SUBOPTIMAL AND HYBRID NUMERICAL SOLUTION SCHEMES FOR ORBIT TRANSFER MANEUVERS

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RESUMO. Este trabalho considera o problema de transferência de órbita de um veículo espacial com mínimo consumo de combustível, em termos de seleção, implementação e testes numéricos das soluções escolhidas. O principal objetivo é obter as leis de controle para as manobras necessárias ao Satélite Brasileiro de Sensoriamento Remoto. Depois de uma pesquisa bibliografica e análise dos trabalhos disponíveis, foram selecionados dois métodos de solução do problema. No primeiro, o problema de controle ótimo associado é tratado numericamente através de um método de busca direta combinado com controle sub-ótimo parametrizado. No segundo, o método híbrido é usado e a determinação dos valores iniciais dos multiplicadores de Lagrange (necessárias na resolução das equações dadas pelas condições necessárias de primeira ordem para um mínimo local) é transformada num problema de busca direta. Nos dois métodos, a solução numérica em cada iteração é reduzida a um problema de programação não-linear, que é resolvida pelo método da projeção do gradiente. O veículo espacial é considerado em movimento Kepleriano controlado apenas pelo empuxo, que é assumido de magnitude constante (alta ou baixa) e operando em modo liga-desliga. Resultados de simulações numéricas são apresentados.

ABSTRACT. In this paper the problem of spacecraft orbit transfer with minimum fuel consumption is considered, in terms of selecting, implementing and testing numerical solutions. The main goal is to obtain the control laws for the maneuvers required by the Brazilian Remote Sensing Satellite. After a search in the literature and analysis of the results available, one selects two schemes of solution to the problem. In the first one the associated optimal control problem is numerically treated by using a direct search approach together with suboptimal parameterized control. In the second one, a hybrid approach is used where the determination of the initial values of the Lagrange multipliers (to solve the equations given by the first order necessary conditions for a local minimum) is transformed in a direct search problem. In both schemes, the numerical solution of the problem in each iteration is reduced to one of nonlinear programming, which is then solved with the gradient projection method. The spacecraft is supposed to be in Keplerian motion controlled by the thrusts, that are assumed to be of fixed magnitude (either low or high) and operating in an on-off mode. Results of simulations are also presented.

1 - INTRODUCTION

The remote sensing satellite is the third Brazilian mission, in a set of four that are scheduled for the 90's. The objective of this first set is to develop technical conditions that can give to Brazil the capability to build, launch and control artificial satellites. The complete set is constituted of two data collection and two remote sensing satellites. The latter will be put in a parking orbit by the launcher, and will have the capability to go to its final orbit with its own propulsion system. Small maneuvers for station-keeping will also be required for this mission.

In this paper, from the analyses of the alternatives of solutions available (Prado, 1989: Prado & Rios-Neto, 1993), results of the implementation and tests of two methods selected to solve the problem of sending a vehicle from one orbit to another with minimum fuel expenditure are shown. The methods can be used either for large orbit transfer (as a geosynchronous satellite launched by the Space Shuttle in a low parking orbit) or for small orbit correction (as the maneuvers required for station-keeping of a space station or a remote sensing satellite). The objective is to find the best way (in terms of minimum fuel expenditure) to accomplish the maneuvers required by the first Brazilian Remote Sensing Satellite.

One of the first solutions of this problem was obtained by Hohmann (1925), using an impulsive approximation, the so called "Hohmann Transfer". There are many solutions propo-

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sed with this type of approximation, like the "Bi-Elliptical Transfer" (Hoelker & Silber, 1959) and the "Parabolic Transfer". Later, a great attention has been given to the more realistic approach, where the thrust is considered finite. Many researchers proposed solutions for this case as Tsien (1953), Lawden (1955), Biggs (1978; 1979), Ceballos & Rios-Neto (1981), Rios-Neto & Bambace (1981).

From the analysis of the alternatives available (Prado, 1989), two choices were made:
i) Sub-optimal parametrization;
ii) Optimal control (hybrid approach);
and they were explored to develop procedures valid for high or low thrust and for large or small transfers.

Numerical results obtained in the simulations of the orbit transfer phase of the first Brazilian remote sensing satellite are presented.

2 - DEFINITION OF THE PROBLEM

The basic problem discussed in this paper is the problem of orbit transfer maneuvers. The objective of this problem is to modify the orbit of a given spacecraft. In the case considered in this paper, an initial and a final orbit around the Earth is completely specified. The problem is to find how to transfer the spacecraft between those two orbits in a such way that the fuel consumed is minimum. There is no time restriction involved here and the spacecraft can leave and arrive at any point in the given initial and final orbits. The maneuver is performed with the use of an engine that is able to deliver a thrust with constant magnitude and variable direction. The mechanism, time and fuel consumption to change the direction of the thrust is not considered in this paper.

3 - MODEL USED

The spacecraft is supposed to be in Keplerian motion controlled only by the thrusts, whenever they are active. This means that there are two types of motion:
i) A Keplerian orbit, that is an orbit obtained by assuming that the Earth's gravity (assumed to be a point of mass) is the only force acting on the spacecraft. This motion occurs when the thrusts are not firing;
ii) The motion governed by two forces: the Earth's gravity field (also assumed to be a point of mass) and the force delivered by the thrusts. This motion occurs during the time the thrusts are firing.

Figure 1 shows this situation. F_E is the gravitational force of the Earth (assumed to be a point of mass) and F_t is the force given by the thrusts.

The thrusts are assumed to have the following characteristics:
i) Fixed magnitude: The force generated by them is always of constant magnitude during the maneuver. The value of this constant is a free parameter (an input for the algorithm developed here) that can be high or low;
ii) Constant Ejection Velocity: Meaning that the velocity of the gases ejected from the thrusts is constant. The importance of this fact can be better understood by examining Prado (1989);

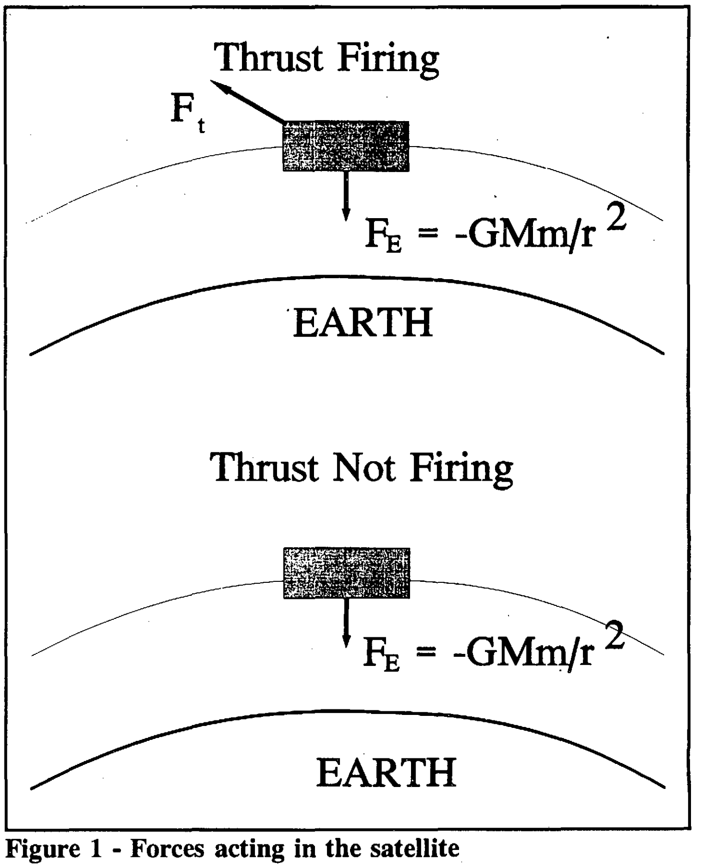

Figure 1 - Forces acting in the satellite

iii) Either free or constrained angular motion: This means that the direction of the force given by the thrusts can be modified during the transfer. This direction can be specified by the angles A and B, called pitch (the angle between the direction of the thrust and the perpendicular to the line Earth-spacecraft) and yaw (the angle with the orbital plane). The motion of those angles can be free or constrained (constant, linear variations, forbidden regions for firing the thrusts, etc.);
iv) Operation in on-off mode: It means that intermediate states are not allowed. The thrusts are either at zero or maximum level all the time.

The solution is given in terms of the time-histories of the thrusts (pitch and yaw angles) and fuel consumed. Several numbers of "thrusting arcs" (arcs with the thrusts active) are tested for each maneuver.

Instead of time, the "range angle" (the angle between the radius vector of the spacecraft and an arbitrary reference line in the orbital plane) is used as the independent variable.

4 - FORMULATION OF THE OPTIMAL CONTROL PROBLEM

This is a typical optimal control problem, and it is formulated as follows:

Objective Function: M_f,
where M_f is the final mass of the vehicle and it has to be maximized with respect to the control $u(.)$, where $u(.)$ is any continuous function;

Subject to: Equations of motion, constraints in the state (initial and final orbit) and control (limits in the angles of "pitch" and "yaw", forbidden region of thrusting and others);

And given: All parameters (gravitational force field, initial values of the satellite and others)

5 - SUBOPTIMAL METHOD

In this approach (Prado, 1989; Biggs, 1978), a linear parametrization is used as an approximation for the control law (angles of pitch (A) and yaw (B)):

$$A = A_0 + A' * (x - x_s) \tag{1}$$

$$B = B_0 + B' * (x - x_s) \tag{2}$$

where A_0, B_0, A', B' are parameters to be found, x is the instantaneous range angle and x_s is the range angle when the motor is turned-on.

These equations are the mathematical representation of the "a priori" hypothesis that A and B vary linearly with the "range angle" x. This is done to explore the possibility of having a model easy to implement in terms of hardware development.

Considering these assumptions, there is a set of six variables to be optimized (start and end of thrusting and the parameters A_0, B_0, A', B') for each "burning arc" in the maneuver. Note that this number of arcs is given "a priori" and it is not an "output" of the algorithm.

By using parametric optimization, this problem is reduced to one of nonlinear programming, which can be solved by several standard methods.

6 - OPTIMAL METHOD

This approach is based on Optimal Control Theory (Bryson & Ho, 1975). First order necessary conditions for a local minimum are used. These equations can give us the following information:

a) One set of differential equations for the Lagrange multipliers. They are called "adjoint equations". Together with the equations of motion they complete the set of differential equations to be integrated numerically at each step;
b) The "Transversality Conditions", that are the conditions to be satisfied by the Lagrange multipliers at the final time. Together with the constraints of the transfer (start in a point that belongs to the initial orbit and finishes in a point that belongs to the final orbit) these end conditions complete the set of boundary conditions to be satisfied. This problem is known as the "Two Point Boundary Value Problem" (TPBVP), because there are boundary conditions to be satisfied at the beginning and at the end of the interval of integration;
c) Maximum Principle of Pontryagin. This principle says that the magnitude of the scalar product of the Lagrange multiplier by the right-hand side of the equations of motion has to be a maximum. Working out the algebra involved we will end up with a condition for the angles of "pitch" and "yaw" that can be solved to give us their numerical values at each time.

Then, the problem becomes a problem of non-linear programming with finite dimension. This problem is then solved using the following algorithm:

i) Choose an estimate for the initial and final "range angle" (the variable that replaces the time as the independent variable) and for the initial values of the Lagrange multipliers;
ii) Integrate the adjoint equations and the equations of motion simultaneously, obtaining the instantaneous values of the "pitch" and "yaw" angles from the Maximum Principle of Pontryagin;
iii) At the end of the maneuver, verify if the boundary conditions are satisfied. If they are not satisfied update the initial values following the procedure described in the next session and go back to step i. If the constraints are satisfied the procedure is finished.

This treatment is called hybrid approach (Biggs, 1979) because it uses direct searching methods for minimization together with first order necessary conditions for a local minimum.

With this approach, the problem is again reduced to parametric optimization, as in the suboptimal method, with the difference that the angle's parameters are replaced by the initial values of the Lagrange multipliers, as the variables to be optimized.

The main difficulty involved in this method is to find good first initial guesses for the Lagrange multipliers, because they are quantities with no physical meaning. This problem can be solved by using the method proposed by Biggs (1979). He proposes a transformation called "adjoint-control", where one guess control angles and its rates at the beginning of thrusting instead of the initial values of the Lagrange multipliers. A set of equations is developed that allow to obtain the Lagrange multipliers from the values of the initial angles of "pitch" and "yaw" and its rates. More details are available in Prado (1989) and Biggs (1979). By performing this transformation it is easier to find a good initial guess, and the convergence is faster. This hybrid approach has the advantage that, since the Lagrange multipliers remain constant during the "ballistic arcs" (arcs that have the thrusts inactive), it is necessary to guess values of the control angles and its rates only for the first "burning arc". This transformation reduces the number of variables to be optimized and, in consequence, the time of convergence.

7 - NUMERICAL METHOD

To solve the nonlinear programming problem, the gradient projection method was used (Bazarra & Sheetty, 1979; Luemberger, 1973).

It means that at the end of the numerical integration, in each iteration, two steps are taken:

i) Force the system to satisfy the constraints by updating the control function according to:

$$\boldsymbol{u}_{i+1} = \boldsymbol{u}_i - \nabla \mathbf{f}^{\mathrm{T}} \cdot \left[\nabla \mathbf{f} \cdot \nabla \mathbf{f}^{\mathrm{T}}\right]^{-1} \mathbf{f} \tag{3}$$

where $\mathbf{f}$ is the vector formed by the active constraints;

ii) After the constraints are satisfied, try to minimize the fuel consumed. This is done by making a step given by:

$$u_{i-1} = u_i + \alpha \frac{\mathbf{d}}{|\mathbf{d}|} \tag{4}$$

where:

$$\alpha = \gamma \frac{J(u)}{\nabla J(u) \cdot \mathbf{d}} \tag{5}$$

$$\mathbf{d} = -\left(\mathbf{I} - \nabla \mathbf{f}^{\mathbf{T}} \left[\nabla \mathbf{f} \cdot \nabla \mathbf{f}^{\mathbf{T}}\right]^{-1} \mathbf{f}\right) \cdot \nabla J(u) \tag{6}$$

where **I** is the identity matrix, **d** is the search direction, J is the function to be minimized (fuel consumed) and γ is a parameter determined by a trial and error technique. The possible singularities in equations (3) to (6) are avoided by choosing the error margins for tolerande in convergence large enough.

This procedure continues until $|u_{i+1} - u_i| < \varepsilon$ in both equations (3) and (4), where ε is a specified tolerance.

The algorithm was coded in single precision FORTRAN IV, and the calculations were performed at INPE's Burroughs 6800 computer.

8 - VALIDATION OF THE ALGORITHM

After deriving and coding the algorithm, several simulations were performed to validate the software. Two examples were used from Biggs (1978; 1979), one for the suboptimal and one for the optimal method.

The detailed results are omitted here to save space, but they are available in Prado (1989). The difference between the results found by the algorithm developed and the literature is usually less than 3% for the parameters involved in the solution and less than 0.8% in fuel consumed, which is the most important parameter.

9 - SIMULATIONS FOR THE BRAZILIAN REMOTE SENSING SATELLITE

For this mission, two kinds of maneuvers will be necessary (in both phases the fuel used is Hydrazine):

i) Initial transfer phase, where the objective is to send the satellite from the parking orbit to the nominal orbit;
ii) Station-keeping, where the objective is to keep the satellite near the nominal orbit.

The transfer phase will occur, in the worst case, with the following data (Rama Rao, 1984):

i) Initial orbit: Semi-major axis of 6768.14, eccentricity of 0.00591, inclination of 97.44 degrees, ascending node of 67.27 degrees, argument of perigee of 97.66 degrees, mean anomaly of 270 degrees;
ii) Final orbit: Semi-major axis of 7017.89, eccentricity of 0.000, inclination of 97.94 degrees, free ascending node, free argument of perigee, free mean anomaly;
iii) Initial mass of 170 kg;
iv) Thrust level of 4.0 N.

The station-keeping phase will correct the semi-major axis only, and this will occur when its value gets 1.26 km below the nominal value (Carrara, 1988). Using these values, a typical maneuver will increase the semi-major axis from 7016.63 km to 7017.89 km and it will keep the eccentricity in zero and the inclination in 97.94 degrees. The initial mass is 150 kg and the thrust level is 4.0 N.

Considering these values, the solutions obtained (Prado, 1989) are compared with Hohmann Transfer (Carrara & Souza, 1988; Carrara, 1988). Initially, the suboptimal method was applied in the transfer phase, with 2, 4 and 8 "thrusting arcs" and no constraints in control. The results are shown in Table 1.

In a second set of simulations the same maneuvers were performed with the additional constraints that the control angles must be fixed ($A' = B' = 0$); and, in a third set, the constraint

Table 1 - Suboptimal initial transfer phase with 2, 4 and 8 "thrusting arcs"

Arc	x_s(deg)	x_e(deg)	A_0(deg)	B_0(deg)	A'	B'	Fuel-kg
1	459.8	722.0	11.6	-60.4	0.028	0.500	.....
2	963.4	1184.7	17.0	49.8	-0.110	-0.050	14.23
1	498.1	603.4	0.6	-25.7	0.019	-0.053	....
2	1025.4	1125.6	10.4	41.0	-0.159	-0.188	....
3	1590.0	1697.8	3.3	-51.5	-0.009	0.497	....
4	2105.8	2206.6	10.2	40.2	-0.150	-0.183	12.16
1	527.4	576.9	1.1	-16.2	-0.001	-0.052	....
2	1055.3	1105.4	6.6	36.0	-0.151	-0.110	....
3	1622.1	1672.8	2.3	-39.6	-0.004	0.560	....
4	2135.5	2187.6	6.3	35.2	-0.139	-0.086	....
5	2327.3	2377.5	1.0	-16.0	0.010	-0.106	....
6	2855.4	2905.7	6.5	35.9	-0.151	-0.110	....
7	3422.2	3473.2	2.2	-39.3	-0.004	0.562	....
8	3935.6	3987.9	6.2	35.0	-0.14	-0.096	11.93

$A_0 = 0$ was added (only B_0 is a free parameter for the control law). The objective is to know how much more fuel is required to compensate a more simple implementation of the control device and to satisfy the constraints of keeping some equipment (antennas, for example) pointed toward Earth. To complete this study, the same maneuvers were simulated with the optimal control approach. The result for the case with four "burning arcs" is shown in Figure 2. This figure shows the control (direction of the thrust) to be applied at each range angle to obtain the maneuver with minimum consumption of fuel. Similar results are available for maneuvers with two and eight "thrusting arcs", but they are not shown here to save space, since they are all very similar to each other. Table 2 shows the comparison in fuel expenditure for all cases studied.

Table 2 - Fuel expenditure (kg) for all maneuvers simulated

Method	2 arcs	4 arcs	8 arcs
Suboptimal	14.23	12.16	11.93
Suboptimal $(A'=B'=0)$	21.38	17.05	12.87
Suboptimal $(A'=B'=A_0=0)$	No solution found	17.96	13.44
Optimal	13.04	12.09	11.87

The value obtained by considering a Hohmann Transfer is about 12.00 kg of fuel.

For the station-keeping phase, the suboptimal and optimal methods were applied with no constraints in control, and with 1, 2, 3 and 4 "thrusting arcs" applied in different positions of the orbit. The results showed that, due to the small magnitudes involved, there is no difference in all methods tested. As an example, the results for the suboptimal and optimal methods with 1 "thrusting arc" are shown in Table 3.

10 - CONCLUSIONS

Suboptimal and optimal control were explored to generate algorithms to obtain solutions for the minimum fuel maneuvers required by the first Brazilian Remote Sensing Satellite.

By comparing the results obtained by the algorithms developed and those found in the literature (Biggs, 1978; 1979) it seems that optimal and suboptimal solutions do not exhibit significant differences in fuel consumed, specially when a large number of "thrusting arcs" is used.

Both methods have a good numerical behavior, but they can not be used in real time. Process time (CPU) is short (1 to 3 minutes, in the Burroughs 6800 computer) for simple maneuvers, but when several constraints and/or "thrusting arcs" are present the process time can be large (more than one hour, in some cases).

Optimization techniques are not required when station-keeping maneuvers are considered.

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Table 3 - Station-keeping with sub-optimal (top) and optimal (bottom) methods (1 "thrusting arc")

Arc	x_s(deg)	x_e(deg)	A_0(deg)	B_0(deg)	A'	B'	Fuel(kg)
1	0.0	1.56	0.0	0.0	0.0	0.0	47.0
Arc	x_s(deg)	x_e(deg)	A(const.)	B(const)			Fuel(kg)
1	0.0	1.56	0.0	0.0			47.0

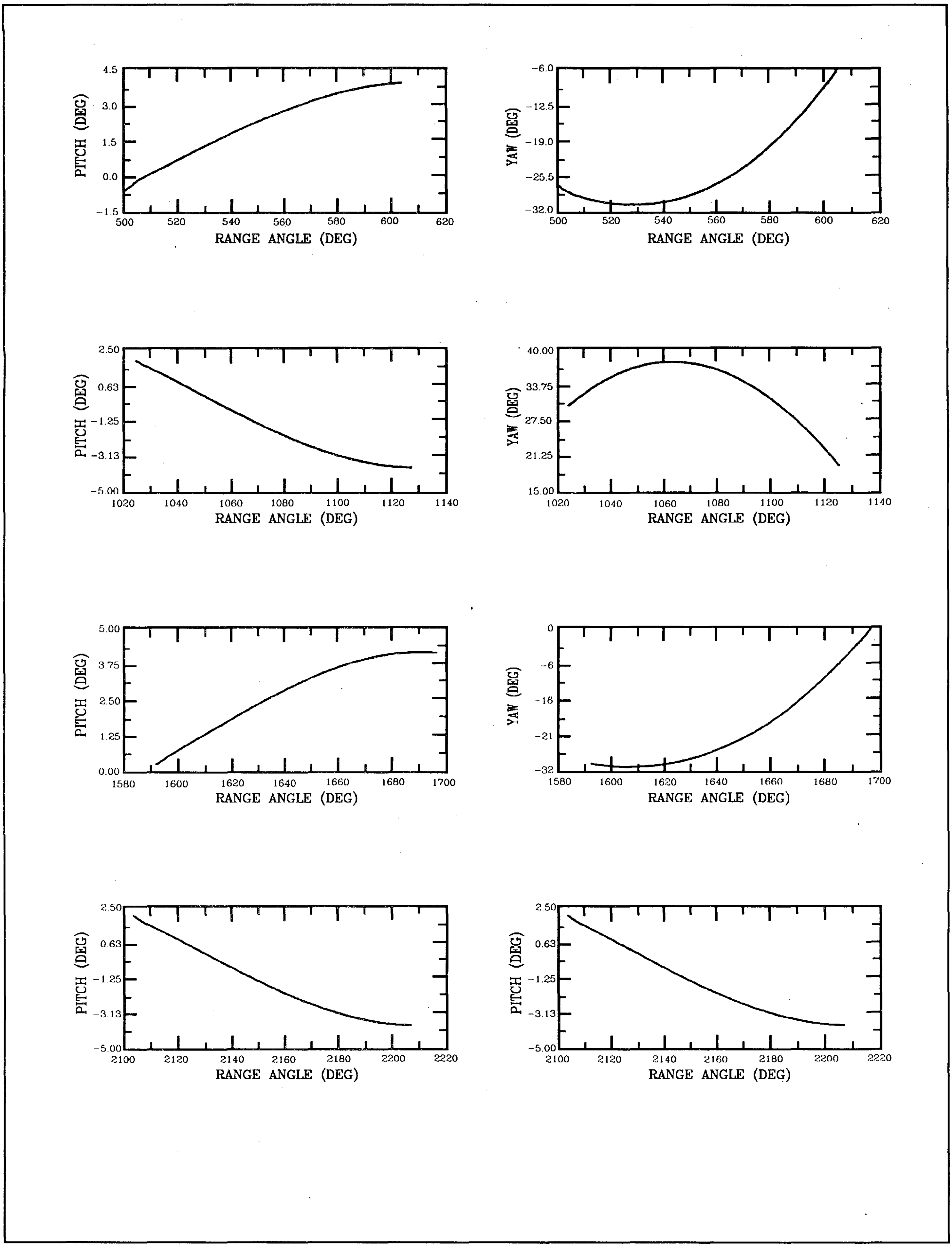

Figure 2 - Optimal Control With 4 "Thrusting Arcs". Fuel Consumed: 12.09 kg.

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