

ADAPTIVE PURE FORCE CONTROL OF ROBOT MANIPULATORS:  
A SINGLE LINK CASE STUDY.

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**ABSTRACT.**- A new adaptive controller to achieve a pure force aim for a single link robot is proposed in this paper. The proposed adaptive controller structure consists of a nonlinear feedback of measured robot end-effector applied forces, positions, and velocities of the manipulator's joints. The adaptive law consists of a standard gradient type estimation algorithm. Closed-loop global stability analysis is performed via Lyapunov approach.

**Keywords:** Adaptive control, robot control, force control, stability.

## 1. INTRODUCTION.

Applications of industrial robots can be categorized in two modes. One is when the robot arm moves in a free space without interacting with the environment following a desired motion trajectory. The other is when the arm interacts with the environment to perform a specified contact force trajectory.

In the last category, so-called force control applications, we would like the robot to contact its workpiece with a specified interaction force. Many implementations of force controller have been reported. When the controller uses the end-effector force and makes no use of position feedback information, is termed explicit force control (Nevins, 1983; Eppiger, 1987). Other methods of accommodating environment constraints include stiffness control (Salisbury, 1980), damping control (Whitney, 1987), and impedance control (Hogan, 1985; 1984). In contrast all these methods use joint position and velocity in addition to end-effector force feedback to achieve the desired response. In the case of hybrid control (Raibert, 1981), directions can be specified for either pure position or force control.

Some controllers for robot applications need an accurate knowledge of the robot dynamic parameters, e.g. the mass, moments of inertia, and position of the center of mass of links and load (Paul, 1981; Craig, 1986). Uncertainty in these parame-

ters leads to degraded performance under robot dynamic parameters unknowledge.

In the last years, adaptive techniques has been used sucessfully in robot motion control. Globally stable adaptive controllers for robot with unknown dynamic parameters has been proposed in (Craig, 1986; Slotine (a), 1987; Sadegh, 1987). In (Slotine (b), 1987) adaptive techniques are considered for motion control in joint space and cartesian space, external control for unknown passive mechanisms, and hybrid control.

In this paper, we consider the adaptive pure force control for a single link robot. The problem can be stated as that of designing a control law and an adaptation law for a robot with unknown dynamic parameters so that its end-effector can accurately apply a desired force over its environment.

It is assumed that the environment is modelled by a passive mechanism with unknown dynamic parameters. Also it is assumed that the robot end-effector is attached to the passive mechanism end-point.

We propose a new adaptive controller to solve the latter problem. The proposed adaptive controller structure consists of a nonlinear feedback of positions, velocities of the manipulator joints, and measured forces at its end-effector. The adaptive law consists of a standard gradient type estimation algorithm (Anderson, 1986).

Using Lyapunov theory, we show global stability

of the closed loop system.

The format of this paper is as follows. In section 2 we present the problem formulation where the robot and passive mechanism dynamic models are described. In section 3 we present a non-adaptive and an adaptive robot force controller. Some simulated examples are given in section 4 illustrating our main result. Concluding remarks are offered in section 5.

## 2. PROBLEM FORMULATION.

In this section we present the formulation of the pure force control problem. Consider a single link rigid robot manipulator whose dynamic model can be written as:

$$H\ddot{q} + C(q, \dot{q}) + G(q) + J(q)F(t) = \tau \quad (2.1)$$

where  $q$  is the joint displacement,  $\tau$  is the applied joint torque (or force),  $H$  is the positive inertia,  $C(q, \dot{q})$  is the friction torque,  $G(q)$  is the gravitational torque,  $J(q)$  is the Jacobian of the robot which is assumed to be a nonsingular function, and  $F(t)$  is the applied force by its end-effector, corresponding to the generalized coordinate  $x$  in a fixed frame of reference  $R_0$ .

Consider a passive mechanism with unknown dynamic parameters attached to the active robot end-effector. Note that the passive mechanism has its own dynamics, in general are nonlinear. It is assumed that the passive mechanism is defined by a holonomic arbitrary mechanical system with a scalar Jacobian. That is the case for a single link manipulator (Craig, 1986).

The dynamic model of a passive mechanism whose endpoint is attached to the robot end-effector is given by

$$H_p^* \ddot{x} + C_p^*(x, \dot{x}) + G_p^*(x) = F \quad (2.2)$$

where  $x$  is the generalized coordinate describing the position of the endpoint of the passive mechanism with respect to fixed frame of reference  $R_0$ ,  $H_p^*$  is a positive inertia,  $C_p^*(x, \dot{x})$  is the friction torque,  $G_p^*(x)$  is the gravitational torque, and  $F$  is the force applied by the manipulator corresponding to the generalized coordinate  $x$ .

The relationship between the robot joint position and its end-effector configuration is

$$x = x(q) \quad (2.3)$$

where  $x(\cdot)$  is the geometric transformation which is assumed to be injective.

The corresponding velocity and acceleration relations are

$$\dot{x} = J(q)\dot{q} \quad (2.4)$$

$$\ddot{x} = J(q)\ddot{q} + \dot{J}(q)\dot{q} \quad (2.5)$$

where  $J(q)$  is the Jacobian of the robot.

Using eq. (2.3)-(2.5) the dynamic model of the passive mechanism (2.2) can be written in terms of the robot joint-space as:

$$H_p(q)\ddot{q} + C_p(q, \dot{q}) + G_p(q) = F \quad (2.6)$$

where

$$H_p(q) = H_p^* J(q) \quad (2.7)$$

$$C_p(q, \dot{q}) = C_p^*(x, \dot{x}) J(q) + H_p^* \dot{J}(q) \dot{q} \quad (2.8)$$

$$G_p(q) = G_p^*(x) \quad (2.9)$$

Now, we are in position to formulate the control problem as follows. Consider the robot manipulator described by (2.1) whose end-effector is attached to the passive mechanism (2.6). The dynamic parameters of the robot manipulator and the the passive mechanism are unknown. The adaptive problem can be stated as that of designing a control law and an adaptive law so that the force applied  $F$ , by the robot end-effector over the passive mechanism can accurately follow a desired force trajectory  $F_d$ .

A similar problem was considered in (Slotine (b), 1987) in the so-called external control of unknown passive mechanism. In this problem a passive mechanism is modeled as a nonlinear differential equations with unknown dynamic parameters and the problem consists of controlling its end-effector along a desired motion trajectory using a known active non-redundant robot. In the problem treated in this paper we pursue a desired force trajectory aim without assuming knowledge on the passive mechanism dynamics and on the dynamic parameters of the active robot.

## 3. ADAPTIVE FORCE CONTROL.

In this section we propose two controllers to solve de force control problem formulated in section 2. First we assume that the dynamic model of the robot (2.1) and the passive mechanism are known.

Second, we consider the case where the dynamic parameters of the robot and passive mechanism are unknown.

### 3.1 Non Adaptive Controller.

Assume that the dynamic model of the robot (2.1) is exactly known. To solve the force control problem we propose the following controller:

$$\begin{aligned} \tau = & HH_p^{-1}(q)F - HH_p^{-1}(q)[C_p(q, \dot{q}) + G_p(q)] \\ & + C(q, \dot{q}) + G(q) + J(q)[F_d + K_i \int_0^t \tilde{F}(\alpha) d\alpha] \end{aligned} \quad (3.1)$$

where  $F_d$  is the desired applied force,  $\tilde{F}(t) = F_d(t) - F(t)$  is the tracking force error and  $K_i$  is an arbitrary positive scalar.

The analysis of the system is carried by combining eq. (2.1) and eq. (3.1):

$$\begin{aligned} H\{-H_p^{-1}(q)F + H_p^{-1}(q)[C_p(q, \dot{q}) + G_p(q)]\} = \\ J(q)[F_d + K_i \int_0^t \tilde{F}(\alpha) d\alpha] - J(q)F \end{aligned} \quad (3.2)$$

Substituting (2.6) yields

$$J(q)[\tilde{F} + K_i \int_0^t \tilde{F}(\alpha) d\alpha] = 0 \quad (3.3)$$

Since by assumption  $J(q)$  is nonsingular then

$$\tilde{F}(t) + K_i \int_0^t \tilde{F}(\alpha) d\alpha = 0 \quad (3.4)$$

this in turn implies that  $\tilde{F}(t) \rightarrow 0$  as  $t \rightarrow \infty$ . Thus, the proposed controller defined by (3.1) is globally stable, and guarantees zero steady error for the applied force.

### 3.2 Adaptive Controller.

We consider now that the dynamic model of the robot is unknown, but its nonlinear dynamic structure is linear in terms of a suitably selected set of robot parameters. It is assumed that the selected set of parameters are unknown. This important property is a key fact used in the most recently works on adaptive motion control reported in the literature (Craig, 1986; Slotine, 1987; Sadegh, 1987).

The follow control law is proposed to solve the control force problem:

$$\tau = \Phi(q, \dot{q}, F)\hat{\theta} + J(q)[F_d + \int_0^t \tilde{F}(\alpha) d\alpha] \quad (3.5)$$

where

$$\begin{aligned} \Phi(q, \dot{q}, F)\hat{\theta} = & \hat{H}H_p^{-1}(q)[\hat{C}_p(q, \dot{q}) \\ & + \hat{G}_p(q)] + \hat{C}(q, \dot{q}) + \hat{G}(q) \end{aligned} \quad (3.6)$$

with  $\hat{H}$ ,  $\hat{C}(q, \dot{q})$ ,  $\hat{G}(q)$ ,  $\hat{H}_p(q)$ ,  $\hat{C}_p(q, \dot{q})$ , and  $\hat{G}_p(q)$  are the estimated of  $H(q)$ ,  $C(q, \dot{q})$ ,  $G(q)$ ,  $H_p(q)$ ,  $C_p(q, \dot{q})$ , and  $G_p(q)$ , respectively.  $\Phi(q, \dot{q}, F)$  is an  $m$ -dimensional vector and  $\hat{\theta}$  is a  $m$ -dimensional vector containing the estimated of the unknown robot and passive mechanism constant parameters.

It is important to remark that there exists a constant vector  $\theta_* \in R^m$  such that:

$$\begin{aligned} \Phi(q, \dot{q}, F)\theta_* = \\ HH_p^{-1}(q)[F - C_p(q, \dot{q}) - G_p(q)] + C(q, \dot{q}) + G(q) \end{aligned} \quad (3.7)$$

To update the parameter estimated vector  $\hat{\theta}$ , we use a gradient type (Anderson, 1986) adaptation law:

$$\dot{\hat{\theta}} = \Gamma \Psi \int_0^t \tilde{F}(\alpha) d\alpha \quad (3.8)$$

where  $\Gamma$  is a  $m \times m$  positive definite matrix, usually diagonal, and  $\Psi$  is a  $m$ -dimensional vector defined by

$$\Psi(t) = J^{-1}(q)\Phi(q, \dot{q}, F) \quad (3.9)$$

The structure of the adaptive controller given by (3.3) and (3.6) is sketched in fig. 1.

To carry out the stability analysis we need to express the closed loop system equations in terms of the so-called error model (Anderson 1986)

From eq. (3.5) and eq. (2.6) we can write.

$$\ddot{q} = H^{-1}[\Phi(q, \dot{q}, F)\hat{\theta} - C(q, \dot{q}) - G(q)] \quad (3.10)$$

The closed loop system composed by the model (2.1) and the control law (3.5) is given by:

$$J(q)[\ddot{F} + K_i \int_0^t \tilde{F}(\alpha) d\alpha] = -\Phi(q, \dot{q}, F)\tilde{\theta} \quad (3.11)$$

where we use (3.1) and  $\tilde{\theta} = \hat{\theta} - \theta_*$  to denote the parameter error vector.

Note that  $\dot{\tilde{\theta}} = \dot{\hat{\theta}}$  since the unknown parameters  $\theta_*$  are constants. Adaptation law (3.8) yields:

$$\dot{\tilde{\theta}} = \Gamma \Psi \int_0^t \tilde{F}(\alpha) d\alpha \quad (3.12)$$

Equations (3.11) and (3.12) define the so-called error model equations. These equations describe the behavior of the adaptive force control system.

We are now in position to present our main result.

**Theorem 3.1.-** Consider the control law (3.5) and the adaptation law (3.8) in closed loop with the robot and passive mechanism (2.1) and (2.6). Then the following hold:

- $\tilde{\theta} \in L_\infty^m$
- $\int_0^\infty \tilde{F}(t) dt \in L_2 \cap L_\infty$

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**Proof.-** Let the nonnegative function of time:

$$V(t) = \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \frac{1}{2} \left[ \left( \int_0^t \tilde{F}(\alpha) d\alpha \right)^2 \right] \geq 0 \quad (3.13)$$

whose derivative time along the trajectories of eqs. (3.11) and (3.12) is:

$$\dot{V}(t) = -K_i \left( \int_0^t \tilde{F}(\alpha) d\alpha \right)^2 \leq 0 \quad (3.14)$$

Expressions (3.13) and (3.14) imply that  $\tilde{\theta} \in L_\infty^m$  and  $\int_0^t \tilde{F}(\alpha) d\alpha \in L_\infty \cap L_2$ .

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#### 4. SIMULATED EXAMPLES.

In this section we present some simulated examples illustrating our main results.

Consider the single link robot shown in figure 2. In this example the passive mechanism consists of a mass  $m$  coupled or not to a linear spring of stiffness  $k$  and damping  $b$ . The robot end-effector is attached to the mass via a force sensor. With notations defined in figure 2, the dynamic model of the single link robot is given by

$$d\ddot{q} - \frac{1}{2} m_r g r \sin(q) + r \cos(q) F = \tau$$

The dynamic model of the passive mechanism can be written as

$$m\ddot{x} + b\delta\dot{x} + k\delta(x + L_m - L_r) = F$$

where

$$x = r \sin(q)$$

$$\dot{x} = r \cos(q) \dot{q}$$

$$\ddot{x} = r \cos(q) \ddot{q} - r \sin(q) \dot{q}^2$$

$$\delta = \begin{cases} 0, & \text{if } x \leq L_r - L_m; \\ 1, & \text{otherwise.} \end{cases}$$

In all examples, the desired applied force  $F_d$  was taken as a step of amplitude 2. The parameter numerical values are  $r = 1$ ,  $m_r = 20$ ,  $L_m = 0.2$ ,  $m = 5$ ,  $d = 20$ ,  $g = 9.81$ ,  $L_r = 1.1$ ,  $b = 10$ , and  $k = 1000$ .

Two controllers were used to perform the force aim control.

#### 4.1 PI Control.

The first simulation was performed with a proportional plus integral (PI) force feedback control. This control law is described by

$$\tau(t) = K_p \tilde{F}(t) + K_i \int_0^t \tilde{F}(\alpha) d\alpha \quad (4.1)$$

where  $K_p$  and  $K_i$  are positive constants. In this example we chose  $K_p = 1$  and  $K_i = 10$ . No information about the link robot and the passive mechanism is needed in the controller design. However, it is not also easy to prove closed loop stability for this first example.

Figure 3 shows the desired and real applied forces obtained from simulation. Due to nonlinear nature of the system, the control aim, i.e.  $F \rightarrow F_d$ , can not be achieved in this example. Notice from figure 4 the oscillatory behavior of the angular position  $q$ .

#### 4.2 Adaptive Control.

The adaptive controller (3.5) and (3.8) applied to force control problem stated in this section is described by

$$\tau = \Phi^T \hat{\theta} + J[F_d + K_i \int_0^t \tilde{F}(\alpha) d\alpha] \quad (4.2)$$

$$\dot{\hat{\theta}} = \Gamma \Psi \int_0^t \tilde{F}(\alpha) d\alpha \quad (4.3)$$

where

$$J = r \cos(q)$$

$$\Phi = \left( \frac{F}{J} \quad -q \quad -\frac{x}{J} \frac{\partial J}{\partial q} \quad \dot{q} \frac{1}{2} r g \sin(q) \quad \frac{1}{J} \right)^T \in R^5$$

$$\Psi = \frac{\Phi}{J}$$

and  $\hat{\theta}$  is the 6-dimensional estimated parameter vector

The non adaptive controller (3.1) can be computed provided that the robot and passive mechanism parameters are known by placing in eq. (4.2)

$$\hat{\theta} = \theta_* = \left( \frac{d}{m} \quad \frac{db\delta}{m} \quad d \quad m_r \quad \frac{k\delta d(L_m - L_r)}{m} \right)^T$$

In the adaptive controller (4.2), (4.3) design, the only a priori information about the system is the knowledge of the robot Jacobian  $J = r \cos(q)$ . The adaptation gain matrix  $\Gamma$  is chosen to be diagonal

$$\Gamma = \text{diag}[10, 10, 1000, 10, 10, 1000]$$

$$K_i = 800 \text{ and } \hat{\theta}(0) = 0.$$

The simulated results are shown in figures 5 and 6. Notice in figure 5 that the control aim is achieved asymptotically i.e.  $F \rightarrow F_d$ . Also good performance on the angular position is shown in figure 6.

## 5. CONCLUSIONS.

An Adaptive controller for pure force robot control for a single link manipulator has been proposed in this paper.

The dynamic parameters of the robot and its environment which is assumed to be a passive mechanism, are unknown. The only a priori information about the system is the knowledge of the Jacobian function of the robot. Position, velocity and force feedback information are used in the controller. Global stability of the closed loop adaptive system is shown via Lyapunov approach. Simulated examples was presented illustrating our main results.

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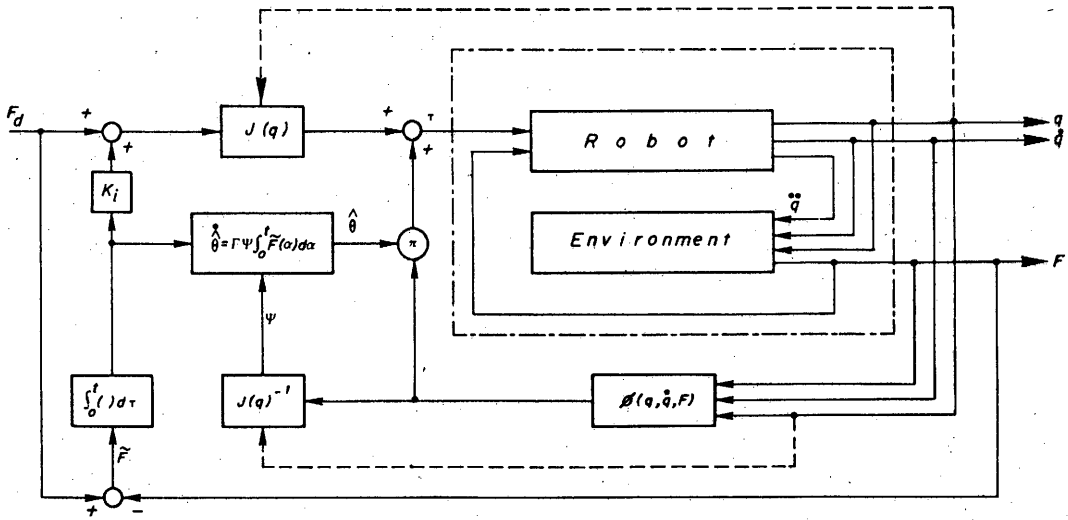


Fig. 1 Structure of the adaptive controller.

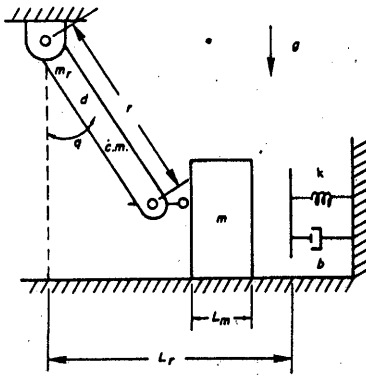


Fig. 2 Single link robot

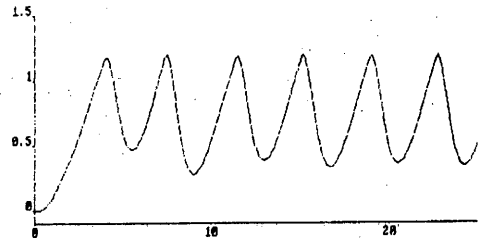


Fig. 4 PI Control. Angular position \$q\$.

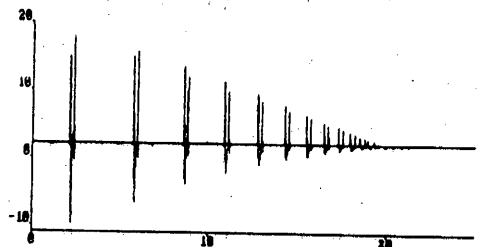


Fig. 5 Adaptive control. Applied and desired forces.

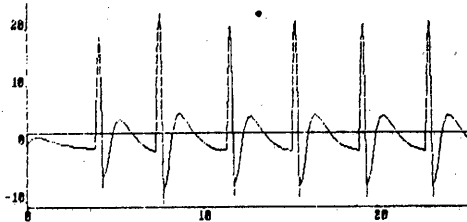


Fig. 3 PI Control. Applied and desired forces.

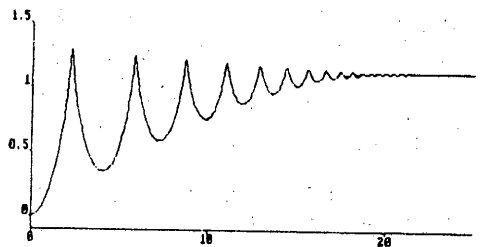


Fig. 6 Adaptive control. Angular position \$q\$.