A SURVEY ON TRAJECTORIES TO THE MOON

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RESUMO: Este trabalho descreve os modelos e métodos utilizados para transferência de um veículo espacial entre a Terra e a Lua. O maior interesse do trabalho é o de descrever e comentar a literatura e métodos já existentes, embora sugestões de melhorias sejam também apresentadas.

ABSTRACT: This paper describes the models and methods used to transfer a spacecraft between the Earth and the Moon. The major goal of this paper is to describe and comment the literature and methods already known, although some suggestions for improvements are made.

INTRODUCTION

The objective of this paper is to present a detailed review of the literature on the topic of transfer orbits, with emphasis on transfers between the Earth and the Moon. Most of the material presented here are of general interest for those studying orbital maneuvers. Several possibilities of mathematical models for the dynamics and for the actuators are explained. Classical and modern methods are shown. Although it is not the main objective of this paper, suggestions for new techniques are presented.

DEFINITION OF THE PROBLEM

In a general formulation, the problem studied here is the problem of transferring a spacecraft between two given orbits with minimum consumption of fuel. In the particular case of trajectories to the Moon, the spacecraft has to be transferred from an initial orbit around the Earth to a final orbit around the Moon. Of course there are more variables involved in this problem than the fuel consumption, such as the time for the transfer, the constraints in the actuators and/or in the state (position, velocity and mass) of the spacecraft, etc. In this paper the attention is focused in minimum fuel transfers.

In a general way, an orbit transfer of a spacecraft with minimum fuel expenditure consists of changing its initial estate (position, velocity and mass) from $\underline{r}_0$, $\underline{v}_0$ and m_0 at the time t_0, to $\underline{r}_f$, $\underline{v}_f$ and m_f at the time t_f ($t_f \geq t_0$) with the minimum fuel expenditure (m_f - m_0). In the most general case one most choose the direction, sense and magnitude of the thrust (the available control) to be applied to get the desired transfer.

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OPTIONS FOR DYNAMICS, ACTUATORS AND OPTIMIZATION METHODS

The literature presents several possibilities for the models involved in this problem. Those choices can be divided basically in three parts: dynamics of the system, types of actuators and methods of optimization. The dynamic is usually based in considering the gravitational forces of the bodies involved (modeled as points of mass) and the forces generated by the control. Usual possibilities are: i) Two-Body Problem, where it is assumed that one main body (Earth, Sun, etc.) governs the motion of a massless spacecraft and the orbits are Keplerian, except for the duration of the application of the thrusters; ii) Two-Body Perturbed Problem, where in the dynamics explained above one or more perturbations are added, like the atmospheric drag, third-body gravitational force, pressure of radiation, etc; iii) Three-body Problem, where the presence of three masses are considered; in particular the restricted version of this problem (Szebehely, 1967), where one massless spacecraft is moving under the gravitation of two other bodies is a very good model for lunar and interplanetary trajectories; iv) N-Bodies Problem, where N points of mass are moving under their own gravitation

For the control to be applied in the system, usually it is possible to distinct between two models: i) the impulsive system, where the control consists in changing instantaneously the velocity of the spacecraft by an amount ΔV; ii) the continuous system, where the control delivers a thrust for a finite amount of time.

When the optimization method is considered, there are basically three choices: i) the direct method, where the problem is reduced to the search of parameters that minimizes a certain objective function; ii) the indirect method, where first-order necessary conditions are written and solved; iii) the so called hybrid approach, where first-order necessary conditions are written and transformed in a search of parameters that minimizes a certain objective function.

IMPULSIVE CLASSICAL MANEUVERS

The classical methods of orbital maneuvers are all based in the impulsive propulsion system. The most important ones are shown briefly in the paragraphs below.

Hohmann Transfer

This is the solution for a bi-impulsive transfer between two circular and coplanar orbits. It was created by Hohmann (1825). It is the most used result in orbital maneuvers. The transfer is as follows:

a) In the initial orbit a $\Delta V_0 = V_0 \left| \sqrt{\frac{2(r_f/r_0)}{(r_f/r_0)+1}} - 1 \right|$ (where r_0 (r_f) is the radius of the initial (final) orbit and V_0 is the velocity of the spacecraft when in its initial orbit) is applied in the direction of the motion. With this impulse the spacecraft is inserted into a elliptical orbit with periapsis r_0 and apoapsis r_f;

b) The second impulse is applied when the spacecraft is at the apoapsis. The magnitude is $\Delta V_f = V_0 \left| 1 - \sqrt{\frac{2}{(r_f/r_0)+1}} \right| \sqrt{r_0/r_f}$ and it circularizes the orbit.

This transfer was generalized to include the circular-elliptic transfer, the elliptic-elliptic-co-axial and out-of-plane transfers.

This result is used as a first model for lunar trajectories, since the Moon is in a circular orbit and the spacecraft usually starts in a LEO (Low-Earth-Orbit).

The Bi-elliptic transfer

Later, Hoelker and Silber (1959) showed that the Hohmann transfer is the optimal transfer between two circular-coplanar orbits only when the ratio r_f/r_0 is less than 11.94. If $r_f/r_0 >$ 11.94 the bi-elliptical tri-impulsive transfer offers a lower total ΔV. This transfer is accomplished in three steps:

a) A first impulse ΔV_0 is applied in the initial orbit that makes the spacecraft goes to an elliptic orbit with periapsis r_0 and apoapsis r_i ($r_i > r_f$); b) When the spacecraft is at the apoapsis, a second impulse ΔV_i is applied to increase the periapsis to r_f; c) Then, a third impulse is applied when the spacecraft is at the periapsis to circularize the orbit.

The ΔV for this transfer decreases when the distance r_i increases. The minimum is in the case $r_i = \infty$, that is the so called bi-parabolic transfer, since the two intermediate transfer orbits are parabolic.

Other variants of impulsive maneuvers are available, like: series of impulses at the apses (Spencer et al., 1982); the transfer with two-impulses of fixed magnitudes (Melton e Jin, 1991); transfers from one body back to the same body (Prado and Broucke, 1993); etc.

Patched Conic

Those classical transfers give results to transfer a spacecraft between two given orbits, but they do not include the phase of insertion into orbit of a second body. This is a very important phase if an Earth-Moon transfer is desired. The patched conic method can solve this problem by splitting the trajectories in two parts: a) The first leg neglects the effect of the Moon and any of the previous methods (Hohmann, bi-elliptic, etc) can be used to transfer the spacecraft from its original parking orbit to an orbit that crosses the Moon's path; b) When the spacecraft reaches a position where the Moon's gravity field dominates its motion (sphere of influence of the Moon), the Earth's effects are neglected and the orbit is studied as a Keplerian lunar orbit.

THE ELLIPTIC-BI-PARABOLIC TRANSFER

The elliptic-bi-parabolic transfer is a new version of the bi-parabolic transfer that takes advantage of an intermediate swing-by with the Moon to reduce the amount of fuel required. To develop the equations involved in this transfer it is assumed that: i) The initial LEO is circular with radius r_0; ii) The space vehicle is in a Keplerian orbit around the Earth, except for the duration of the swing-by at the Moon; iii) The swing-by at the Moon can be modeled by the two-body scattering; iv) The propulsion system is the usual impulsive system, able to delivery an instantaneous increment of velocity ΔV; v) The Moon is in a circular orbit with radius r_B, coplanar with the initial orbit of the spacecraft; vi) The final orbit desired for the spacecraft is a circular orbit with radius r_f around the Moon. With those hypotheses, the complete transfer follows the steps:

i) From the initial circular parking orbit an impulse is applied to send the spacecraft to an elliptic Hohmann transfer to the Moon. The time to apply this impulse is chosen such that the spacecraft reaches the apoapsis of its transfer orbit at the same time that the Moon is passing by that point, to have a near-collision encounter;

ii) In this point, the spacecraft makes a swing-by with the Moon to transform its elliptic orbit around the central body to a parabolic orbit. In a typical swing-by, there are three independent free parameters that can be varied to achieve the purposes of the maneuver: V_∞ (the velocity of the spacecraft relative to the Moon, when it is entering its sphere of influence); r_p (the distance during the moment of the closest approach); the approach angle ψ (the angle between the velocity of the spacecraft during the moment of the closest approach and the velocity of the planet). In this particular case, the values for V_∞ and ψ are not free, since it is decided to approach the Moon from a Hohmann transfer (to achieve the minimum ΔV for the first impulse). What is left to choose is r_p, and it has to be chosen in a such way that the orbit after the encounter is parabolic.

iii) Then, the same principle used in the bi-parabolic transfer is applied here. Theoretically, it is necessary to wait until the spacecraft reaches the infinity to apply a near-zero impulse to transfer the spacecraft to a new parabolic orbit, with periapsis distance equals to r_B. This maneuver has a near-zero ΔV, because it is performed at infinity, where the gravitational force from the central body is zero;

iv) The last step is the insertion of the spacecraft in orbit around the Moon. The same principle from the bi-parabolic transfer is used again. The V_∞ (the velocity of the spacecraft relative to the Moon when entering its sphere of influence) is calculated; then a conic around the Moon with periapsis at r_f is constructed and an impulse at the periapsis of this conic is applied, opposite to the motion of the spacecraft, to reduce its velocity to the circular velocity at r_f.

All those steps can be combined to offer an expression for the savings in ΔV between the standard Hohmann transfer and the elliptic-bi-parabolic transfer. The expression is:

$$\Delta V_{SAV} = \left| \sqrt{\left(\sqrt{\frac{2\mu_C r_0}{r_B (r_B + r_0)}} - \sqrt{\frac{\mu_C}{r_B}} \right)^2 + \frac{2\mu_T}{r_f}} - \sqrt{\frac{\mu_C}{r_B}\left(\sqrt{2}-1\right)^2 + \frac{2\mu_T}{r_f}} \right|$$

where μ_C is the mass parameter of the Earth and μ_T is the mass parameter of the Moon.

As an example, it is calculated the ΔVs involved to transfer a spacecraft with the following data:

$r_0 = 6545$ km; $r_B = 384400$ km; $r_f = 1850$ km; $\mu_C = 398600.44$ km^3/s^2; $\mu_T = \mu_C/81.3$

The results are:

$\Delta V_1 = 3.140$ km/s; $V_i = 0.1863$ km/s; $V_\infty = 0.832$ km/s; $V_o = 1.440$ km/s; $\delta = 39.13°$;

$r_p = 4139.0$ km; $\Delta V_2 = 0.713$ km/s

The total ΔV involved in this maneuver is $\Delta V_T = \Delta V_1 + \Delta V_2 = 3.853$ km/s.

To give an idea of the savings obtained, Table 1 shows the standard results available, obtained from Sweetser (1991).

Table 1 - DV for several models in km/s

	DV_1	DV_2	DV_{total}
Hohmann	3.140	0.819	3.959
Bi-parabolic	3.232	0.714	3.946
Elliptic-Bi-Parabolic	3.140	0.713	3.853

To explore better the possible savings in more generic cases, Fig. 1 shows contour plots for the savings obtained. The canonical system of units is used in those graphs, what means that $\mu_C = r_0 = V_0 = 1$, where the unit for velocity is chosen to be V_0 (the velocity of a spacecraft in a circular orbit with radius r_0). To make those results more general, the values of μ_T used are 0.001, 0.01 and 0.1, respectively (the value for the Earth-Moon system is 0.0123). The vertical axis is used for the variable r_f and the horizontal axis is used for r_B.

Of course, this maneuver is not practical since the time required for the complete transfer is infinity. It should be considered as a limiting case of a more practical maneuver that performs the step iii in a finite time (as large as possible) with $\Delta V \neq 0$ (but still very small). More details are available in Prado (1993).

CONTINUOUS THRUST

The next stage on trajectories research is to consider the case where the thrust is continuous. It means that a finite force is applied during a finite interval of time. There are many results for this case, starting with Tsien (1953), Lawden (1955), Biggs (1979), etc. A collection of those results and references are available in Prado (1989) and Prado and Rios-Neto (1993). The most used approach is based on optimal control theory. It is called "primer-vector" theory and it was developed by Lawden (1953 and 1954). To show it briefly, let us define the state of the spacecraft as the position ($\underline{r}$), velocity ($\underline{v}$) and the characteristic velocity ($c = \int \Gamma dt$), that replaces the mass. Then, the equations of motion of the spacecraft are:

$$\dot{\underline{r}} = \underline{v}, \qquad \dot{\underline{v}} = -\mu \underline{r}/r^3 + \underline{\Gamma}, \qquad \dot{c} = \Gamma$$

The first-order necessary conditions of the associated optimal control problem are:

$H = \underline{p}.\underline{f} = \underline{p}_r.\underline{v} + \underline{p}_v.(\underline{\Gamma} - \mu \underline{r}/r^3) + p_c\Gamma$, for the Hamiltonian,

$\Gamma^* = \Gamma_{max}(c).U(p_v + p_c)$, for the optimal thrust (from the Maximum Principle of Pontryagin), where: $U(x) = (1 + \text{sign}(x))/2$ is 1 if $p_v + p_c > 0$ and 0 if $p_v + p_c < 0$.

That states the "bang-bang" control, that consists of alternating arcs of maximum thrust and ballistics arcs, depending on the sign of $p_v + p_c$. Then, it is necessary to study the adjoint equations. They are:

$$\dot{\underline{p}}_r = -\partial H/\partial \underline{r} = -\underline{p}_v.\underline{G}$$

$$\dot{\underline{p}}_v = -\partial H/\partial \underline{v} = -\underline{p}_r$$

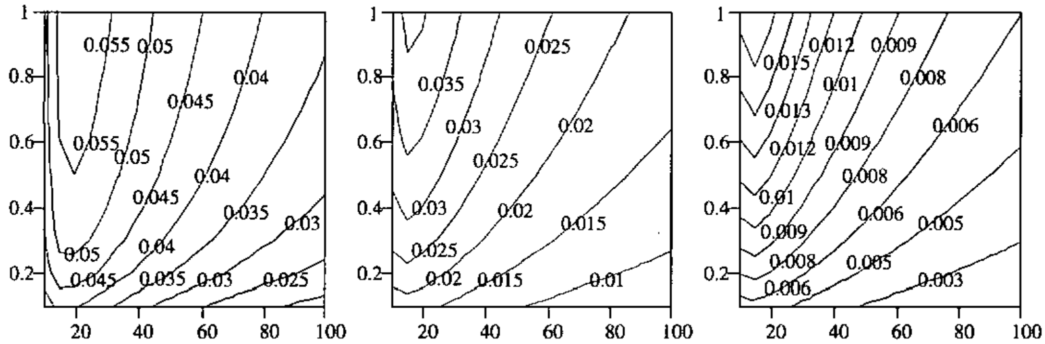
$$\dot{p}_c = -\partial H/\partial c = -(\Gamma^*/W).(p_v + p_c)$$

where $\underline{G}$ is the gravity gradient tensor ($\partial g/\partial \underline{r}$).

The first two equations can be combined into:

$$\ddot{\underline{p}}_v = -\underline{p}_v.\underline{G}$$

where $\underline{p}_v$ (the Lagrangean multiplier associated with $\underline{v}$) is the so called "primer-vector". The problem now is reduced to the



$\mu_T = 0.001$ $\mu_T = 0.01$ $\mu_T = 0.1$

Fig. 1 - Contour-Plots Showing ΔV_{SAV} as a Function of r_B (Horizontal) and r_f (Vertical).

integration of the adjoint equations to verify the sign of $p_v + p_c$ in every instant. More details about this topic can be found in Marec (1979), Prado (1989) and Prado and Rios-Neto (1993). A numerical example of this approach applied to an Earth-Moon transfer can be found in Prado (1990).

TRANSFERS USING GRAVITATIONAL CAPTURE

One of the most interesting and modern approaches of lunar transfers is the transfer using gravitational capture. The idea is to use the effects of the Earth (sometimes the Sun is also included in the model) to reduce the ΔV for insertion into lunar orbit. This idea appears first in a series of articles by Belbruno and Miller (Belbruno 1987 and 1990; Miller and Belbruno, 1991). After those first ideas, a more complete study appears in the excellent paper written by Yamakawa et. al. (1993). To find those orbits we study orbits that start close to the Moon with some hyperbolic excess velocity by using numeric integration. For some special positions (see Yamakawa et. al., 1993) the two-body (Moon-spacecraft) energy becomes negative after some time. It means that a hyperbolic orbit became elliptic and it requires a smaller ΔV to reach the desired final orbit. This is the core engine of the savings obtained in the lunar insertion phase. It is important to emphasize that a definitive capture does not exist in the restricted-three-body-problem. What occurs is a temporary capture, and an impulse is applied during this time to complete the capture with some savings in ΔV.

The Belbruno-Miller Trajectories

This idea is used in the Belbruno-Miller trajectories together with the basic ideas of the gravity-assist maneuver and the bi-elliptic transfer. The maneuver consists of the following steps: i) the spacecraft is launched from an initial circular orbit with radius r_0 to an elliptic orbit that crosses the Moon's path; ii) a Swing-By with the Moon is used to increase the apoapsis of the elliptic orbit. This step completes the first part of the bi-elliptic transfer, with some savings in ΔV due to the energy gained from the Swing-By; iii) With the spacecraft in the apoapsis, a second very small impulse is applied to rise the periapsis to the Earth-Moon distance. Solar effects can reduce even more the magnitude of this impulse; iv) The transfer is completed with the gravitational capture of the spacecraft by the Moon, as described above. This transfer is one step ahead of the Elliptic-Bi-Parabolic Transfer, because it uses the Sun to reduce the second impulse and the gravitational capture to reduce the third impulse.

CONCLUSIONS

This paper studied Earth-Moon trajectories. It showed several types of strategies used to make those transfers with minimum consumption of fuel. The classical trajectories are the ones using impulsive thrust and two-body dynamics (Hohmann, Bi-elliptic, etc...). They are the class of transfers more used in the literature. It gives fast results, good enough for a first estimate of the maneuver. Numerical integration with better models for the dynamics has to be used to get more accurate results after this first estimate. Another possibility is the class of transfers that use continuous thrust. This class generates a more complex problem (an optimal control problem), that usually increases the transfer time, but it gives large savings in fuel. The most modern approach is to use the gravitational capture to make the transfer. This possibility comes from the dynamics of the three and four bodies. This class of transfers uses the perturbation of the third and fourth bodies to help the capture, decreasing the fuel expenditure. The transfer time is also larger for this transfer, compared with the classical maneuvers.

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