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REAL TIME DATA SMOOTHING TO IMPROVE THE PERFORMANCE OF A STRAPDOWN NONGYROSCOPIC ATTITUDE PROPAGATION INERTIAL UNIT

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RESUMO

Analisa-se a possibilidade de se usar unidade inercial do tipo "strapdown", não giroscópica, para propagação de atitude. Um projeto utilizando-se apenas acelerômetros lineares é considerado. O uso de acelerômetros conduz à alternativa mais barata e simples, mas implica em problemas relacionados com a precisão obtida. Este problema pode ser atenuado pela eliminação da maioria dos erros sistemáticos, através de calibração. Entretanto, o erro puramente aleatório pode somente ser eliminado com o alisamento das saídas dos acelerômetros. Neste trabalho apresenta-se um esquema para fazê-lo em tempo real, em que se adota ajuste de curvas para o alisamento dos dados, usando-se filtro de Kalman e uma técnica de estimação do ruído no estado. Testes com dados simulados digitalmente indicam a viabilidade de uso da concepção "strapdown" proposta, no caso de missões satélite que demandem baixa precisão no controle de atitude.

Abstract

The possibility of using a strapdown nongyroscopic inertial unit for attitude propagation is analysed. A design made up exclusively of linear accelerometers is considered. The use of accelerometers leads to a cheaper and simple design but poses problems related to the level of the attainable accuracy. This can be attenuated by eliminating most of the systematic errors with the help of calibration. However, the purely random errors can only be eliminated if smoothing of the accelerometers output data is done. In the paper a scheme to do this in real time is proposed, where data smoothing by curve fitting is adopted using Kalman filtering and a state noise estimation technique. Tests with digitally simulated data indicate feasibility of using the proposed strapdown design for satellites missions demanding low accuracy in attitude control.

1. INTRODUCTION

The present state of the art in data processing techniques, digital hardware and instrumentation opens new possibilities in terms of solutions to materialize an inertial navigation measurement unit (IMU). Among these new possibilites are the IMU's where only accelerometers are used in a strapdown design (Krishnan, 1965; Schuler et al, 1967; Merhav, 1982). This nongyroscopic scheme leads to a cheaper and simpler solution, but still poses problems related to the level of attainable accuracies. Existing limitations in a strapdown solution of this kind are due to both sensor hardware performance and lack of satisfactory random errors compensation techniques.

Assuming that systematic errors can be eliminated by calibration, the paper proposes a scheme to attenuate the effect of random errors and analyses the feasibility of using such a nongyroscopic strapdown IMU for satellite attitude propagation. One of the configurations made up only of accelerometers, as proposed by Schüler et al (1967) is adopted together with a stochastic data smoothing procedure recently developed (Orlando, 1983; Orlando et al, 1986). The averaging out of high frequency errors is done by curve fitting in a scheme where Kalman filtering (see for example Sorenson, 1966; Jazwinski, 1970) is used together with a state noise estimation technique (Rios Neto & Kuga, 1982, 1985). The result is a procedure that can sequentially process the data, thus allowing smoothing in real time.

The analysis and tests (Trabasso, 1985) using simulated satellite data (Moro, 1983; Tilgner, 1971) indicate the feasibility of the proposed accelerometric strapdown IMU in applications of interest; for example in situations with low accuracy requirements (about one tenth of degree) and with short attitude propagation intervals between points where the aid of external noninertial information is available (of about ten minutes).

2. INERTIAL NAVIGATION WITH ACCELEROMETERS

The possibility of having an IMU exclusively employing accelerometers is a direct consequence of the following kinematic relation (see for example Greenwood, 1965):

$$\vec{A}_P = \vec{A}_0 + \dot{\vec{\Omega}} \times r + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) \qquad (1)$$

Where (Figure 1) $\vec{A}p$ is the acceleration of a generic body fixed point P; $\vec{A}_0$ is the acceleration of the body fixed reference system origin 0; $\vec{\Omega}$ is the body angular velocity; $\vec{r}$ is the position vector of P with respect to the body fixed reference system; the over dot is to indicate derivative with respect to time, and:

$$\dot{\vec{\Omega}} \triangleq \dot{\Omega}_x \vec{i} + \dot{\Omega}_y \vec{j} + \dot{\Omega}_z \vec{k} \qquad (2)$$

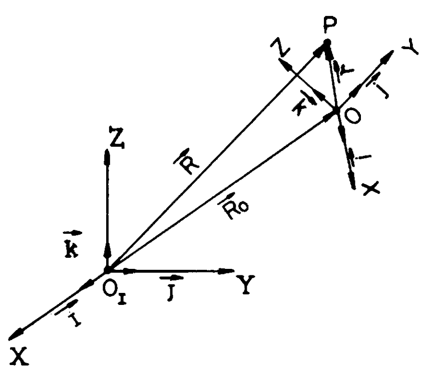

Fig. 1 - Inertial (O_I,X,Y,Z) and body $(0,x,y,z)$ reference systems.

If one considers a configuration of nine body fixed accelerometers, as in Figure 2, it is possible to determine velocity or the angular acceleration components in the body fixed reference system (Schuler et al, 1967). In this configuration, A_{ij} is to indicate the value measured by an accelerometer with sensitive axis parallel to the ith coordinate axis which is placed in the j th axis. By using the result of Eq. 1, the measured value can be directly related to the variables characterizing the body motion. For point a, for example, one has that:

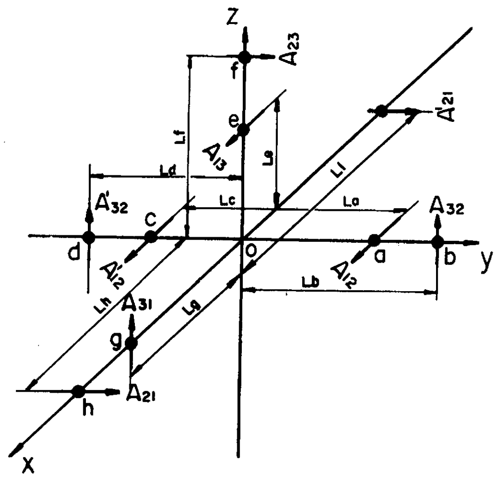

Fig. 2 - Configuration of body fixed accelerometers.

$$\vec{A}_a = (A_{0x} - (\dot{\Omega}_z - \Omega_x\Omega_y)l_a)\vec{i} + (A_{0y} - (\Omega_x^2 + \Omega_z^2)l_a)\vec{j} + (A_{0z} + (\dot{\Omega}_x + \Omega_y\Omega_z)l_a)\vec{k} \qquad (3)$$

resulting that in the absence of measurement errors, the output of the accelerometer located at point a is:

$$A_{12} = A_{0x} - (\dot{\Omega}_z - \Omega_x\Omega_y)l_a \qquad (4)$$

For the remaining accelerometers one can easily get the other eight correspondent equations, from which it results that:

$$\dot{\Omega}_x = (A_{32}/(l_b + l_d) - A'_{32}/(l_b + l_d) - A_{23}/l_f + A_{21}(l_l/l_f)/(l_h + l_l) + A'_{21}(l_h/l_f)/(l_h + l_l))/2 \qquad (5)$$

$$\dot{\Omega}_y = (A_{13}/l_e - A_{12}(l_c/l_e)/(l_a + l_c) - A'_{12}(l_a/l_e)/(l_a + l_c) - A_{31}/l_g + A_{32}(l_d/l_g)/(l_b + l_d) + A'_{32}(l_b/l_g)/(l_b + l_d))/2 \qquad (6)$$

$$\dot{\Omega}_z = (A_{21}/(l_h + l_c) - A'_{21}/(l_h + l_l) - A_{12}/(l_a + l_c) + A'_{12}/(l_a + l_c))/2 \qquad (7)$$

$$\Omega_x\Omega_y = (A_{21}/(l_h + l_l) - A'_{21}/(l_h + l_l) + A_{12}/(l_a + l_c) - A'_{12}/(l_a + l_c))/2 \qquad (8)$$

$$\Omega_y\Omega_z = (A_{32}/(l_b + l_d) - A'_{32}/(l_b + l_d) +$$
$$+ A_{23}/l_f - A_{21}(l_l/l_f)/(l_h + l_l) -$$
$$- A'_{21}(l_h/l_f)/(l_h + l_l))/2 \qquad (9)$$

$$\Omega_x\Omega_z = (A_{13}/l_e - A_{12}(l_c/l_e)/(l_a + l_c) -$$
$$- A'_{12}(l_a/l_e)/(l_a + l_c) + A_{31}/l_g -$$
$$- A_{32}(l_d/l_g)/(l_b + l_d) -$$
$$- A'_{32}(l_b/l_g)/(l_b - l_d))/2 \qquad (10)$$

where $l_.$ are the accelerometers location arms, as indicated in Figure 2.

The analysis of Eqs. 4-10 shows that the effect of the gravitational acceleration does not need to be considered, since it cancels out in the determination of $\vec{\Omega}$ and $\dot{\vec{\Omega}}$

Once $\vec{\Omega}$ is known, the attitude can be propagated using, for example, the formalism of quaternions (see, for example, Ref. 14):

$$\dot{q} = \frac{1}{2}\{\Omega\}\, q \qquad (11)$$

where $q^T \triangleq [q_1, q_2, q_3, q_4]$ is the vector formed by four Euler symmetric parameters or quaternions; and $\{\Omega\}$ assumed constant in the integration along each interval of discretization is the matrix defined as

$$\{\Omega\} \triangleq \begin{bmatrix} 0 & \Omega_z & -\Omega_y & \Omega_x \\ -\Omega_z & 0 & \Omega_x & \Omega_y \\ \Omega_y & -\Omega_x & 0 & \Omega_z \\ -\Omega_x & -\Omega_y & \Omega_z & 0 \end{bmatrix} \qquad (12)$$

3. ACCELEROMETERS DATA SMOOTHING

3.1 GENERAL SCHEME

The inertial unit is assumed to be used for attitude propagation in an interval $[T_I, T_F]$. Inside this interval two levels of discretization are involved, as indicated in Figure 3. The subintervals $[T_k, T_{k+1}]$ correspond to the discretization in the control loop; and the subintervals $[\tau_l, \tau_{l+1}]$ correspond to the sampling rate of the accelerometers.

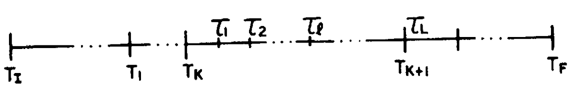

Fig. 3 - Attitude inertial propagation interval

The accelerometers measurements are contaminated by errors. Assuming that corrections are done to eliminate systematic errors, the data obtained from a given accelerometer is of the type:

$$Y_k(\tau_l) = A(T_k + \tau_l) + v_k(\tau_l) \qquad (13)$$

where $Y_k(\tau_l)$, $l = 0,1,2,\ldots,L$, is the value measured at time $(T_k + \tau_l)$; $A(T_k + \tau_l)$ is the actual or true value of the specific force measured by the given accelerometer; and $v_k(\tau_l)$ is the measurement noise, assumed a zero mean Gaussian process, with

$$E[v_k(\tau_l)]=0, \quad E[v_k(\tau_l)v_k(\tau_m)] = R_k(\tau_l)\delta_{lm} \qquad (14)$$

where δ_{lm} is the Kronecker symbol; and the variance $R_k(\tau_l)$ is given.

In the proposed scheme, the data of each accelerometer are smoothed before being used to determine $\vec{\Omega}$ in the intervals $[T_k, T_{k+1}]$. This is done employing a polynomial fitting to approximate $A(T_k + \tau_l)$:

$$A(T_k + \tau) \cong x_{0k} + x_{1k}\tau + \ldots + x_{npk}\tau^{np} + \varepsilon_k \qquad (15)$$

where $0 \leq \tau \leq \tau_L$; np is adjusted to lead to a negligible truncation error ε_k; and the coefficients x_{ik}, $i = 0,1,2,\ldots,np$ are sequentially estimated processing the data $Y_k(\tau_l)$, as shown in what follows.

3.2 - POLYNOMIAL FITTING PROCEDURE

The coefficients x_{ik} in Eq. 15 are viewed as the components of a state vector X_k and estimated with the Kalman filter (see for example Jazwinshi, 1970) in the solution of the following state estimation problem (Orlando, 1983; Orlando et al, 1986).

$$X_k(\tau_{l+1}) = X_k(\tau_l) + W_k(\tau_l) \qquad (16)$$

$$Y_k(\tau_l) = M_k(\tau_l)\, X_k(\tau_l) + v_k(\tau_l) \qquad (17)$$

where from Eq. 13 it is seen that $M_k(\tau_l) \triangleq [1, \tau_l, \ldots, \tau_l^{np}]$; and $W_k(\tau_l)$ is a zero mean, Gaussian white noise sequence with:

$$E[W_k(\tau_l)\, W_k^T(\tau_m)] = Q_k(\tau_l)\delta_{lm} \qquad (18)$$

and $Q_k(\tau_l) = \text{diag}[q_{ki}(\tau_l),\ i = 1,2,\ldots,np]$ is estimated together with the state X_k, using a state noise estimation procedure (Rios & Kuga, 1982, 1985), summarized in the Appendix.

4. SIMULATION OF ACCELEROMETRIC DATA

To digitally simulate the accelerometers outputs and test the proposed scheme, the TD-1A satellite (Tilgner, 1971) was considered, acted upon by environmental torques, with the following initial data (Moro, 1983):

. *Orbital elements at 05/16/82, Oh min epoch*

$a = 6910$ km; $e = 0.0027$; $i = 97.6^{o}$

$\Omega = 317.9^{o}$; $\omega = 90.6^{o}$; $M = 15.3^{o}$

. *Angular velocity and Euler angles*

$\Omega_x = \Omega_y = 1.13$ E-3 rad/s; $\Omega_z = 1.74$ E-3 rad/s

$\phi = 2.85$ rad; $\theta = 1,57$ rad ; $\psi = 2.56$ rad.

For the results presented in Section 4.3 and the configuration depicted for the accelerometers locations:

$l_a = .36$m; $l_b = .60$m; $l_c = .14$m; $l_d = .40$m

$l_e = .40$m; $l_f = .50$m; $l_g = .40$m; $l_h = .45$m;

$l_1 = .05$m

The attitude motion is sampled each 30 milliseconds (interval $(\tau_{1+1} - \tau_1)$, in Fig.3) and it is assumed for the accelerometers a performance equivalent to that of a Q-Flex Servo Accelerometer, QA1300 (Sundstrand Data Control Inc.), for which $R_k(\tau_1)$ is:

$$R_k^{1/2}(\tau_1) = 1.0\text{E-}7 \quad m/s^2 = R$$

4.1 - INITIALIZATION OF SMOOTHING PROCEDURE

The calibration of initial values was done along the simulation tests, leading to the values that follows, for any given accelerometer.

. *Polynomial:* $\Delta T_k \triangleq T_{k+1} - T_k = 3$ s; np=2 (see Fig. 3 and Eq. 15)

. *State filter (see Eqs. 15, 16 and 17):* $\bar{x}_{00}(T_0) = Y_0(T_0)$, $\bar{x}_{0k}(T_k) = \hat{A}(T_{k-1} + \tau_L)$, $\bar{x}_{j0}(T_0) = \bar{x}_{jk}(T_k) = 0, j = 1,2,\ldots,np$ and $k \geq 1$; $\bar{P}_{11}(T_0) = = P_{11}(T_k) = R$, $\bar{P}_{jj}(T_0) = \bar{P}_{jj}(T_k) = (\sigma/(\Delta T_k)^{(j-2)})^2$, $k \geq 1$, $j=2,\ldots,np+1$, $\sigma = .02$ for A_{12}, A_{12}^1, A_{13}, $\sigma = .006$ for A_{21}, A_{21}, A_{21}^1, A_{23} and $\sigma = .016$ for A_{31}, A_{32}, A_{32}^1, adjusted in the simulation, where the over bar means a priori estimates, the hat means filter estimates and thus:

$$\hat{A}(T_{k-1}+\tau_L) \triangleq \hat{x}_{0k-1}(\tau_L) + \hat{x}_{1k-1}(\tau_L) \cdot \tau_L + \ldots + \hat{x}_{npk-1}(\tau_L) \cdot \tau_L^{np} \qquad (19)$$

. *Noise filter (see Appendix):* $\bar{Q}_k(T_k) = 0$ $\bar{P}_k^q(T_k) = \text{diag}\,[\bar{P}_{jj}^2(T_k), j=1,2,\ldots,np+1]$, $k \geq 0$, where the meaning of $\bar{P}_k^q(T_k)$ is defined in the Appendix.

To reinforce $A(T_k + \tau_1)$ in Eq. 15 to behave as a convergent series, one only estimates the first two diagonal terms of $Q_k(T_k + \tau_1)$, and for the other terms one takes:

$$\hat{q}_i(T_k+\tau_1) = \hat{q}_2(T_k+\tau_1)/(\Delta T_k^{2(i-2)}),$$

$$i=3,\ldots,np+1 \qquad (20)$$

4.2 - RESULTS OF TESTS

The improvement in accuracy given by the proposed procedure is illustrated in Figure 4. For one of the acceleromenters, the errors in the accelerometric outputs without smoothing are plotted against the errors obtained when these same outputs are previously submitted to smoothing. One can see that an improvement of about one order of magnitude is obtained.

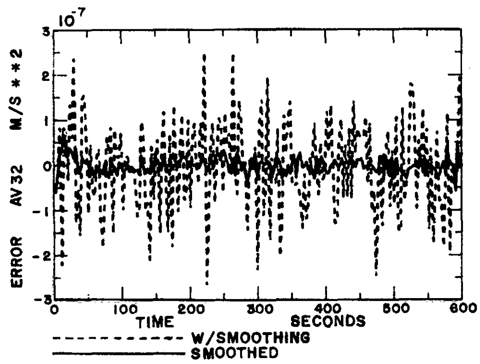

Fig. 4 - Errors in the measured and smoothed accelerometric outputs.

Figures 5 and 6 illustrate the effects of this improvement in the angular velocities and Euler angles. In these figures one again compares the errors in the results with and without smoothing. To obtain the angles in Figure 6, Wilcox (1967) analytical approximation was adopted to integrate Eq. 11.

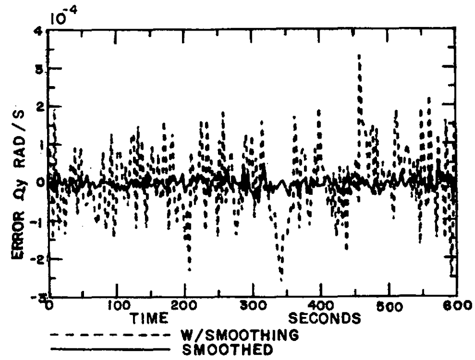

Fig. 5 - Effects of smoothing in angular velocity (Ωy component) errors.

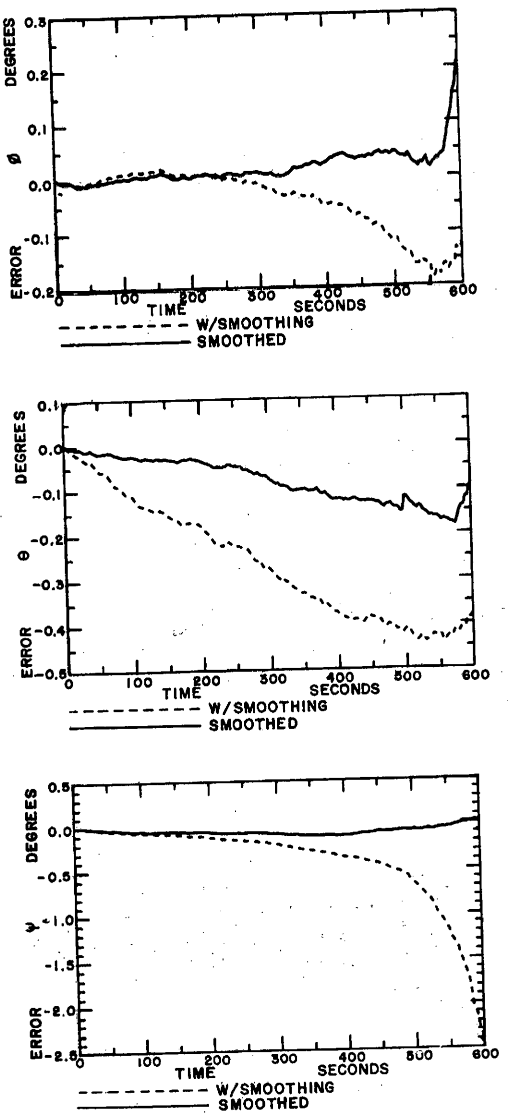

Fig. 6 - Effects of smoothing in propagated Euler angles errors.

Notice, in Figure 6, that for the case tested an accuracy better than or of the order of 0.1^{o} was maintained for about 10 minutes.

5 - CONCLUSIONS

The possibility of using a nongyroscopic inertial unit made up exclusively of accelerometers, for attitude propagation, was analysed. Data smoothing was done to improve the results, using a procedure which can process the data sequentially, in real time. This allowed the averaging out of the high frequency errors present in the accelerometers measurements, improving the accuracy of the propagated attitude.

The simulation tests results indicate the feasibility of adopting the proposed solution as a cheap and simple one in situations where one has: (i) short duration missions with required accuracies in the engineering level; or (ii) externally aided inertial navigation with short intervals between the instants of time when noninertial information is available.

Further studies are recommended to: (i) better analyse the influence of the accelerometers arm lenghts (l_a, l_b,..., l_l) and the number of accelerometers in configurations like that of Figure 2; and (ii) improve the performance of the smoothing procedure, specially concerning the calibration of initial values and the other parameters needed in the filters.

6. APPENDIX: STATE NOISE ESTIMATION TECHNIQUE

To estimate the diagonal elements of $Q_k(\tau_l)$ in Eq. 18 one imposes consistency between the observations residues and their statistics. To do so, the diagonal terms, which correspond to the variances of the state noise, are imposed to assume those values that maximize the probability occurrence of the true residue of each observation (Rios Neto & Kuga, 1985). Dropping for the sake of simplicity, the subindex k and replacing τ_l by l Eqs. 16 and 17, one defines the observation residue as:

$$r(l+1)=Y(l+1)-M(l+1)\bar{X}(l+1) \qquad (A.1)$$

and the true residue as:

$$r^{v}(l+1)=r(l+1)-v(l+1) \qquad (A.2)$$

Then, under the hypothesis of the residue having normal distribution, the criterion of statistical consistency is realized by imposing

$$[r^{v}(l+1)]_o^2 = E\{[r^{v}(l+1)]^2\} \qquad (A.3)$$

which is the condition to maximize the probability of occurrence of the value $[r^{v}(l+1)]_o$ of the random variable $r^{v}(l+1)$. Using Eqs. 16 and 17 of section 3.2 and after some algebraic manipulations, it results:

$$[r^2(l+1)-2r(l+1)v(l+1)+v^2(l+1)]_o = M(l+1)P(l)M^T(l+1)+M(l+1)Q(l)M^T(l+1) \qquad (A.4)$$

where $P(l)\triangleq E[(X(l)-\hat{X}(l))(X(l)-\hat{X}(l))^T]$

It is thus reasonable to define the noise:

$$\eta(l+1)\triangleq -2[r(l+1)]_c\, v(l+1)+v^2(l+1)-R(l+1) \qquad (A.5)$$

$$E[\eta(l+1)]=0,\ E[\eta^2(l+1)]=N(l+1) = 4[r^2(l+1)]_c\, R(l+1)+2R^2(l+1) \qquad (A.6)$$

where $[r(l+1)]_c$ is the calculated value, corresponding to the accurred value of the random variable $r(l+1)$.

Having $\eta(l+1)$ defined and assuming a diagonal $Q(l)$ matrix whose elements $q(l+1)_i$, are components of the $q(l+1)$ vector, the following model can be taken to represent the condition of Eq. A.3:

$$z(l+1)=h(l+1)+\eta(l+1) \quad (A.7)$$

where

$$h(l+1)=[m_1^2(l+1) \ : \ m_2^2(l+1) \ : \ \ldots \ : \ :m_{np+1}^2(l+1)]$$

$$z(l+1)=r^2(l+1)+R(l+1)-M(l+1)P(l)M^T(l)$$

The vector $q(l+1)$ is then estimated using the following algorithm:

(i) prediction or characterization of a information for l > 1.

$$\bar{q}(l+1) = \hat{q}(l) \quad (A.8)$$

$$\bar{P}^q(l+1)=P^q(l) + \text{diag}\ (\tfrac{1}{9}\ \hat{q}_j^2(l), \quad j = 1,2,\ldots,np+1) \quad (A.9)$$

(ii) filtering

$$\hat{q}(l+1)=\bar{q}(l+1)+K^q(l+1)[z(l+1) - -h(l+1)\bar{q}(l+1)] \quad (A.10)$$

$$P^q(l+1)=[I - K^q(l+1)h(l+1)]\bar{P}^q(l+1) \quad (A.11)$$

$$K^q(l+1)=\bar{P}^q(l+1)h^T(l+1)[h(l+1)\bar{P}^q(l+1)\cdot h^T(l+1)+N(l+1)]^{-1} \quad (A.12)$$

The matrix $Q(l)$ is then approximately given by:

$$Q(l) \cong \text{diag}\ \{q(l+1)=\hat{q}(l+1)\geqq 0\}$$

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