et alii*, 1992) was further improved by Barquín *et alii* (1995);
- load margins to voltage collapse, which are distances from the current operating point to voltage collapse given in terms of system parameters, like MW or MVAr. In Alvarado *et alii* (1994) a method for the computation of the closest bifurcation point was presented. The direction of load increase was adjusted based on the left eigenvector associated to the zero eigenvalue of the Jacobian matrix. In Flatabø *et alii* (1990) and Flatabø *et alii* (1993) the load margin to voltage collapse was obtained for a predefined load increase direction. An iterative method was proposed in which load increases were defined based on sensitivity analysis. The maximum loading of a system can also be computated by the continuation power flow (Ajjarapu *et alii*, 1992). By using this method the ill-conditioning of the Jacobian matrix near the critical point is avoided. The continuation power flow, which is a predictor-corrector method based on the conventional Newton power flow, has

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two drawbacks: (a) major modifications in the conventional Newton power flow must be performed, and (b) its Jacobian matrix ends up being different from the one of the conventional power flow. Another approach to computing load margins is by using optimization methods. More recently interior point methods have been proposed (Irisarri *et alii*, 1997). Even though optimization methods are very powerful and accurate, they are still recognized as impractical from the standpoint of real time aplications.

This paper presents an efficient method for determining load margins to voltage collapse according to a static approach. Load margins are distances in the load parameter space from the system's current operating point to the point of voltage collapse. This distance is obtained by gradual load increases determined by sensitivity analysis. The method represents an improved version of the method proposed by Flatabø *et alii* (1990), and Flatabø *et alii* (1993). Load increases take into account their localized effects over generators' reactive power limits. Also some heuristics based on the knowledge of the systems' behavior were used. Finally, a new power flow method using step size optimization (Castro and Braz, 1997a; Castro and Braz, 1997b) was used in order to improve convergence characteristics and to provide important information in the case the system has no feasible operating point after some load increase.

Section 2 presents some fundamentals of sensitivity analysis. Existing load margin calculation methods are presented in 3 along with a discussion of some of their problems. In section 4 the proposed method is presented and discussed. The simulation results are presented in section 5. The conclusions are shown in section 6.

2 SOME FUNDAMENTALS OF SENSITIVITY ANALYSIS

In this section some fundamentals of sensitivity analysis necessary to the understanding of the proposed method will be briefly presented. Most of it has been already shown in (Flatabø *et alii*, 1990; Flatabø *et alii*, 1993), where more details can be obtained. The non-linear set of power flow equations for a n_b-bus system can be written as:

$$\mathbf{g}\left(\mathbf{x},\mathbf{u},\mathbf{p}\right)=\mathbf{0} \tag{1}$$

where $\mathbf{g}$ is the $(2n_b \times 1)$ vector of real and reactive power mismatches, $\mathbf{x}$ is the $(2n_b \times 1)$ vector of state variables (voltage magnitudes and phase angles), $\mathbf{u}$ is the $(n_u \times 1)$ vector of control variables (voltage magnitudes at generation buses, real power generation, reactive compensation) and $\mathbf{p}$ is the $(n_p \times 1)$ vector of parameters (real and reactive load powers). It is assumed that $\mathbf{x}$ is a $(2n_b \times 1)$ vector for the sake of simplicity only. Of course some elements of $\mathbf{x}$ are not unknowns, but constitute inicial data for the problem. For instance, voltage magnitude and phase angle are previously defined for the slack bus. As for generation buses, the voltage magnitude is also defined previously. Though these aspects alter the set of power flow equations (1), they do not affect the basic ideas of the method. The same considerations are also valid for the power mismatch equations.

Expanding (1) in Taylor series around the current operating point and assuming that this point is feasible, one gets:

$$\Delta\mathbf{g}=\mathrm{G}_x\cdot\Delta\mathbf{x}+\mathrm{G}_u\cdot\Delta\mathbf{u}+\mathrm{G}_p\cdot\Delta\mathbf{p}=\mathbf{0} \tag{2}$$

where $\mathrm{G}_x=\partial\mathbf{g}/\partial\mathbf{x}$ is the $(2n_b \times 2n_b)$ Jacobian matrix and $\Delta\mathbf{x}$ is the vector of changes in the state variables. The definition of the other terms of (2) is straightforward. From (2), changes in the state variables due to changes either in control variables or in the parameters are given by:

$$\begin{aligned}\Delta\mathbf{x} &= -\mathrm{G}_x^{-1}\mathrm{G}_u\cdot\Delta\mathbf{u}-\mathrm{G}_x^{-1}\mathrm{G}_p\cdot\Delta\mathbf{p}\\ &= \mathrm{S}_{xu}\cdot\Delta\mathbf{u}+\mathrm{S}_{xp}\cdot\Delta\mathbf{p}\end{aligned} \tag{3}$$

where S_{xu} $(2n_b \times n_u)$ and S_{xp} $(2n_b \times n_p)$ are sensitivity matrices of the state variables to control variables and parameters, respectively. For load changes ($\Delta\mathbf{p}$) the state variables can be updated by:

$$\Delta\mathbf{x}=\mathrm{S}_{xp}\cdot\Delta\mathbf{p} \tag{4}$$

Dependent variables, such as reactive powers at generation buses and real power generation at the slack bus, can be written as:

$$\mathbf{w}=\mathbf{w}\left(\mathbf{x},\mathbf{u},\mathbf{p}\right) \tag{5}$$

Expanding (5) in Taylor series similarly to what was done for (1):

$$\Delta\mathbf{w}=\mathrm{W}_x\cdot\Delta\mathbf{x}+\mathrm{W}_u\cdot\Delta\mathbf{u}+\mathrm{W}_p\cdot\Delta\mathbf{p} \tag{6}$$

Substituting (3) in (6) and considering that $\mathrm{W}_p=0$:

$$\begin{aligned}\Delta\mathbf{w} &= \left(\mathrm{W}_x\mathrm{S}_{xu}+\mathrm{W}_u\right)\cdot\Delta\mathbf{u}+\left(\mathrm{W}_x\mathrm{S}_{xp}\right)\cdot\Delta\mathbf{p}\\ &= \mathrm{S}_{wu}\cdot\Delta\mathbf{u}+\mathrm{S}_{wp}\cdot\Delta\mathbf{p}\end{aligned} \tag{7}$$

where S_{wu} $(n_w \times n_u)$ and S_{wp} $(n_w \times n_p)$ are sensitivity matrices of the dependent variables to control variables and parameters, respectively. For instance, by using (7) it is possible to compute the change in the reactive power generated at a generation bus ($\Delta\mathbf{w}$) due to a load change at a certain load bus ($\Delta\mathbf{p}$), given the sensitivity matrix S_{wp}. This would be done by:

$$\Delta\mathbf{w}=\mathrm{S}_{wp}\cdot\Delta\mathbf{p} \tag{8}$$

3 LOAD MARGIN CALCULATION METHODS

Most load margin calculation methods are based on gradual load increases till the system collapses (Alvarado *et alii*, 1994; Flatabø *et alii*, 1990; Flatabø *et alii*, 1993). In this paper it is considered that the critical point (point of collapse) is the point where real solutions of the power flow equations (feasible operating points) are no longer found. This point is usually identified as the point of saddle-node bifurcation and the Newton's method Jacobian matrix is singular. In this section the idea of operating regions and a discussion of existing load margin calculation methods are presented.

3.1 Operating regions and load margins

The multidimensional parameter (load) space can be divided into three operating regions. Figure 1 (Overbye, 1993) shows a two-dimensional parameter space.

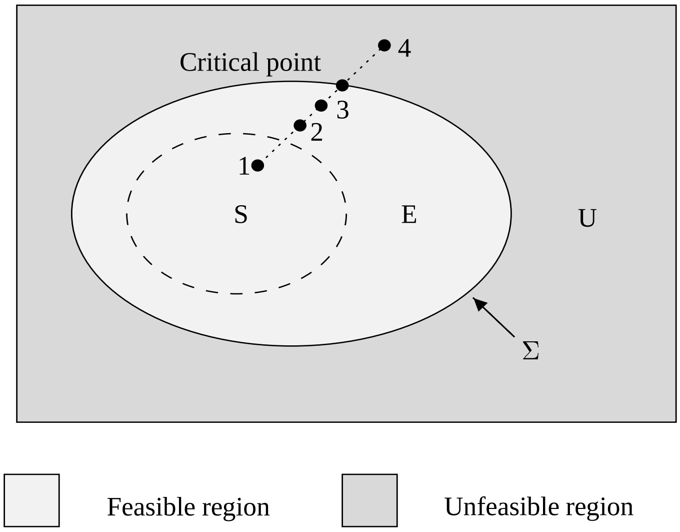

Figure 1: Multidimensional parameter space.

The two parameters represented in figure 1 could be, for instance, the real and reactive load powers at a bus supplied by a generation bus (operating as a slack bus) through a transmission line. Each point in the parameter space corresponds to one operating state. The *feasible region* comprises the points in parameter space where system operation is possible, whereas in the *unfeasible region* (U) operation is not possible. There is no power flow solution (feasible operating point) for points in the unfeasible region. These two regions are separated by a boundary (Σ). Within the feasible region the system can operate either in a *secure* (S) way, that is, there are no violations of any of its operating variables, or in an *emergency* (E) way, that is, some limits are violated. Operation in the secure region is always preferred. In many cases though the system can stand emergency operation for some time, till corrective measures are taken to pull it back to secure operation.

The load margin calculation methods assume that both an initial feasible operating point (point 1 of figure 1) and a direction of load increase (dotted line in figure 1) are known. The idea is to follow the load increase direction till a point on Σ is found (critical point of figure 1). The load margin is the distance in parameter space between the initial point and the critical point on Σ.

3.2 Existing load margin calculation methods (Flatabø *et alii*, 1990; Flatabø *et alii*, 1993)

The main idea of those methods is to gradually increase the load till the operating point reaches the boundary Σ. Beyond Σ, the power flow equations have no real solutions and the system is considered to have collapsed.

The load increase at each iteration is defined in principle as the smallest one that will result in some generator reaching its reactive power generation limit. The load increase can be obtained by sensitivity analysis, in particular by equation (8). The relationship between a load increase ΔS_i at a load bus i and changes in the reactive power ΔQ_j at a generation bus j is:

$$\Delta Q_j = \sigma \cdot \Delta S_i \tag{9}$$

where σ is a sensitivity factor. In this paper load margins will be computed only for (a) real power (MW) change at one bus, (b) reactive power (MVAr) change at one bus, and (c) complex power change (MVA) at one bus keeping a constant power factor. The method is also applicable to more general load changes, involving any power factors and any number of buses.

In case the load increase is defined as real power, $\sigma = \mathrm{S}_{wp}(j_q, i_p)$ and $\Delta S_i = \Delta P_i$. For reactive power increase $\sigma = \mathrm{S}_{wp}(j_q, i_q)$ and $\Delta S_i = \Delta Q_i$. In those cases, ΔS_i is an element of the $(2n_b \times 1)$ singleton vector $\mathbf{\Delta S}$. For load increase with constant power factor (pf_i), $\sigma = \mathrm{S}_{wp}(j_q, i_p) \cdot pf_i + \mathrm{S}_{wp}(j_q, i_q) \cdot \left(1 - pf_i^2\right)^{1/2}$ and $\Delta S_i = \left(\Delta P_i^2 + \Delta Q_i^2\right)^{1/2}$. j_q corresponds to the row of the sensitivity matrix related to reactive power of bus j. i_p and i_q have similar meanings. The necessary load change at bus i so that the reactive power limit of generator j is reached is given by:

$$\Delta S_i = \frac{\Delta Q_j^{lim}}{\sigma} = \frac{Q_j^{lim} - Q_j}{\sigma} \tag{10}$$

where Q_j is the reactive power currently being generated at bus j. Superscript $lim = max$ for $\sigma > 0$ and $lim = min$ for $\sigma < 0$. For a system with n_g generators, n_g values of ΔS_i may be computed. In (Flatabø *et alii*, 1990; Flatabø *et alii*, 1993) the amount of load increase at an iteration ν is obtained by:

$$\Delta S_i^\nu = \min_j \left[\left(Q_j^{lim} - Q_j\right)/\sigma\right] \qquad \text{for} \qquad j = 1 \ldots n_g' \tag{11}$$

where n_g' is the number of available generators at iteration ν. At the first iteration $n_g' = n_g$. In the following iterations n_g' may assume different values, since in previous iterations some generators may had reached their reactive power limits. After a generator has reached its limit, it is regarded as a load, that is, voltage magnitude will vary and a reactive power injection Q_j^{lim} is assumed at the corresponding bus. The load increase ΔS_i^ν is defined as the smallest change in load so that one generator reaches its reactive power generation limit. A basic algorithm for the calculation of the load margin at bus i is (Flatabø *et alii*, 1990; Flatabø *et alii*, 1993):

(1) Initialize iteration counter $\nu = 0$; initialize load margin $LM = 0$;

(2) Compute the necessary elements of sensitivity matrices S_{xp} and S_{wp};

(3) Obtain ΔS_i^ν according to (11);

(4) Update load margin $LM = LM + \Delta S_i^\nu$;

(5) Compute the new system's operating point for the new loading condition. In Flatabø *et alii* (1990) sensitivity equations (4) and (8) are used. In Flatabø *et alii* (1993) power flow calculations are performed;

(6) If the system is still stable, increment the iteration counter $\nu = \nu + 1$ and go to step (2). If the system becomes unstable, consider the load margin equal to LM and stop. In Flatabø *et alii* (1990) stability is evaluated by checking the following elements of matrix S_{xp}:

$$vs_i = \frac{d}{dQ_i} V_i \qquad i = 1 \ldots n_b \tag{12}$$

The system is considered unstable in case any vs_i changes its sign. In Flatabø *et alii* (1993) the system is considered unstable in case the power flow equations no longer have a real solution.

A number of problems related to the basic algorithm above have been identified:

(1) the exact load margin in fact is given by $LM + \varepsilon$, where $0 \leq \varepsilon \leq \Delta S_i^\nu$. However, it is very difficult to evaluate ε using the basic algorithm, since it is found that the system is unstable *after* the power flow has failed. Once that happened, no useful information can be obtained from the power flow output. In addition, ΔS_i^ν may be large in some cases (that will be discussed later), leading to a significant error in assuming LM as the load margin;

(2) in (Flatabø *et alii*, 1990) voltages and dependent variables updates after a load increase are done by using linearized equations, namely equations (4) and (8). These updates have shown to provide very poor results from the accuracy standpoint, since they are valid for small changes around the current operating point. The basic method may define large amounts of load increase at each iteration, depending on the sensitivities and the reactive power availability $\left(Q_j^{lim} - Q_j\right)$ at generation buses;

(3) in (Flatabø *et alii*, 1993) power flows are run after load increases for the evaluation of new operating points. However, load increases may be defined such that the resulting operating point is located at the unfeasible region (point beyond boundary Σ). This means that the load increase for voltage collapse is smaller than the one defined by the method. In these cases, conventional power flow methods diverge and do not provide any useful information about the unfeasible operating point, as for example the distance from this point to the boundary Σ;

(4) the power flow may diverge before any generator reaches its reactive power limits. This situation may happen for already heavily loaded systems when the first load increase defined leads the system to the unfeasible reagion. This situation falls into case (3);

(5) generators may have large reactive power availabilities, which results in large load increases (see equation (10)). This situation may fall into case (3);

(6) generators may have very small sensitivities of their generated reactive power with respect to load increases at a certain load bus. This situation is likely to occur for large systems with load buses electrically far away from generation buses. Small sensitivities result in large load increases (see equation (10)) and, again, this situation may fall into case (3). Moreover, for realistic systems voltage collapse may occur well before the limits of these generators are reached (other generators may have been chosen before, according to equation (11)). This means that at each iteration computational time may be wasted with such calculations;

(7) if the system is still stable after all generators have reached their power limits but the slack bus, then a load increase must be defined so that its real or reactive power limit (whichever is smaller) be reached. Due to the slack bus own characteristics, this load increase may be too large making the linearized voltage update meaningless or the power flow diverge. In this case, there is no way to evaluate how far this unfeasible operating point is from Σ. This situation is rather rare for realistic systems, tough it has happened for some test systems.

4 PROPOSED METHOD

In order to overcome the difficulties discussed in the previous section, a new efficient method for load margin calculation is proposed. Basically, a special Newton's power flow with step size optimization is used and some heuristics are added to the basic algorithm. The basic aspects of both improvements will be discussed below.

4.1 Newton's power flow with step size optimization

Step size optimization has long been recognized as an efficient way of improving power flow convergence characteristics and handling a number of difficult situations found in practice, such as ill-conditioned systems or unfeasible operating points. In (Iwamoto and Tamura, 1981) a very efficient power flow method using step size optimization was proposed. Its main disadvantage was that voltages were represented in rectangular coordinates, which is uncommon is production grade power flow programs. Scudder and Alvarado (1981) proposed an alternative method based on (Iwamoto and Tamura, 1981) for voltages in polar coordinates, in an attempt to adapt the method to practical power flow programs. However, the method presented a poorer performance if compared to Iwamoto and Tamura's. Dehnel and Dommel (1989) have also proposed a step size optimization method, however, their results were not encouraging either, as far as performance was concerned. Castro and Braz (1997a,1997b) proposed a new version of a step size optimization method based on Iwamoto and Tamura's ideas with voltages in polar coordinates. The method performed very well and showed to be both efficient and robust, being able to handle the most different and difficult practical situations. Among those, there are situations in which the operating point falls into the unfeasible region. This is of special interest for voltage stability analysis methods. The details of the method used in this paper are presented in (Castro and Braz, 1997a; Castro and Braz, 1997b) and are briefly described here, for the sake of completeness. The idea is to update voltages at iteration ζ by:

$$\begin{aligned} \mathbf{x}^{\zeta} &= \mathbf{x}^{\zeta-1} + \mu \Delta \mathbf{x}^{\zeta-1} \\ &= \mathbf{x}^{\zeta-1} + \mu \mathrm{J}^{-1} \Delta \mathbf{s}\left(\mathbf{x}^{\zeta-1}\right) \end{aligned} \tag{13}$$

where $\mathrm{J} = \mathrm{G}_x$ is the Jacobian matrix, $\Delta \mathbf{s}\left(\mathbf{x}^{\zeta-1}\right)$ is the vector of real and reactive power mismatches and μ is a scalar multiplier computed so as to minimize the following quadratic function:

$$\begin{aligned} F\left(\mathbf{x}^{\zeta}\right) &= \frac{1}{2}\left[\Delta \mathbf{s}\left(\mathbf{x}^{\zeta}\right)\right]^T \cdot \left[\Delta \mathbf{s}\left(\mathbf{x}^{\zeta}\right)\right] \\ &= \frac{1}{2}\left[\Delta \mathbf{s}\left(\mathbf{x}^{\zeta-1} + \mu \Delta \mathbf{x}^{\zeta-1}\right)\right]^T \cdot \\ &\quad \left[\Delta \mathbf{s}\left(\mathbf{x}^{\zeta-1} + \mu \Delta \mathbf{x}^{\zeta-1}\right)\right] \end{aligned} \tag{14}$$

The solution of the power flow equations is regarded as a nonlinear programming problem. It is shown (Castro and Braz, 1997a; Castro and Braz, 1997b) that μ can be obtained by solving the following cubic equation:

$$g_0 + g_1 \mu + g_2 \mu^2 + g_3 \mu^3 = 0 \tag{15}$$

where the coefficients g_i are:

$$\begin{aligned} g_0 &= \sum_{i=1}^{2n_b} (a_i b_i) \qquad & g_2 &= 3 \sum_{i=1}^{2n_b} (b_i c_i) \\ g_1 &= \sum_{i=1}^{2n_b} \left(b_i^2 + 2 a_i c_i\right) \qquad & g_3 &= 2 \sum_{i=1}^{2n_b} \left(c_i^2\right) \end{aligned} \tag{16}$$

It is also shown that vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are defined as:

$$\begin{aligned} \mathbf{a} &= -\mathbf{b} = \Delta \mathbf{s}\left(\mathbf{x}^{\zeta-1}\right) \qquad (17) \\ \mathbf{c} &= \frac{1}{2} \cdot \left[\sum_{m \in \mathcal{K}} \Delta x_m \cdot \frac{\partial}{\partial x_m}\right]^2 \cdot \Delta \mathbf{s}\left(\mathbf{x}^{\zeta-1}\right) \end{aligned}$$

where $\mathcal{K}$ is the set of buses directly connected to some bus i plus bus i itself. Vector $\mathbf{c}$ corresponds to the second order term of the expansion of the power flow equations in Taylor's series.

The basic iterative process of the special Newton's power flow with step size optimization follows.

(i) Initialize iteration counter $j = 0$.

Choose voltage initial guess $\mathbf{x}^j$.

(ii) Compute power mismatches $\Delta \mathbf{s}\left(\mathbf{x}^j\right)$.

Obtain vectors $\mathbf{a}$ and $\mathbf{b}$.

(iii) Compute voltage correction vector:

$$\Delta \mathbf{x}^j = \left(\mathbf{J}^j\right)^{-1} \Delta \mathbf{s}\left(\mathbf{x}^j\right)$$

(iv) Compute vector $\mathbf{c}$.

(v) Compute coefficients g_0, g_1, g_2 and g_3.

(vi) Find μ.

(vii) Compute new voltages:

$$\mathbf{x}^{j+1} = \mathbf{x}^j + \mu \Delta \mathbf{x}^j$$

(viii) Increment iteration counter $j = j + 1$ and go to step (ii).

In case the system is well-conditioned, μ assumes values close to one. For systems with no feasible operating point μ tends to assume very low values (theoretically μ tends to zero), indicating that the current voltage vector cannot be changed further in order to minimize function F (14). In case the load increase leads to an operating point in the unfeasible region, the results provided by the power flow with step size optimization provides useful information regarding the distance from the unfeasible point to the boundary Σ in parameter space. Moreover, if the load increase is in one bus only, it can be verified that the final power mismatch at that bus corresponds approximately to the distance from the operating point to Σ along the load increase direction (Castro and Braz, 1997a; Castro and Braz, 1997b). This feature suggests a way of moving back from an unfeasible operating point to Σ taking into account the load increase direction previously chosen. In this paper, if an unfeasible operating point is reached, the largest power mismatch is taken (usually it is the mismatch of the bus whose load margin is to be calculated) and subtracted from the load increase defined by sensitivity analysis. Then a new power flow has to be solved for checking whether a point in Σ has been reached. The process of decreasing the load is repeated until Σ is reached.

Therefore, the distance from an initial operating point to the final point on Σ can be easily evaluated and it is in fact the load margin to voltage collapse. This feature plays a very important role as far as the efficiency of the method is concerned. Of course, if the load increase involve multiple buses, more elaborated methods to move back to Σ should be used (Overbye, 1994). It should be pointed out, however, that in (Overbye, 1994) the movement from the unfeasible point towards Σ is performed such as to go through the smallest distance in load space, whereas in the proposed method this movement takes into account the load increase direction previously chosen (such as MW, MVAr or MVA with constant power factor).

4.2 Heuristics

The main goal of the addition of heuristics to the basic algorithm is to make use of the particular characteristics of power systems to increase the overall algorithm's efficiency. The heuristics implemented are discussed below.

- A threshold for the sensitivities of reactive power at generation buses with respect to load power at load buses was defined. So a generator j is considered as a candidate for further analysis only in case its sensitivity to the load bus i ($\mathrm{S}_{wp}\,(j_q, i)$) is greater than a pre-defined value. As a result, n_g' of equation (11) is no longer regarded as the number of available generators but only those with sensitivities greater than the threshold. A threshold of 0.1 was found to be a reasonable value, though it is in fact dependent on the characteristics of each system.
- Rather than computing the sensitivities of all generators in the system, a search for generators electrically close to the load bus is performed. This is done based on the idea that generators located electrically close to the load bus will have larger sensitivities. Starting at the load bus, a tier counter is initialized and a tier by tier search for neighbour buses is performed. Whenever a neighbour bus that contains a generator is found at a certain tier and its sensitivity is large (larger than the threshold), this generator is flagged, the tier counter is reset, and the search continues. If no generators with large sensitivities are found after a predefined number of tiers (in this paper this number was 4 and it will be hereafter referred to as *tiermax*), the search is interrupted, since chances are that all other generators farther away from the load bus have even smaller sensitivities. In order for the algorithm to work, at least one generator must be selected. In case no generators are selected at all after *tiermax* tiers, two situations may have occurred: (a) no generators were found after *tiermax* tiers were searched for, or (b) generators were actually found, however their sensitivities were below the threshold. In the first case, *tiermax* is increased and the search process is started over. In the second case, the threshold is decreased and the process is also started over. Only the flagged generators are considered for the computation of ΔS_i^{ν} according to equation (11) at step (3) of the basic algorithm. This heuristic significantly improves the efficiency of the method in terms of computational time savings, specially for large systems, since a number of sensitivities are not computed.
- If the sensitivity is too small or the reactive power availability is too large, the load increase ΔS_i^{ν} may be too large. Then the operating point may lie too far away from the boundary Σ deep into the unfeasible region, causing the final mismatch of the power flow with step optimization to be as large as the load increase. In this case, the idea of reducing the load by the same amount as the mismatch (section 4.1) may fail. Therefore, very large load increases must be avoided. So, in case a generator falls into this category (small sensitivity or large availability), the load increase is not computed by (10) but rather by:

$$\Delta S_i = \frac{\Delta Q_j}{\sigma} = \pm \frac{\mid (\kappa - 1) \cdot Q_j \mid}{\sigma} \qquad (18)$$

where the plus sign is used when $\sigma > 0$ and the minus sign is used when $\sigma < 0$. In this paper, $\kappa = 2$ was used and good results were obtained. The smaller κ, the larger the number of iterations in order to reach Σ. However, κ cannot be too large so it becomes uneffective in solving the problem of large load increases.

5 TEST RESULTS

The proposed method has been tested for several power systems. Simulation results will be shown for four systems, namely (a) system I, with 118 buses and 179 branches, (b) system II, with 662 buses and 1017 branches, (c) system III, with 904 buses and 1283 branches, and (d) system IV, with 14 buses and 20 branches (Freris and Sasson, 1968). Table 1 shows the performance of the proposed method for system I as far as accuracy is concerned.

Table 1: System I – accuracy evaluation.

Bus		3	45	75	118
Power factor		0.97	0.92	0.97	0.91
ΔP	(MW)	506	320	693	436
ΔP^{PF}	(MW)	506	321	693	436
V_{cr}	(pu)	0.758	0.640	0.650	0.630
V_{cr}^{PF}	(pu)	0.758	0.676	0.650	0.629
ΔQ	(MVAr)	363	242	528	322
ΔQ^{PF}	(MVAr)	363	242	528	322
V_{cr}	(pu)	0.545	0.565	0.543	0.521
V_{cr}^{PF}	(pu)	0.545	0.564	0.542	0.522
ΔS	(MVA)	439	255	601	336
ΔS^{PF}	(MVA)	441	257	600	344
V_{cr}	(pu)	0.702	0.658	0.615	0.640
V_{cr}^{PF}	(pu)	0.692	0.614	0.630	0.580

Load margins for MW (ΔP), MVAr (ΔQ), and MVA (ΔS) power changes computed by the proposed method are shown, as well as the voltage magnitudes at the critical (collapse) point (V_{cr}). The power factors of the MVA changes are shown in the table. They correspond to the base case power factors of the loads. ΔP^{PF}, ΔQ^{PF}, and ΔS^{PF} are the MW, MVAr, and MVA load margins obtained by changing the system's data base and performing power flow calculations on a trial and error basis. V_{cr}^{PF} are the respective bus voltage magnitudes after the power flow is performed. The table shows that the results obtained are accurate.

Table 2 shows the results for system II. Once again the results show that the proposed method is accurate. For both tables 1 and 2 a sensitivity threshold of 0.1 and a *tiermax* of 4 were used. Figure 2 shows the iteration by iteration load increases for bus 3 of system I.

The bullets indicate the final maximum loading. The load margin is calculated by the difference between maximum and base case loadings. The dashed lines show the maximum loadings obtained by manually changing the bus load in the system's data base and performing power flow calculations, as it was discussed earlier. Figure 2 also shows that the power flow with step size optimization is used when a load increase defined in one itera-

Table 2: System II – accuracy evaluation.

Bus	67	293	449	884
Power factor	0.99	0.98	0.90	0.98
ΔP (MW)	123	90	720	1780
ΔP^{PF} (MW)	125	91	721	1788
V_{cr} (pu)	0.664	0.631	0.736	0.760
V_{cr}^{PF} (pu)	0.627	0.640	0.732	0.759
ΔQ (MVAr)	100	75	379	1158
ΔQ^{PF} (MVAr)	106	77	380	1166
V_{cr} (pu)	0.590	0.693	0.520	0.608
V_{cr}^{PF} (pu)	0.570	0.588	0.516	0.587
ΔS (MVA)	117	82	510	1575
ΔS^{PF} (MVA)	120	83	511	1592
V_{cr} (pu)	0.611	0.621	0.598	0.653
V_{cr}^{PF} (pu)	0.615	0.614	0.637	0.700

tion makes the operating point move into the unfeasible region (above dashed lines). Additional corrective iterations are performed in order to pull the system back into the feasible region (dotted lines). The amount of load to be shed is determined by the final mismatches provided by the special power flow. It may be necessary to perform more than one corrective iteration, depending on the system's condition. For instance, the base case real power at bus 3 is 39 MW. The first load increase is defined so as a generator reaches its reactive power limit and the load goes up to 137 MW. After 9 iterations load at bus 3 reaches 546 MW which is a point in the unfeasible operating region (see zoom window of figure 2). Then a corrective iteration is performed and load is decreased to 545 MW. The amount of load decrease was determined by the final mismatches of the step size optimization based power flow run at iteration 9. The power margin is given by the difference between the maximum loading (545 MW) and the base case load (39 MW), resulting in 506 MW. For MVA changes one corrective iteration was also necessary. For MVAr changes, two corrective iterations were performed.

Tables 3, 4, and 5 show the importance of a proper choice of the sensitivity threshold. Three buses of system III were chosen to illustrate that, namely buses 19, 20, and 326. These tables show the average number of tiers per iteration that are searched for, the average number of generators per iteration that are flagged, the number of load increase iterations, the number of corrective iterations, the average number of sensitivities per iteration that are computed, and the final load margin.

Let us consider the particular case of MW load changes at bus 19 of system III (table 3). Starting from bus 19, 23 tiers are counted up to all buses of the system are accounted for. By using a threshold of 0.001, almost all tiers are searched for (20), whereas only 9 are searched for in case the threshold is set to 0.1. From a total of 155 existing generators, 62 are flagged when the threshold is 0.001. Only 2 generators are flagged for a threshold of 0.1. Even though 62 generators are flagged (threshold of 0.001), 154 sensitivity factors have to be computed. This means that 92 generators have sensitivities smaller than 0.001. For a threshold of 0.1, only 61 sensitivity factors are computed. The number of iterations and the final load margins are not affected by the threshold chosen.

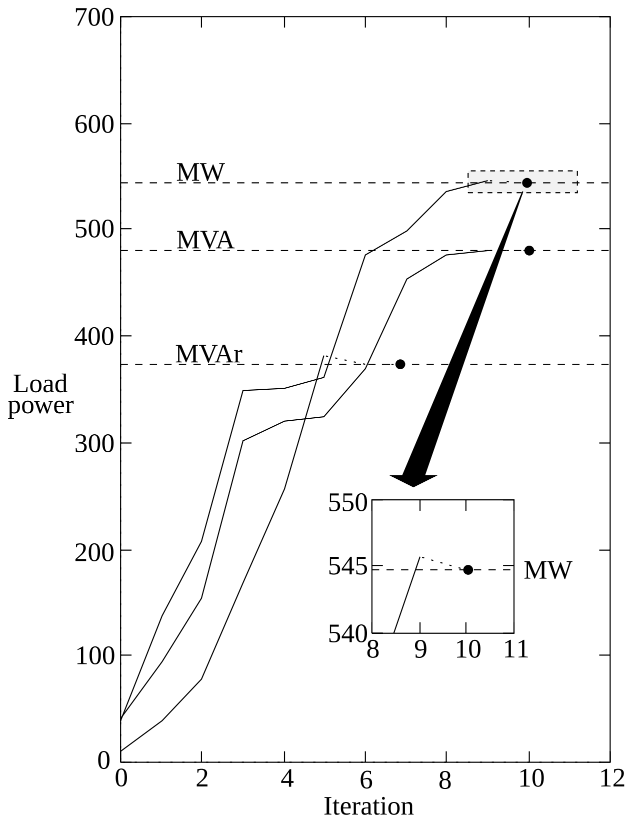

Figure 2: Load increases for bus 3 of system I.

Results show that an appropriate choice of the threshold can result in significant processing time savings, since less tiers are searched for and less sensitivities are computed. At the same time the performance in terms of number of iterations and final results (load margins) are not significantly affected. In fact, the processing time savings depend on the system, on the bus being analyzed, and on a proper choice of the threshold and *tiermax*. An appropriate combination of those two values can result in important gain as far as the efficiency of the method is concerned. The simulations performed have shown that a threshold of 0.1 and a *tiermax* of 4 may work very well for most systems.

As mentioned before, the processing times depend on the system, on the bus being analyzed, and on a proper choice of the threshold and *tiermax*. As an illustration, the following three tables show some processing times obtained by the proposed method using a Sun Sparc Ultra 1 workstation connected to a local area network under light load condition. Table 6 shows CPU times for three buses of system I. The idea is to show how CPU times vary for different types of load increase.

Table 7 shows CPU times for different power systems.

Table 8 shows the importance of adding heuristics to the algorithm. Two situations are shown. In the first situation, the threshold is set to zero and *tiermax* is set to 50. This means that all generators are to be considered and the whole system is searched for. In this case the heuristics have no effect on the algorithm's performance. In the second situation, the threshold and *tiermax* are set to the usual values 0.1 and 4, respectively. For the cases shown in table 8, the addition of heuristics to the algorithm resulted in dramatic CPU time savings (up to 50%, approximately), while the load margin errors were kept smaller than 5%.

Table 3: Bus 19 of system III – performance as a function of the sensitivity threshold.

		Load change		
	Threshold	MW	MVAr	MVA
Avg. tiers	0.001	20	16	20
(max. 23)	0.1	9	7	9
Avg. generators	0.001	62	23	60
(max. 155)	0.1	2	2	4
Load increase	0.001	1	1	1
iterations	0.1	1	1	1
Corrective	0.001	7	2	4
iterations	0.1	7	2	4
Sensitivities	0.001	154	142	155
(max. 155)	0.1	61	50	61
Load margin	0.001	254	166	199
	0.1	254	166	199

Table 4: Bus 20 of system III – performance as a function of the sensitivity threshold.

		Load change	
	Threshold	MW	MVAr
Avg. tiers	0.001	21	5
(max. 24)	0.1	12	5
Avg. generators	0.001	74	1
(max. 155)	0.1	9	1
Load increase	0.001	3	4
iterations	0.1	2	4
Corrective	0.001	1	1
iterations	0.1	2	1
Sensitivities	0.001	155	10
(max. 155)	0.1	69	10
Load margin	0.001	2762	1407
	0.1	3047	1407

It is important to point out the the code used for the simulations can be further improved, resulting in better processing times as well as time savings.

It is worth discussing some aspects as far as the corrective iterations are concerned. A corrective iteration is performed whenever (a) the power flow with step size optimization is run and no real solution is found (system has been led to the unfeasible region), and (b) the resulting final mismatches are greater than a pre-defined value (α). In this case, the load at the bus of interest is decreased by an amount equal to its final mismatch and another power flow is run. The process is repeated till (a) the system goes back into the feasible region, or (b) the system is still in the unfeasible region but the final mismatches are smaller than α. This means that the final maximum loading may correspond to an operating point sufficiently close to Σ, being feasible or slightly unfeasible. Depending on the value of α, the distance from the final operating point to Σ is very small if compared to the load margin. A slightly unfeasible final operating point does not constitute a problem from a practical standpoint. The main goal here is to provide system operators with a good estimate of the load margin. Naturally, preventive control actions ought to be taken before the maximum loading is reached.

Table 5: Bus 326 of system III – performance as a function of the sensitivity threshold.

		Load change	
	Threshold	MW	MVAr
Avg. tiers	0.001	22	19
(max. 22)	0.1	7	7
Avg. generators	0.001	76	28
(max. 155)	0.1	2	2
Load increase	0.001	2	3
iterations	0.1	2	3
Corrective	0.001	4	1
iterations	0.1	3	3
Sensitivities	0.001	155	129
(max. 155)	0.1	15	15
Load margin	0.001	793	1230
	0.1	793	1230

Table 6: CPU times for system I (times in seconds).

	Load change		
Bus	MW	MVAr	MVA
14	0.83	0.54	0.73
45	0.52	0.34	0.44
117	0.48	0.35	0.49

The results shown in tables 3, 4, and 5 were obtained for α equal to 0.3 MW/MVAr/MVA. In general, this value resulted in a small number of corrective iterations. Tests have shown that the number of corrective iterations depends upon the value of α, so that larger values of α can result in smaller number of iterations, without affecting significantly the accuracy of the method. However, there certainly is a tradeoff between the number of corrective iterations (value of α) and accuracy. Table 3 shows that for MW changes, the number of corrective iterations was large. In this particular case, any attempt of reducing the number of corrective iterations resulted in very large values of α, affecting the final load margin. A large number of simulations were done for several test and real life power systems and this case constituted an atypical situation.

As mentioned earlier, in (Flatabø *et alii*, 1990) it is proposed that both the state variables (voltages) and dependent variables (reactive power of generating units) be updated by using linear relationships (equations (4) and (8)). Also, it is proposed that the stability condition be verified by checking the following elements of matrix S_{xp}:

Table 7: CPU times for different power systems.

System	Bus	CPU time (sec)
I	11	1.54
II	568	4.70
III	633	6.47

Table 8: Evaluation of the effectiveness of the heuristics.

System/Bus	II/884		III/764	
Threshold	0	0.1	0	0.1
tiermax	50	4	50	4
CPU time (sec)	8.10	4.13	7.62	4.40
Load margin (MVA)	1592	1569	1184	1128
Voltage (pu)	0.702	0.753	0.709	0.774

$$vs_i = \frac{d}{dQ_i} V_i \qquad i = 1 \ldots n_b \tag{19}$$

The system becomes unstable if any vs_i changes its sign after a certain load increase. Even though the linearized approach results is a very fast calculation process, it is not accurate. This can be shown in figure 3, where reactive load increases are defined for obtaining the load margin for bus 14 of system IV.

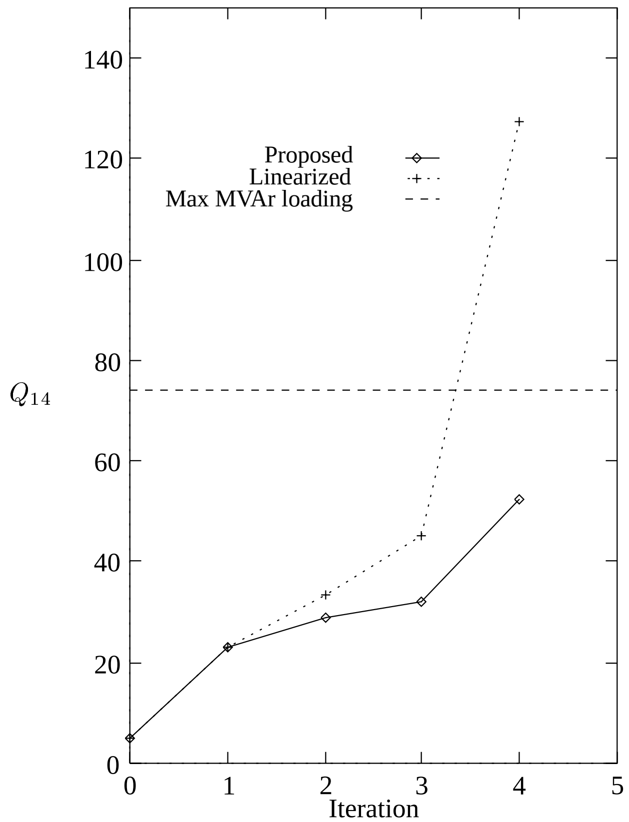

Figure 3: Load increases for bus 14 of system IV.

By using the linearized approach (Flatabø *et alii*, 1990) the system becomes unstable after 4 iterations, when vs_{14} changes its sign (dotted lines). The maximum loading in this case would be 127 MVAr (resulting in a load margin of 122 MVAr, since the base case reactive load is 5 MVAr). By solving power flows for different values of reactive power at bus 14 (Q_{14}) on a trial and error basis as discussed earlier, a maximum loading of 74 MVAr was obtained (dashed lines), which shows that the linearized approach can result in large errors in estimating load margins. For the sake of comparison, figure 3 also shows the performance of the proposed method up to iteration 4, and the difference between the two approaches can be easily inferred. It can be noted that the linearized approach becomes less accurate the closer the system gets to the critical point.

6 CONCLUSIONS

In this paper an efficient method for the calculation of load margins to voltage collapse was presented. It was based on gradual load increases till the system's operating point reaches the boundary Σ between the feasible and unfeasible operating regions. Load increase amounts are defined through sensitivity analysis calculations. The efficiency of the algorithm was significantly improved by using a special power flow with step size optimization and adding heuristics to the basic algorithm that make use of the particular characteristics of power systems in order to save computation time.

Test results have shown that the method is efficient, accurate and robust. It can be an important tool in the analysis of power systems with respect to proximity of voltage collapse.

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REFERENCES

Ajjarapu, V., and C. Christy (1992). The continuation power flow: a tool for steady state voltage stability analysis. *IEEE Transactions on Power Systems*, vol. 7, n. 1.

Alvarado, F., I. Dobson, and Y. Hu (1994). Computation of closest bifurcations in power systems. *IEEE Transactions on Power Systems*, vol. 9, n. 2.

Barquín, J., T. Gómez, and F.L. Pagola (1995). Estimating the loading limit margin taking into account voltage collapse areas. 1995 IEEE/PES Winter Meeting, New York, NY, USA.

Castro, C.A., and L.M.C. Braz (1997a). A new approach to the polar Newton power flow using step size optimization. Proceedings of the 29th North American Symposium, Laramie, Wyoming, USA.

Castro C.A., and L.M.C. Braz (1997b). Uma nova abordagem para a solução do problema de fluxo de carga pelo método de Newton com otimização de passo. Revista Controle & Automação, SBA, vol. 8, no. 3.

Chiang, H.-D., I. Dobson, R.J. Thomas, J.S. Thorp, and L. Fekih-Ahmed (1990). On voltage collapse in electric power systems. *IEEE Transactions on Power Systems*, vol. 5, no. 2.

Dehnel, M., and H.W. Dommel (1989). A method for identifying weak nodes in nonconvergent load flows. *IEEE Transactions on Power Systems*, vol. 4, no. 2.

Flatabø, N., R. Ognedal, and T. Carlsen (1990). Voltage stability condition in a power transmission system calculated by sensitivity methods. *IEEE Transactions on Power Systems*, vol. 5, no. 4.

Flatabø, N., O.B. Fosso, R. Ognedal, T. Carlsen, and K.R. Heggland (1993). A method for calculation of margins to voltage instability applied on the Norwegian system for maintaining required security level. *IEEE Transactions on Power Systems*, vol. 8, n. 3.

Freris L.L., and A.M. Sasson (1968). Investigation of the loadflow problem. *Proceedings of the IEE*, vol. 115, no. 10.

Gao, B., G.K. Morison, and P. Kundur (1991). Voltage stability evaluation using modal analysis. *IEEE Transactions on Power Systems*, vol. 7, no. 4.

Irissari, G.D., X. Wang, J. Tong, and S. Mokhtari (1997). Maximum loadability of power systems using interior point nonlinear optimization method. *IEEE Transactions on Power Systems*, vol. 12, n. 1.

Iwamoto, S., and Y. Tamura (1981). A load flow calculation method for ill-conditioned power systems. *IEEE Transactions on Power Apparatus and Systems*, vol. 100, n. 4.

Löf, P.-A., G. Andersson, and D.J. Hill (1992). Voltage stability indices for stressed power systems. IEEE PES Winter Meeting, paper WM 101-6 PWRS.

Overbye, T.J. (1993). A power flow measure for unsolvable cases. IEEE PES Summer Meeting, paper SM 492-9, Vancouver, B.C., Canada.

Overbye, T.J. (1994). Computation of a practical method to restore power flow solvability. IEEE PES Winter Meeting, paper WM 245-1, New York.

Rajagopalan, C., B. Lesieutre, P.W. Sauer, and M.A. Pai (1992). Dynamic aspects of voltage/power characteristics. *IEEE Transactions on Power Systems*, vol. 7, no. 3.

Scudder, J.M., and F.L. Alvarado (1981). Discussion of (Iwamoto and Tamura, 1981).

Sekine, Y., A. Yokoyama, K. Takenaka, and T. Kumano (1989). Mode transition of multiple load flow solutions, In: Bulk Power System Voltage Phenomena – Voltage Stability and Security, EPRI - EL 6183.

Sekine, Y., and H. Ohtsuki (1990). Cascaded voltage collapse. *IEEE Transactions on Power Systems*, vol. 5, no. 1.